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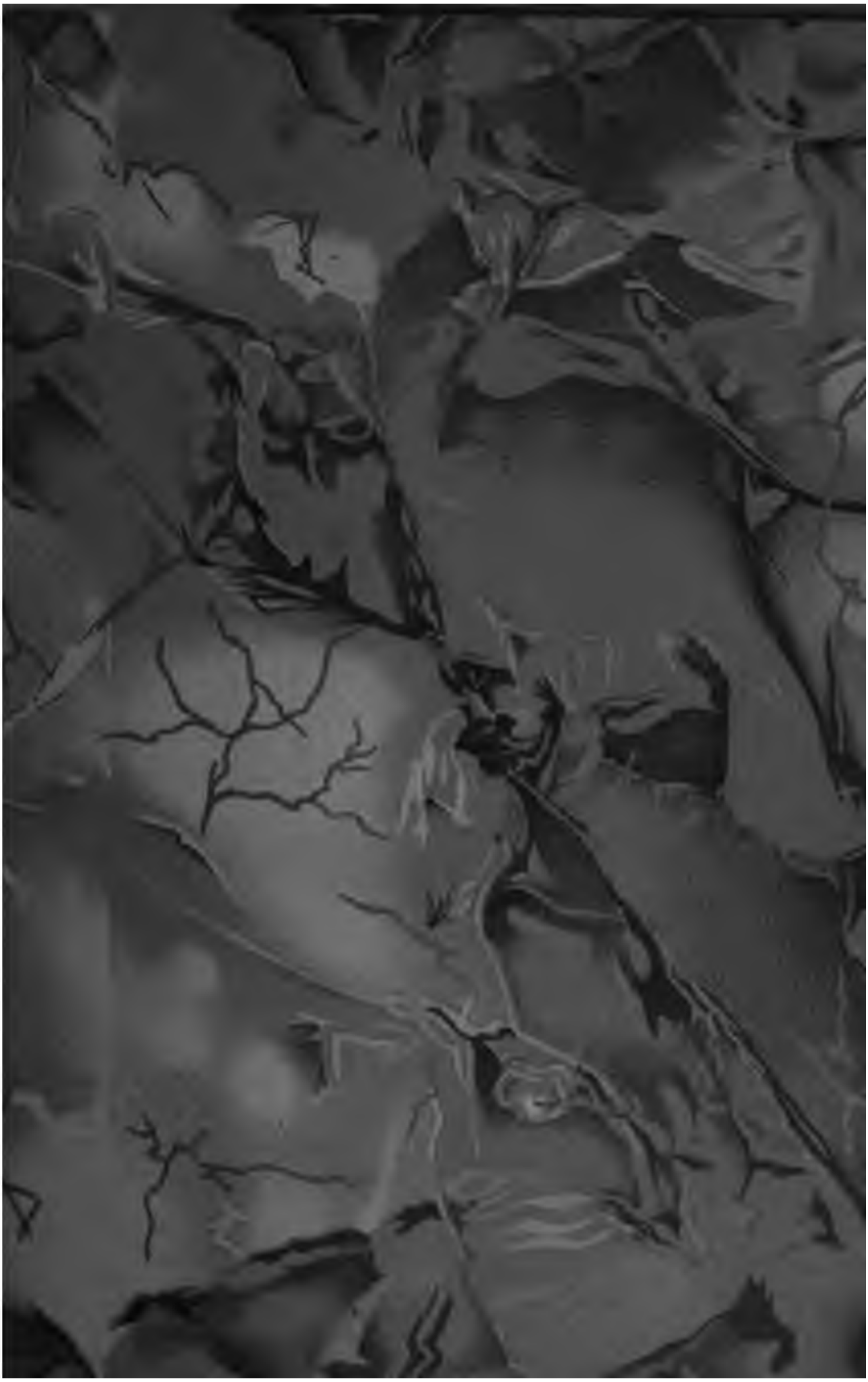

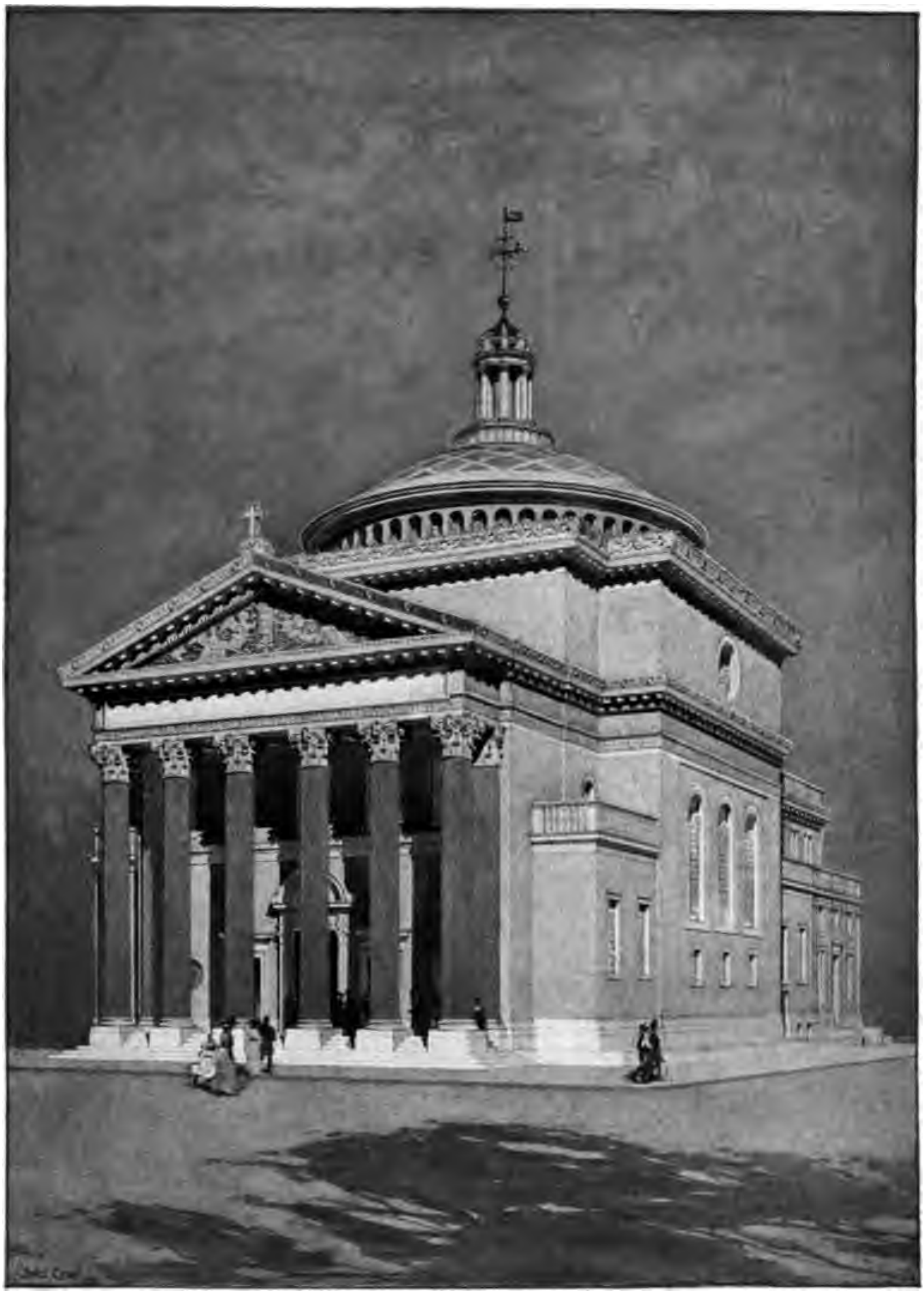

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McKim, Mead & White, Architects, New York. For plan, see next succeeding plate.

Cyclopedia
of
Architecture, Carpentry, and Building

A General Reference Work on
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1917

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Grateful acknowledgment is here made also for the invaluable co-operation of the foremost architects, engineers, and builders in making these volumes thoroughly representative of the very best and latest practice in the design and construction of buildings; also for the valuable drawings and data, suggestions, criticisms, and other courtesies.

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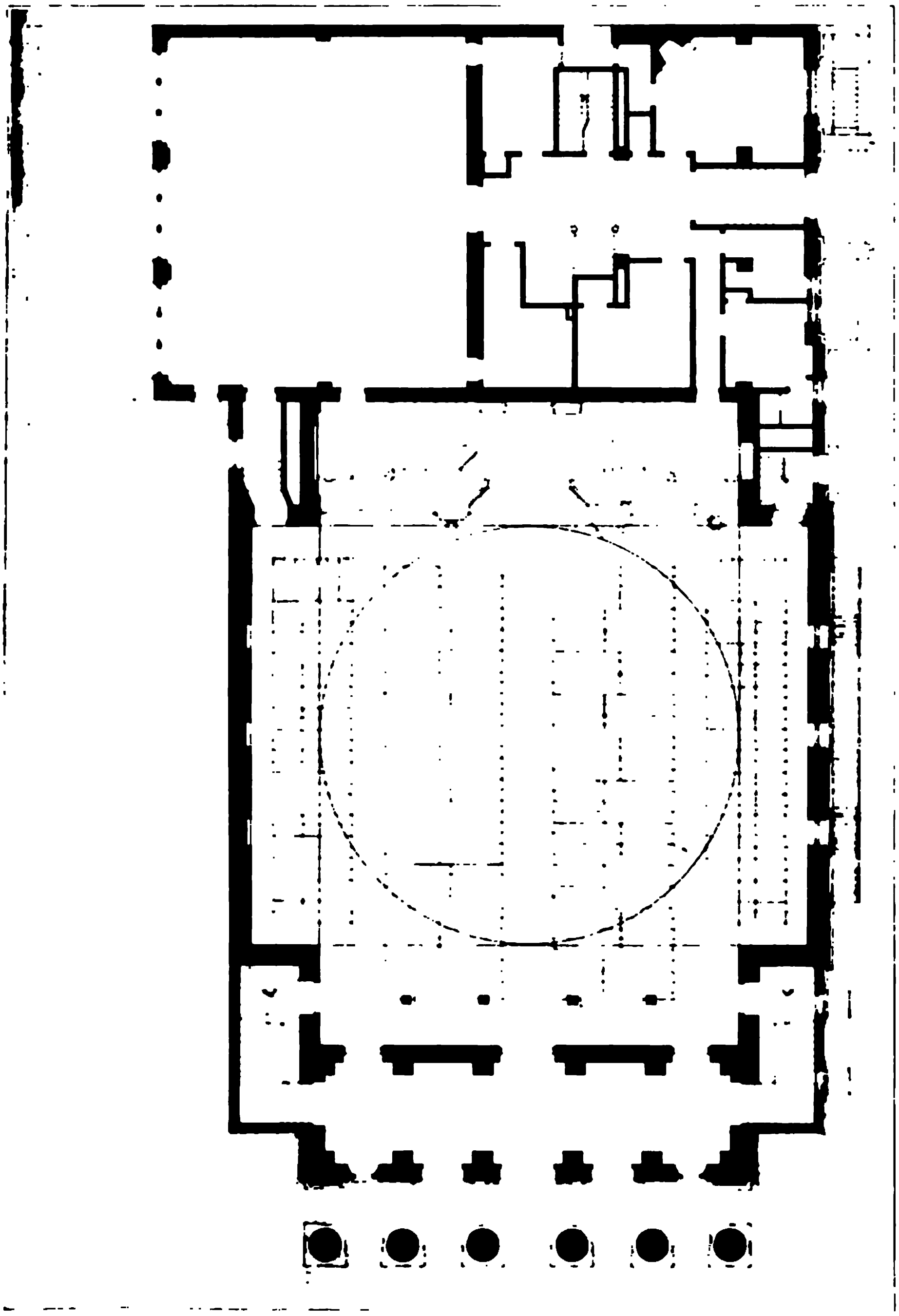

PLAN OF MADISON SQUARE PRESBYTERIAN CHURCH, NEW YORK CITY

McKim Mead & White, Architects, New York

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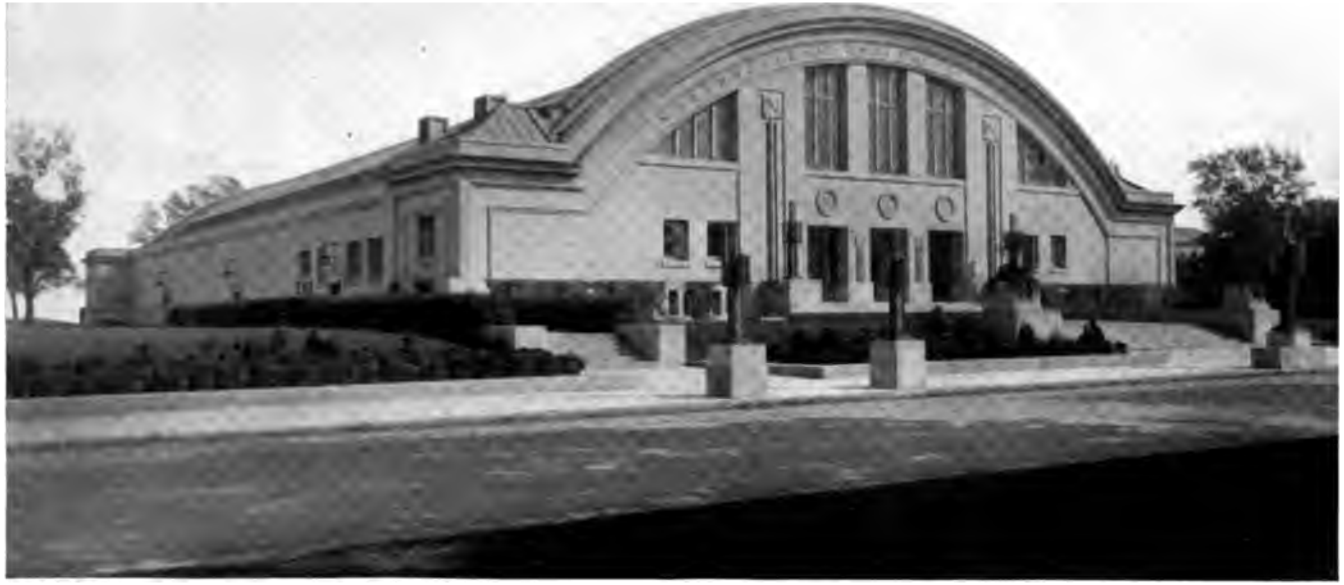

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Foreword

THE rapid evolution of constructive methods in recent years, as illustrated in the use of steel and concrete, and the increased size and complexity of buildings, has created the necessity for an authority which shall embody accumulated experience and approved practice along a variety of correlated lines. The Cyclopedia of Architecture, Carpentry, and Building is designed to fill this acknowledged need.

¶ There is no industry that compares with Building in the close interdependence of its subsidiary trades. The Architect, for example, who knows nothing of Steel or Concrete construction is today as much out of place on important work as the Contractor who cannot make intelligent estimates, or who understands nothing of his legal rights and responsibilities. A carpenter must now know something of Masonry, Electric Wiring, and, in fact, all other trades employed in the erection of a building; and the same is true of all the craftsmen whose handiwork will enter into the completed structure.

¶ Neither pains nor expense have been spared to make the present work the most comprehensive and authoritative on the subject of Building and its allied industries. The aim has been, not merely to create a work which will appeal to the trained expert, but one that will commend itself also to the beginner and the self-taught, practical man by giving him a working knowledge of the principles and methods, not only of his own particular trade, but of all other branches of the Building Industry as well. The various sections have been prepared especially for home study, each written by an acknowledged authority on the subject. The arrangement of matter is such as to carry the student forward by easy stages. Series of review questions are inserted in each volume, enabling the reader to test his knowledge and make it a permanent possession. The illustrations have been selected with unusual care to elucidate the text.

❡ The work will be found to cover many important topics on which little information has heretofore been available. This is especially apparent in such sections as those on Steel Construction; Reinforced Concrete Construction; Building Superintendence; Estimating; Contracts and Specifications, including the principles and methods of awarding and executing Government contracts; and Building Law.

❡ In conclusion, grateful acknowledgment is due the staff of authors and collaborators, without whose hearty co-operation this work would have been impossible.

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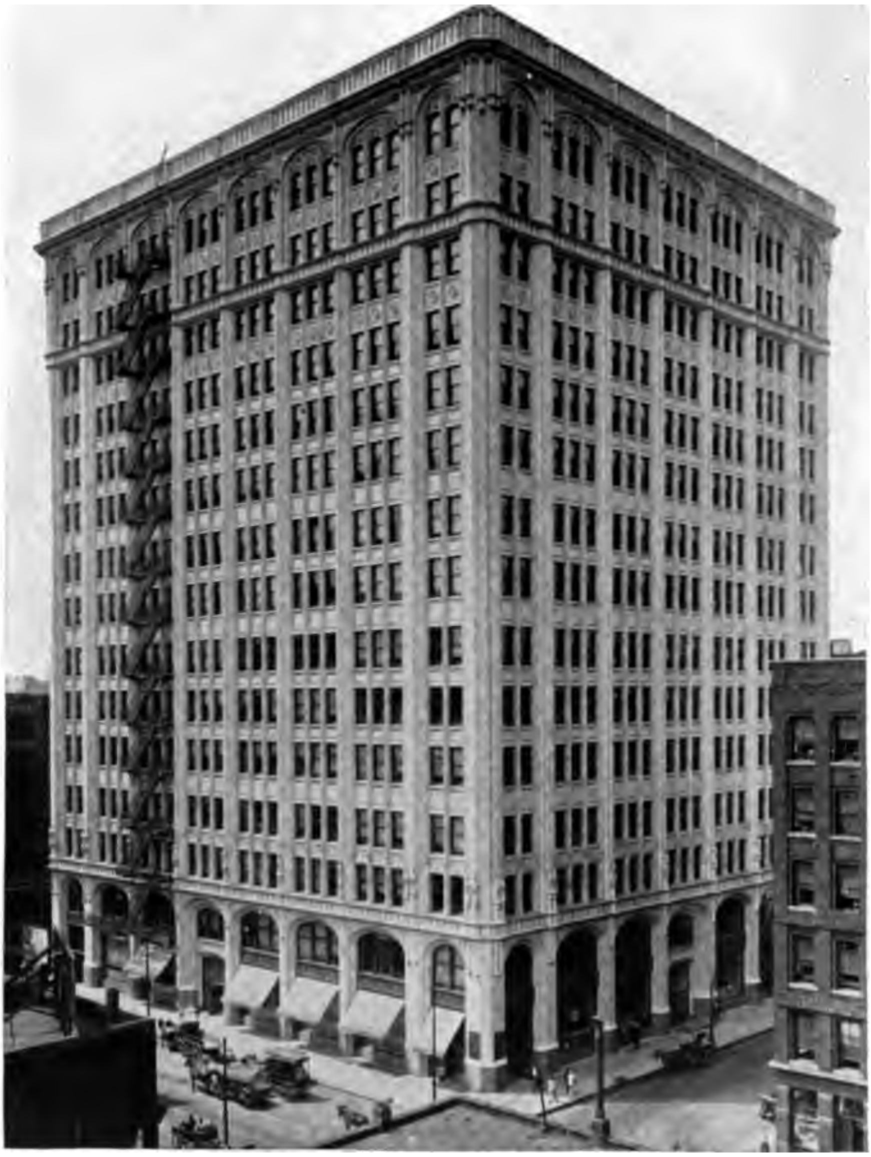


BURLINGTON BUILDING, CORNER CLINTON AND JACKSON STREETS, CHICAGO
Marshall and Fox, Architects

STRUCTURAL DRAFTING

PART I

DRAFTING ROOM EQUIPMENT AND PRACTICE

Introduction. Structural drafting may be defined as the art of making drawings of certain objects and placing thereon dimensions and other notes which when taken together will convey the necessary information for the manufacture and in some cases the erection of the structure under consideration.

In the making of these drawings great accuracy in drafting is not necessarily required. The chief requisites are that the lettering and dimensions should be so clear that no misunderstanding is possible. Dimensions not given should never be scaled by the draftsman or workman, but the actual value should be ascertained by consulting some one familiar with the work.

Classification of Drawings. The classes of drawings which are made in a structural drafting room are: the stress sheet; the assembly, or general detailed, drawings; and the shop drawings, or, as they are more often called, the detailed drawings.

The stress sheet is a tracing upon which is usually shown a skeleton outline of the structure upon the lines of which are marked the stresses which are caused by the traffic or other forces to which the structure is subjected, and also the size and shape of the member designed to withstand these stresses.

The assembly or general detailed drawings usually give several views of the structure as it appears after it has been erected. On these views are shown to scale the members as they appear in the finished structure together with all the rivets and other details necessary for its completion. The overall dimensions are usually given and also any other dimensions which are necessary for the draftsman to complete the shop drawings. While the size of the members and their connections, as well as the number of rivets required, are always given, yet in a few cases the length of the member or shape and the individual spacing of the rivets are also given.

The shop drawings, or detailed drawings as they are more often called, consist of views of a certain member of the finished structure so dimensioned that it may be constructed by the men in the shop. It requires greater skill and more experience to make the assembly drawings than it does the detailed drawings, but in each case the men must be familiar not only with the drafting practice but also with that of the mill and the shop.

Drafting-Room Personnel. A drafting-room force consists of an engineer, a chief draftsman, squad boss, checkers, draftsmen, and tracers.

The *engineer* has charge of the plant as well as of the drafting room and is directly responsible for the ordering of all material, the manufacturing of the structure and its shipping to the place of erection. He conducts the correspondence, keeps track of the work in the drafting room and in the shop, and, in case his plant is one of many of a large corporation, makes weekly or monthly reports to his superior officers. In case his plant is a small one, the engineer usually does most of the work of designing and estimating.

The *chief draftsman* is directly responsible to the engineer for the getting out of the detailed plans or shop drawings and also ordering of the material.

The *squad boss* reports to the chief draftsman and his duty is to keep track of and to get out the drawings of any particular structure which is assigned to him by the chief draftsman. The squad bosses usually have from three to four to as many as twenty *draftsmen* under them, according to the magnitude or the number of structures which they are responsible for.

In addition to the draftsmen are the *checkers,* certain men usually of great experience in matters relative to mill and shop as well as drafting-room practice. It is the duty of these checkers to go over the draftsmen's work, see that all errors are corrected, and then finally sign it as approved. The checker only is held responsible for mistakes which then may be left upon the sheet.

The *tracers* are for the most part young college graduates or apprentices, and their office is simply to trace the drawings which are handed to them by the draftsmen.

A fireproof vault is always a part of the equipment of every well-equipped drafting office. In it are kept the notebooks in which the computations necessary for the design and detailing of the structures are kept, and also the tracings which have been made in the drafting room. In case the drawing of any particular structure is required, the tracing is taken out of the vault, blue prints are made, and the tracing returned as soon as possible. The vault should be so equipped that whenever the door is opened the interior becomes lighted. Aside from the mechanical convenience of this arrangement, it avoids the possibility of any person being accidentally locked in, since the rule is that in case of fire the vault should be immediately closed by the one nearest to it.

Assignment of Work. When the engineer of the plant received a stress sheet from his head officer or from the designing department in his own work, he hands it to the chief draftsman. The chief draftsman makes a record of it and gives it to the squad boss who is most accustomed to that class of work. The squad boss in turn hands it to the checker or checkers and these men make details for the various parts of the structure and make layouts for the various joints. The engineer now orders the material which will be required to build the structure or assigns a checker to do so and then returns the stress sheet to the squad boss who assigns certain draftsmen to prepare the shop drawings for the structure. Draftsmen make the drawings and turn them over to the tracers to trace them.

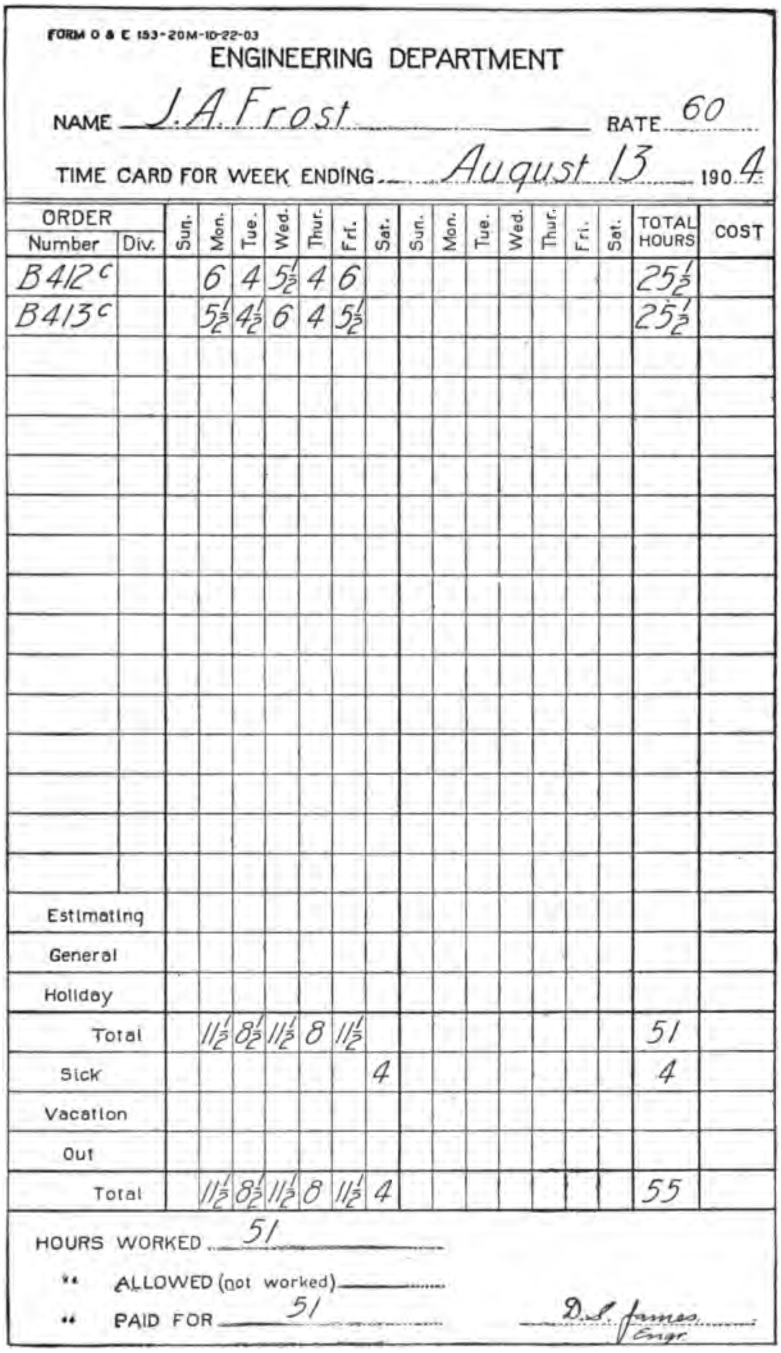
After the tracer has finished the tracings of the sheets, he passes them to the checker who in the first place made out the details and layout and ordered the material. The checker goes over these tracings very carefully and sees that all dimensions are correct, that all material used is that which he ordered, and if the drawings are correct he signs his name to the sheet. If the dimensions or any other matter upon the drawing is found to be incorrect, the checker places a ring around it with his blue pencil which is used in checking and off to one side places the correct value. After all the apparent errors have been corrected in this manner, a consultation between the checker and the draftsman who made the drawing is held. The errors are pointed out to the draftsman who in turn checks the work to prove the checker's results. The draftsman then takes the drawing and makes the necessary changes and returns it to the checker.

Great care should be taken in making the changes that no dimensions or other notations written upon the drawing by the checker are rubbed off. The checker then examines the drawing carefully to see that all the errors which he has pointed out have been corrected. He then cleans the tracing, signs his name to it, and returns it to the squad boss. The squad boss in turn has the necessary blue prints made and turns the tracing together with the prints over to the chief draftsman, who in turn files the tracing in its proper place and gives the blue prints to the engineer of the plant who sees that they are distributed to the foremen of the various shops where they are required.

Records. A job is known by the order number which is given it when it comes into the hands of the engineer of the plant. This order number should go on all papers upon which anything concerning that structure is placed. Failure to do this will result in great confusion and much time will be lost. The penalty for persistent failure to comply with this very important method of procedure is usually dismissal.

Since the draftsman, or in fact any of the office force, may work upon more than one order during the day or week, and since it is important that the cost of the drafting or engineering work for any particular order should be known, it is essential that the men keep time cards upon which the order and the time placed upon that order is noted. Usually fractions of an hour less than one-fourth are not reported. Fig. 1 shows one of these time cards upon which is noted the work of a checker for one week. It shows that he has worked upon several orders and also shows the exact amount of time he has placed upon each one and also the rate per hour which he received. In this way it is possible to obtain the cost of engineering of any particular order when it has finally been finished.

An orderly record of the passage of the work from the time the stress sheet enters the engineer's office until the material has been shipped, and also a record of the progress of the work during erection, should be kept. This is usually kept on 3×5 cards in the engineer's office. In addition to this card-index record, a monthly report in blue print form is kept showing the progress of the various orders. For instance, the progress report would contain such items as these: Order received, layouts made, material ordered, detailed sheets



FORM O & C 153-20M-10-22-03

ENGINEERING DEPARTMENT

NAME J. A. Frost RATE 60

TIME CARD FOR WEEK ENDING August 13 1904

ORDER																TOTAL HOURS	COST
Number	Div.	Sun.	Mon.	Tue.	Wed.	Thur.	Fri.	Sat.	Sun.	Mon.	Tue.	Wed.	Thur.	Fri.	Sat.		
$B412^{c}$			6	4	5½	4	6									25½	
$B413^{c}$			5½	4½	6	4	5½									25½	
Estimating																	
General																	
Holiday																	
Total			11½	8½	11½	8	11½									51	
Sick								4								4	
Vacation																	
Out																	
Total			11½	8½	11½	8	11½	4								55	

HOURS WORKED 51

" ALLOWED (not worked)

" PAID FOR 51

D. S. James
Engr.

Fig. 1. Draftsman's Time Card Showing Hours Spent on Order Indicated

finished, shop bills made, templet work finished, work fabricated, work shipped; and in addition to this progress report, which is made out in the office, is the report of the erector on a job in the field. The erector's form of report contains such headings as tend to indicate the progress in the false work; the erection of the trusses and floor system; and the amount of field riveting and painting completed.

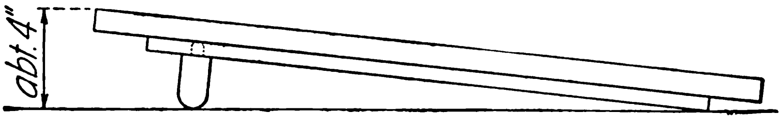


Fig. 2. Side View of Drawing Board, Having Elevating Pegs

Drafting Materials. *Instruments.* The drafting instruments required are: A drawing board, T-square, triangles of various kinds as noted below, pencils, scales, erasers and erasing shields, a set of drawing instruments, a large linen cover, and half sleeves.

The drawing board should be made of soft pine with battens upon the back in order to prevent the warping of the board. Since few drawings in structural engineering are larger than 24×36 inches, it is not necessary to have the drafting board larger than 26×38 inches. A drafting board should not lie close to a table, but should be raised from the table by small legs placed at its upper edge as indicated in Fig. 2.

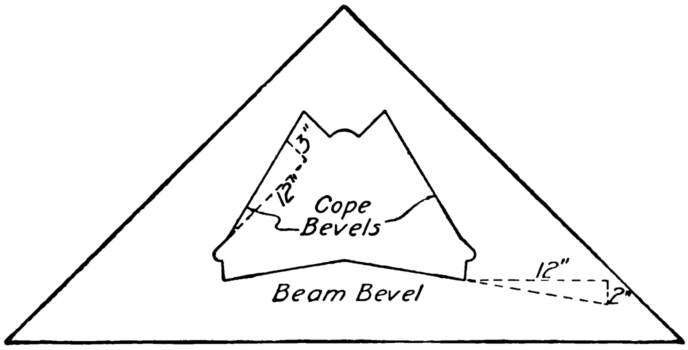


Fig. 3. 45° Triangle with Cope and Beam Bevels

The T-square should be about 40 inches in length and should be of good quality with an amber edge upon each side. The amber edge is of great advantage since it will allow the draftsman to see lines below that one which he is drawing and, therefore, prevent him from overrunning by drawing one line past its limiting point. Such a T-square may be procured for about $2.25.

The triangles should be of amber or celluloid, and should consist of the following: One 45-degree triangle with 10- or 12-inch sides; one smaller, say with 6-inch sides; two 60-degree triangles with 10-inch sides; and two with 4-inch sides. One or more of these triangles should have the beam and coping bevels fixed upon it as in Fig. 3; this will have to be done by the draftsman, since no such triangles are on the market.

The pencils used by the draftsman should be such as will make clear and black lines upon paper in case the drawing is to be traced. If the drawing is not to be traced, a harder pencil will suffice. In case a drawing is made directly upon tracing cloth, a soft pencil should be used and it should be kept sharpened. This will necessitate frequent rubbing over the sand paper pad which every draftsman should have close at hand in order to keep a good point upon his pencil. The pencil recommended for detailing where a tracing is to be made is "Koh-I-Noor, 3 H," although some draftsmen prefer 4 H or 5 H. The latter are, in the writer's opinion, to be recommended for detailing where a tracing is not required from the original. In case drafting is done directly upon tracing cloth, a 2 H pencil is the correct one to use.

A red pencil should be kept for marking upon blue prints and a blue pencil for making checks on tracings. Never use a red pencil upon tracing cloth, since it will not be easy to erase, whereas the blue-pencil mark may be washed off with gasoline or erased with a pencil eraser.

The scales required are the architect's and the engineer's. The former has certain divisions upon it and each of these divisions is divided into twelve parts which indicate inches, and these parts are in turn divided into halves or quarters or other small divisions denoting the fraction of the inch. The architect's scale which best serves the purpose is the one which has the 2-inch, $1\frac{1}{2}$-inch, 1-inch $\frac{1}{2}$-inch, $\frac{3}{4}$-inch, $\frac{3}{8}$-inch, $\frac{1}{4}$-inch, $\frac{1}{8}$-inch, $\frac{3}{16}$-inch, and $\frac{3}{32}$-inch scale. A special scale for the making of drawings to a large size or for the making of layouts is a great convenience. Such a scale is on the market and is divided so that half of an inch is equal to one inch. This scale should be in the outfit of all checkers. The engineer's scale is one on which the inches are divided into certain decimal divisions. The best scale for this is that which has its edges divided into 10, 20, 40, 50, and 60 parts of an inch. This scale is of use only in laying off bevels and natural functions of angles or in drawing outlines upon which details will be constructed with the use of the architect's scales. The tendency of young engineers to use the engineer's scale, allowing a certain decimal to equal a certain fraction of an inch, is to be discouraged because of the liability of error, and a severe penalty imposed for a second offense. Care should be taken in the use of scales such as the architect's which have different scales on the same edge in order not to get the feet which belong to the wrong scale.

A small paper clamp should be attached to the scale a short distance from the center opposite the end where the scale which the draftsman is using is situated. This will prevent the scale from being turned over, hence avoiding any other scale, but the one the draftsman is using at the time, turning up. Also when the draftsman picks up the scale by the paper clamp, the end on which the scale he is using is situated will tilt downward and at once indicate to him which end he should place in position to measure what he wishes. Fig. 4 shows one of these clamps in position for the scale as indicated.

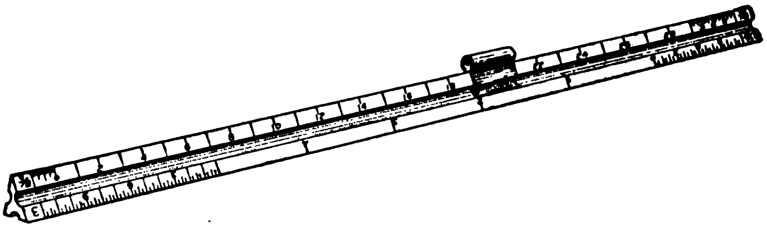


Fig. 4. Triangular Boxwood Scale, with Scale Guard or Clamp in Position

A good ink eraser, together with a metal sheet called an eraser shield in which are various shaped holes, is an indispensable adjunct of the draftsman. In all cases where it is necessary to erase, the ink eraser and the shield should be used. Never use a knife to erase either upon a paper or upon a tracing cloth, for no matter how sharp the knife is, the sheet will be rubbed and ink will not run smoothly upon the place so worked over. A good soft rubber may be used for erasing pencil marks upon either the paper or the tracing cloth, although benzine, turpentine, or gasoline is much better for erasing pencil marks and cleaning off other dirty spots on the tracing cloth. Care should be taken to investigate the status of the insurance and fire laws on this point, since in many cases it is not allowable to use such inflammable materials in houses of the character of the drafting office.

An expensive set of instruments is not necessary in order to do good drafting. A good pen, a bow pen, a pair of dividers, and a compass with pencil and pen point, are all that are necessary. In many cases it is advisable to have two or more pens, one of which should be quite large, one medium, and one rather small.

Many good drafting inks are sold in the open market, and it is no longer necessary for the draftsman to make his own ink by combining India ink with water. In fact this is a distinct disadvantage, since many of the drafting inks on the market are waterproof and while tracings should not be placed so as to become wet, nevertheless it is quite an advantage to use waterproof ink upon them, so that in case they should be accidentally wetted, it will not injure them so that they can not be used.

A sheet of cambric of dark color the size of the drafting board or better still the size of the entire table and drafting board should be used to cover up the work when no one is working, since dust accumulates very readily upon the drafting board and produces much undesirable dirt and, therefore, a very dirty drawing. It is also advisable upon starting work in the morning to brush off the desk and drawing board and to wipe off the T-square and triangles with a cloth. This will prevent dirty marks appearing on the drawing when they are first placed upon them.

Detail Paper. Detail paper is the paper upon which a drawing is made before it is traced or upon which drawings are made to be used by the detailers in making up the details of the structure. Detail papers should be of buff color in order to prevent the showing of dirt upon them too easily, and also to be restful to the eye, and they should present a surface which will take a pencil or ink mark equally well, and they should not be so thin that they will not stand a great amount of erasing.

Many good papers may be bought in the open market. They may be purchased in sheets of a desired size or they may be purchased in rolls of a certain weight, and any width. When sold in sheet form they are usually sold by number of sheets; when sold in roll form, by weight. An inspection of the trade catalogue or letters of inquiry to any of the manufacturing concerns will bring further information if desired.

The standard size of a detailed sheet is 24×36 inches. Inside of this are drawn two borders each ½ inch from the other. In the lower right-hand corner is the place for the title. The size of this block is 4×5½ inches. The 24×36-inch size is the outside dimension of the brown paper or detailed sheet. The 23×35 inch, which is the size of the first border line, is the line upon which the blue prints are cut. The second border line is the real border line of the drawing, and remains upon the blue print. Fig. 5 indicates the dimensions indicated above.

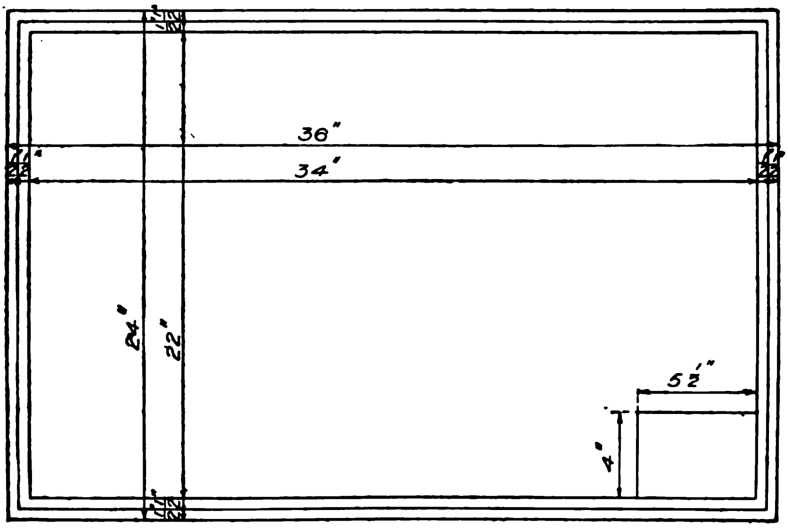


Fig. 5. Standard Detail Sheet with Dimensions

Tracing Cloth. Tracing cloth is used on account of the fact that the prints may be made from it and, therefore, any number of duplicate copies may be made available for distribution to the various departments. The drawings should be made upon the rough side of the cloth since this takes the pencil mark and also the ink better than does the smooth side. The rough side is also of a great advantage when it comes to reproducing the figure by photography. In order to make the ink take readily upon the tracing cloth and flow easily, the cloth should first be cleaned by rubbing over with powdered chalk or wiping it off with gasoline. This removes all trace of grease. Before placing the tracing cloth upon the drawing, the pink border or edge which appears upon the cloth should be torn off. If this is not done the sheet will be affected unevenly by changes in temperature, and dampness will cause the cloth to wrinkle up on account of the fact that the border is not affected by dampness and the remainder of the cloth is. This will make it difficult for the draftsman to complete his drawing in good form in case he has let it lay over for a considerable time, since the lines which he made at first will be moved from their original position by the wrinkled condition of the sheet. In case it has been forgotten to tear off this border and the sheet becomes wrinkled it is advisable totear off the border and leave the sheet until it becomes straightened out before further drafting is done.

In some cases tracing paper is used in making small unimportant drawings. This paper should be of good quality in order that it may stand erasing, since mistakes are liable to occur and these must necessarily be corrected. The best tracing paper is brittle and will not stand much handling. For this reason its use for expensive drawings is not to be recommended.

Stress Sheet. The stress sheets for various structures are usually not made in the drafting room, but are made in the designing room of the company. Much data and many computations are made by the designer which would be of use to the draftsman in detailing. All of this information should be placed upon the stress sheets. The making of a stress sheet should be and usually is done by men of considerable experience. Plates I, II, V, and VI show stress sheets of a truss bridge, roof truss, and a plate girder, respectively, and while these can not be said to be perfect, yet they indicate the engineering practice of our larger bridge corporations and may be taken for examples. (For Plates, see pages 63, 82, 88, and 89.)

ORDERING OF MATERIAL

Since the ordering of material is of great importance it will be discussed here somewhat at length. Although this is usually done by men of considerable experience, yet it is advisable that the draftsman should know the method of procedure in order that he may be able to make the detail drawings more advisedly.

Layout. *Typical Case.* As has been mentioned before, the checker makes details to a large size scale from which he determines the size and amount of material required for certain members. In order to illustrate this, let it be required to determine the size of the plates and the length of the angles used in the cross frame of the plate girder shown on Plate VI. Here the checker first lays off the center to center of the girder to a small scale, say, 3″ to 1′. These lines are marked 1 in Fig. 6. He next draws a web to the proper thickness and then follows in turn the flange angles and the bracing angles as indicated by the numbers 2, 3, etc., on the figure. The number of rivets in the top and bottom angle and in the diagonal

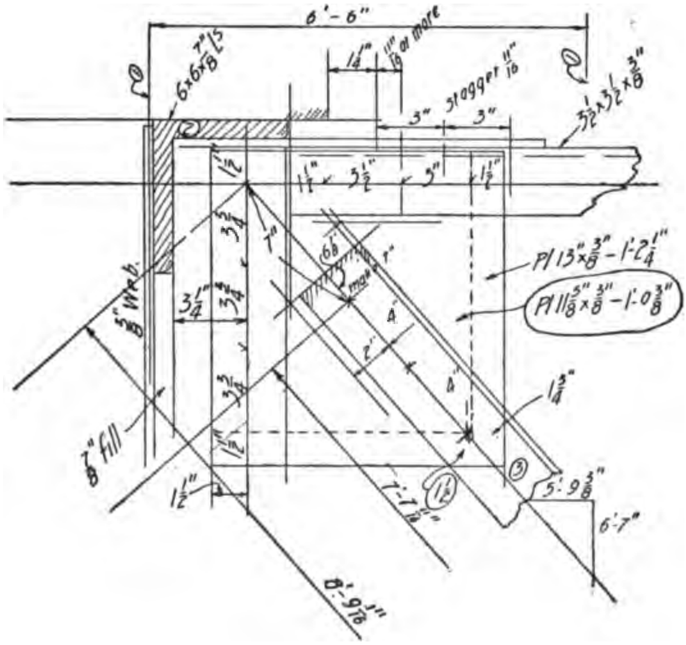


Fig. 6. Layout for Detail in Cross-Frame Connection

should be given on the stress sheet. These should be laid off on the "layout," which is the name for the drawing that has just been made, any spacing, preferably 3 inches, being used so that the plate may be kept as small as possible. It is as a usual thing not possible to have the rivet spacing in both the diagonal and the top and the side angles of equal spacing. The number of rivets is usually put in the diagonal at about a 3-inch spacing, and the spacing of the rivets in the top and the side angles is so varied as to fill out the plate as indicated. No rivets should come closer to the edge of the plate than 1½ inches nor further from the edge than 2 inches, and no plate should be less than an even number of inches in width although its length may be in feet, or in inches to an eighth of an inch. It is not policy to place the length of the plate in sixteenths of an inch, since the shopmen are unable to cut that close. Therefore, in determining the size of the plate the rivets should be so placed that a sufficient number should go in and the size of the plate be kept an even number of inches in width. If the rivets alone governed the size of the plate, it would be as indicated by the dotted lines in Fig. 6, and the dimensions would then be as indicated by the dimensions with a line drawn around it. The correct size of the plate is as indicated by the full line.

The length of the line from intersection to intersection point is [illegible] as indicated upon the drawing. In order to have the length of the diagonal to come out the nearest sixteenth of an inch, the distance of the first rivet from the intersection is taken arbitrarily and is as indicated here, 7 inches. It is not necessary to give this dimension to a thirty-second of an inch, since if the diagonal varies that much from the computed length, it can be drawn up into place by using a drift pin and can be riveted up without injuring the material.

[illegible] and Drafting. The checker has now determined the size of the plate and the length of the diagonal angle and he [illegible] them upon the material [illegible] to the mills as an order for material. The [illegible] of the material bill should be given to the draftsman [illegible] to detail the girder. He [illegible] size of a plate and the size of an angle for that part [illegible] the material which has been ordered, probably months before, and has arrived before

TABLE I

Allowances for Single Lengths

Description of Material or Rule	Allowance Inches
Web plates when ends are planed	Add $\frac{5}{8}$
Web plates when one end only is planed	" $\frac{1}{2}$
Web plates over 24″ wide, ends not planed	" $\frac{3}{8}$
Web plates under 24″ wide	" $\frac{5}{8}$
Cover plates and all other plates that must be full length when in work	" $\frac{3}{8}$
All angles where full length must be maintained	" $\frac{5}{8}$
All channels when ends are planed	" $\frac{5}{8}$
All channels when ends are not planed	" 0
All I-beams when ends are planed	" $\frac{3}{8}$
All I-beams when ends are not planed	" 0
All Z-bars when ends are planed	" $\frac{5}{8}$
All Z-bars when ends are not planed	" 0
All plates over $\frac{7}{8}$″ thick (except when ends must be planed)	" 0
Order width of all sheared plates $\frac{1}{4}$″ greater than finished width when planed edges are specified	
Order all end connection angles which must be planed or faced $\frac{1}{16}$″ thicker than specified thickness	
Order sole plates planed one side $\frac{1}{16}$″ thicker than specified	
Order sole plates planed both sides $\frac{1}{8}$″ thicker than specified	
Order Tees when ends are not planed	Add 0

the draftsman starts the detail, can be used and will be used in that girder. In case the draftsman details the cross frame without consulting the layouts and bills of material, he is liable to draw up a detail which will demand a plate larger or smaller than that ordered for that particular plate; in the first case a new plate will be required, the ordered plate being placed in the stock pile until some other job comes up in which it can be used; and in the second case the plate ordered will have to be cut down to the size of plate the draftsman has used, thus necessitating extra expense and loss of material.

In accordance with the method above stated, layouts are made, then material ordered for all details, and these layouts and copies of material bills are laid aside to be placed in the hands of the draftsman who detailed the subject. Before the material is ordered from the mills, these bills should of course be checked by another checker or by the squad boss.

In making layouts where angles are placed so that one of their legs is vertical, care should be taken to see that the horizontal leg is at the top in all cases where the angle is exposed to the action of rain and snow. If it is not in this position the angle, in case it is on a slant, will serve as a little trough down which the rain and melted snow will run into the joint at the lower end. In case the angle is not on a slant it forms a pocket-like arrangement so that the snow and ice may lodge upon it to a greater extent than if it had the vertical leg downward. Rust will result and the angle will, therefore, deteriorate. In cases such as lower chords and diagonals of roof trusses, the vertical leg of the angle should extend upward, since here the angles are not exposed to the elements and it is somewhat of an advantage that the angle should catch any dust which falls upon it, and should hold it in order to keep it from dropping to the floor beneath.

Allowances for Planing and Cutting. *Single Lengths.* When material is ordered it should be so ordered that it will be sure to be of the correct length when it gets to the shop. If the material is ordered in single lengths, that is, the length ordered to go into the finished structure without being cut in two or more pieces after it gets to the shop, it is customary to make some allowance for planing off the ends or for chance errors in the mills where the men may not be careful enough in cutting and may accidentally make the cut a short distance on one side or the other of the mark which would give the exact length The customary allowances for single lengths are given in Table I.

Multiple Lengths. In cases where there are several pieces of the same size and length, they may, for convenience in handling, be ordered in one piece at the mills and cut into lengths after they reach the shop. In this case, however, care must be taken that the multiple length is not too long to ship on an ordinary freight car. The allowances to be made in such cases and the general rules are given in Table II.

Allowances for Pin Material. In case material is ordered for pins, it is necessary that certain allowances be made for turning and for ordering in multiple. The following very general rules are given in Table III.

TABLE II

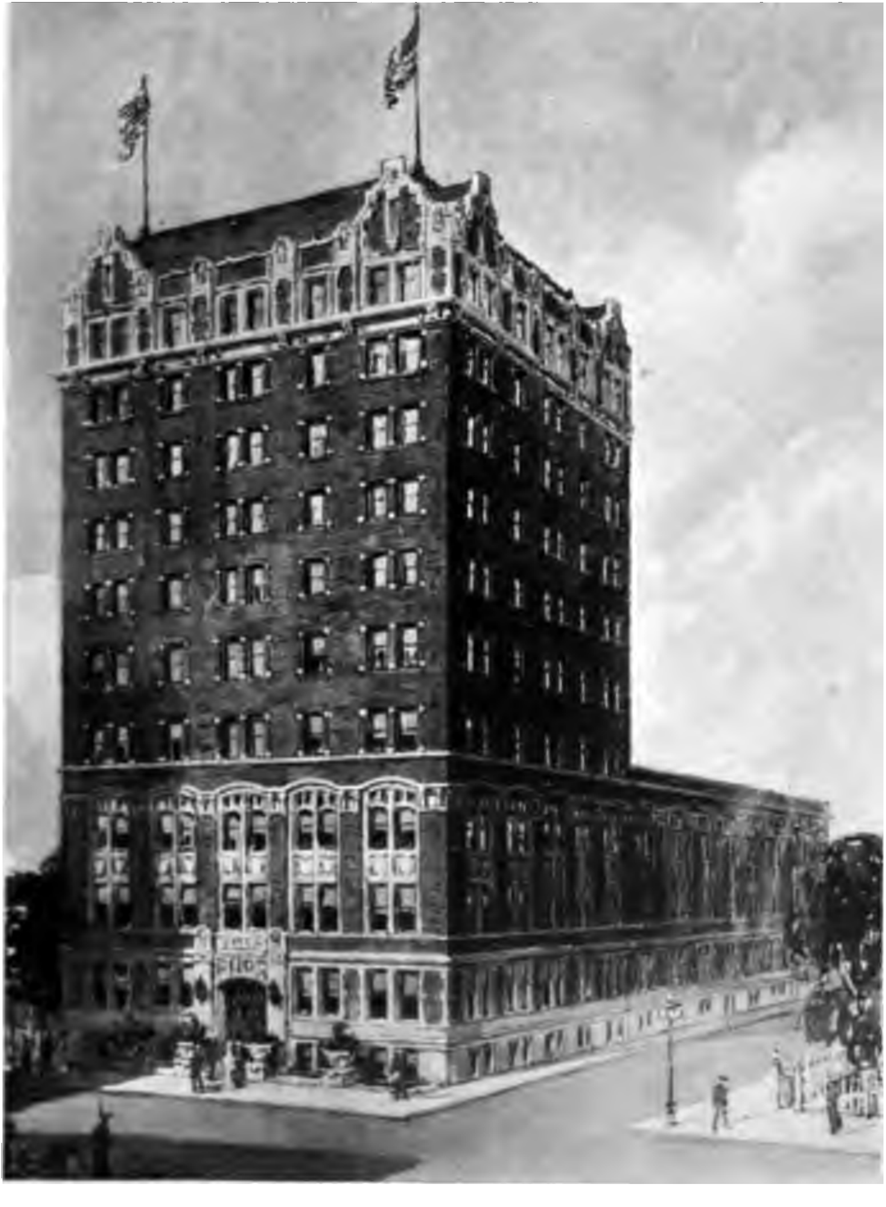
Allowances for Multiple Lengths

No.	Rule
1	No pieces more than 7 ft. long are to be ordered in multiple lengths unless under special instructions
2	In arranging multiple lengths make lengths about 30 ft. and never exceed 32 ft.
3	Never order plates over 24″ wide in multiple lengths
4	Never order plates ⅞″ thick in multiple lengths
5	Never order channels in multiples unless specially instructed
6	Never order I-beams in multiples unless specially instructed
7	Never order Z-bars in multiples unless specially instructed
8	Plates and shapes to be sheared to length without finishing, add 1″ to product of length times number required
9	When planed ends are required add specified amount to each piece multiplied and add 1″ to multiple lengths so found
10	Stiffeners with fillers, add ¼″ to net length of each for planing and 1″ to multiple length so found
11	Stiffeners when crimped, order same as *b-b* of girder angles plus ¼″ for planing and add 1″ to multiple length so found
12	When 4 or less shapes not over 3 ft. long are ordered in multiple lengths, add ⅝″ to multiple and add for planing when required.
13	When ordering fillers, allow ⅛″ clearance at ends when necessary, and add for multiple as for plates
14	Make all multiples end with nearest ¼″
15	Tees under 7 ft. long may be ordered in multiple lengths. Add 2″ to length times number required and make longest multiple 24 ft.
16	If I-beams or channels are cut from long lengths allow loss of 3½″ for each cut
17	7″×3½″ angles can be multiplied up to and including ¾″ in thickness

Allowances for Bending. In all cases where angles have to be bent, additional material is required. In such cases the following rules are applicable:

(1) In the case of Fig. 7a. Figure length on c.g. line of angles and add 1″ for each bend when the angle of bend is not more than about 30°; add 2″ for each bend when the angle is between 30° and 60°; over 60° ask for special instructions from the forge shop.

(2) In the case of Figs. 7b and 7c. In the case of sharply curved end angles or when sharp bends are made near ends, add to the length figured on the c.g. line as follows: 3-inch angles add 4″; 4-inch angles add 5″; 5-inch angles add 6″; 6-inch angles add 7″; 7-inch angles add 8″; and 8-inch angles add 9″.



NEW Y. M. C. A. BUILDING AT CANTON, OHIO

Shattuck and Hussey, Architects, Chicago

One of the largest Y. M. C. A. Buildings in the country for a city of 65,000 people. The building covers a full city block, is of dark red brick with terra cotta and stone trimming. The equipment of the building is very complete and well designed.

TABLE III

Allowances for Pin Material

No.	Rule
1	Pins up to and including 4" in diameter, add $\frac{1}{8}$" to finished diameter for turning
2	Pins 4" to 6" in diameter, add $\frac{1}{4}$" to finished diameter for turning
3	Pins over 6" in diameter, add $\frac{3}{8}$" to finished diameter and order them rough turned unless specially instructed to the contrary
4	Pins up to and including 6" in diameter shall be ordered in multiple length of about 12 ft. Add $\frac{1}{16}$" for each tool cut and 1" to multiple length thus found
5	Pins over 6" in diameter shall be ordered in single pieces and to exact length required
6	When pins are over 4" diameter, ordered diameters must end in no fractions smaller than quarter inches

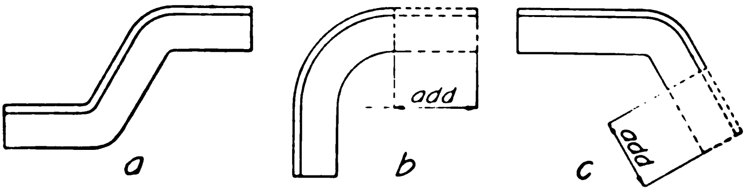


Fig. 7. Illustration Showing Angle Bends

Shop Bills. In order to facilitate the getting out of certain articles which are of the same general form but of different dimensions, and for convenience in tabulating information relative to certain material either before or after it has been assembled into members for structures, certain bills called "shop" bills are used. These bills, which save much drafting and much letter writing, may be of almost any character to suit the practice of the plant. Figs. 8 to 26 give the headings of various bills and Fig. 27 gives the heading of a bill which is used in case it becomes desirable to change an order which has been sent in. The lower part of Fig. 27 is suitable for all of the other bills.

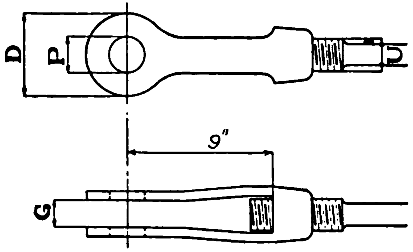
These bills are made on thin paper so that prints may be made from them and sent to the various shops concerned. A copy of each should also be filed in the engineer's office, and all bills of each job should be kept together by binding in some way.

ORDER NO.................

SALINE BRIDGE COMPANY

SHEET NO.................

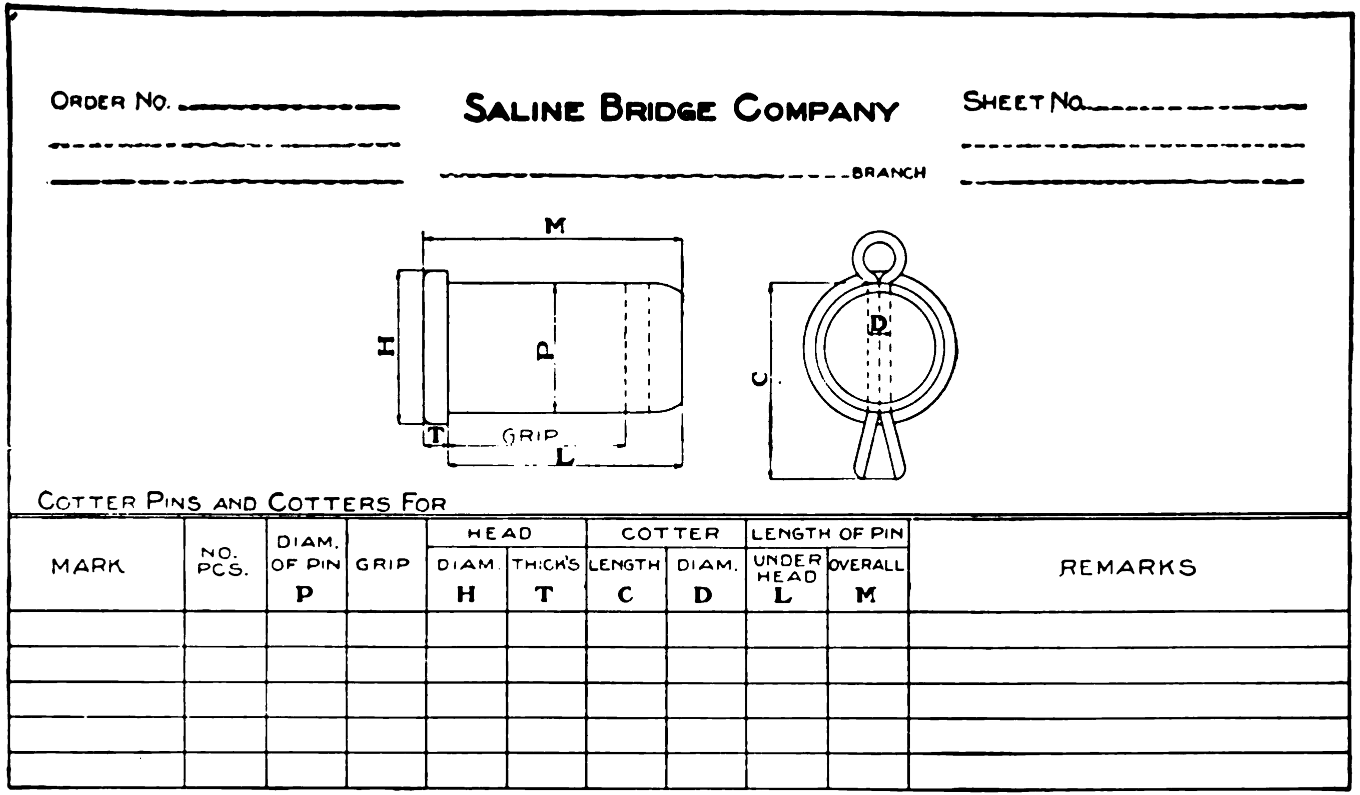
..............................BRANCH



CLEVISES FOR

MARK	NO. OF PIECES	DIAM. D	SCREW U	PIN P	GRIP G	REMARKS

Fig. 10. Shop Bill for Clevises


M
H
P
T
GRIP
L
C
D

ORDER NO. ______________

SALINE BRIDGE COMPANY

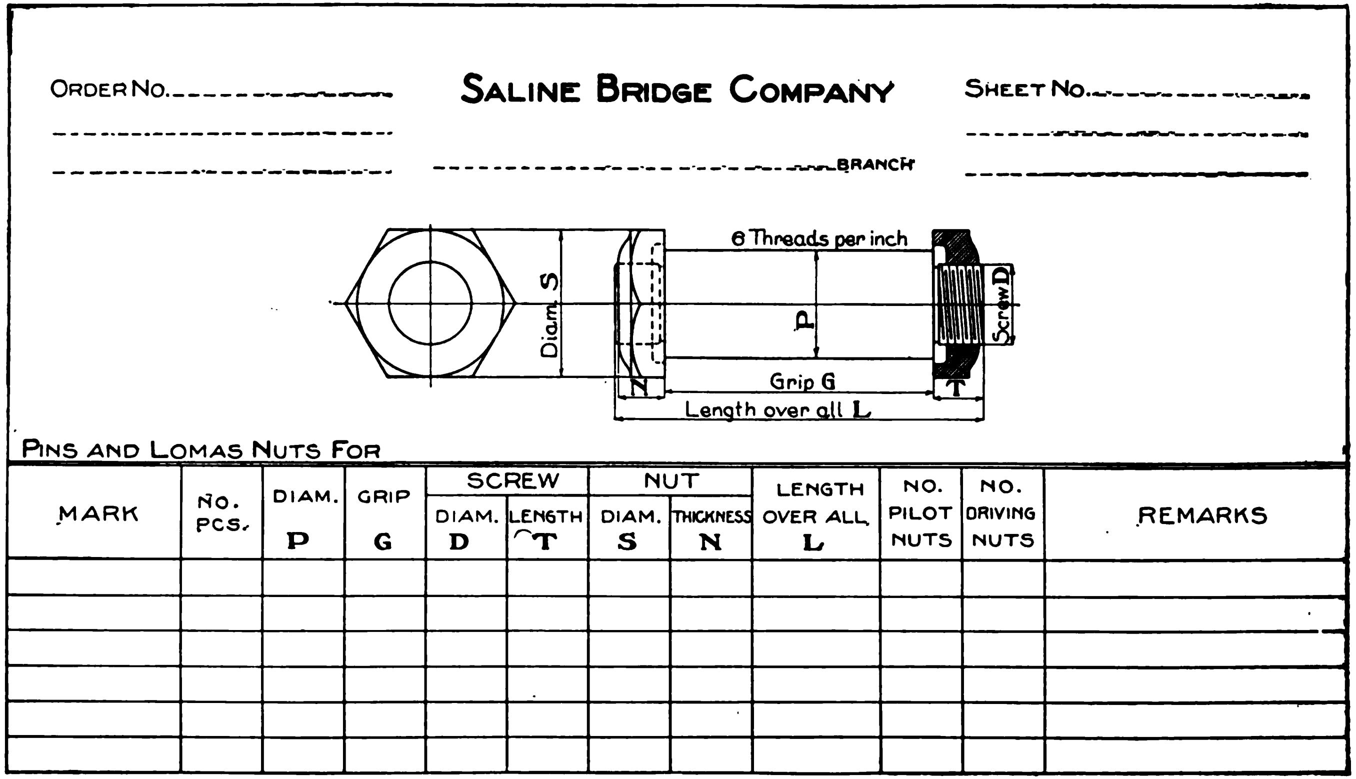
______________ BRANCH

SHEET NO. ______________

COTTER PINS AND COTTERS FOR

MARK	NO. PCS.	DIAM. OF PIN P	GRIP	HEAD		COTTER		LENGTH OF PIN		REMARKS
				DIAM. H	THICK'S T	LENGTH C	DIAM. D	UNDER HEAD L	OVERALL M	

Fig. 11. Shop Bill for Cotters and Cotter Pins



ORDER NO.---------------- **SALINE BRIDGE COMPANY** SHEET NO.----------------

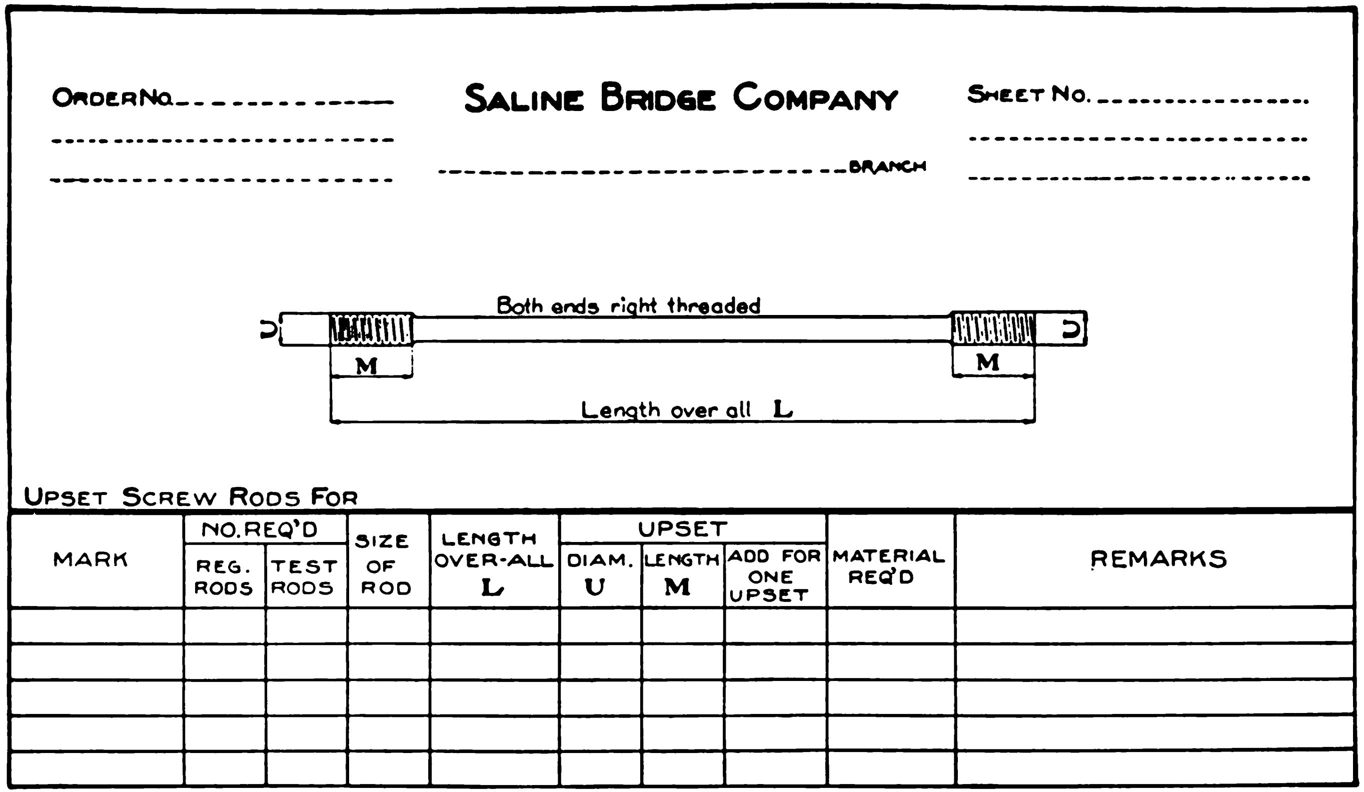
---------------------------BRANCH

6 Threads per inch
Diam. S
P
Screw D
N
Grip G
T
Length over all L

PINS AND LOMAS NUTS FOR

MARK	NO. PCS.	DIAM. P	GRIP G	SCREW		NUT		LENGTH OVER ALL L	NO. PILOT NUTS	NO. DRIVING NUTS	REMARKS
				DIAM. D	LENGTH T	DIAM. S	THICKNESS N				

Fig. 12. Shop Bill for Pins and Lomas Nuts



ORDER No.

SALINE BRIDGE COMPANY

BRANCH

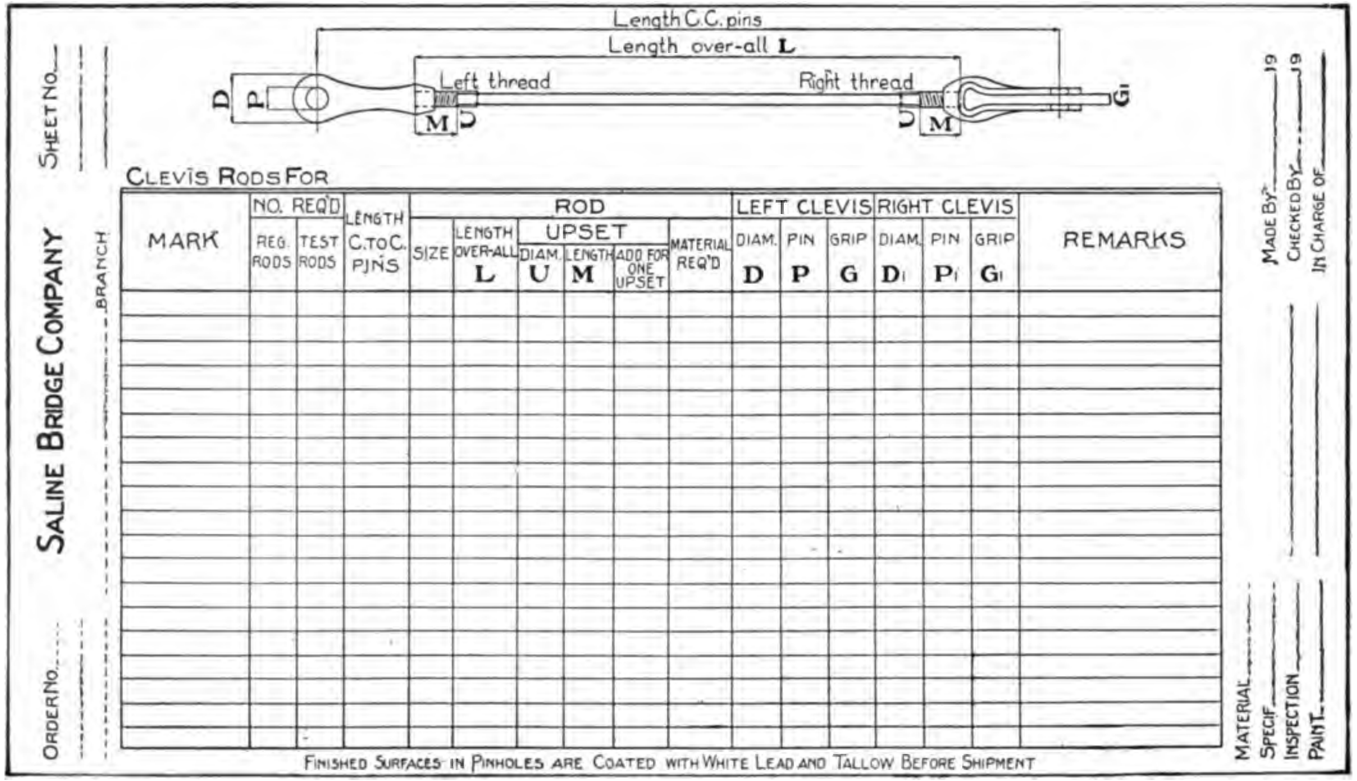
SHEET No.

Both ends right threaded
M
M
Length over all L

UPSET SCREW RODS FOR

MARK	NO. REQ'D		SIZE OF ROD	LENGTH OVER-ALL L	UPSET			MATERIAL REQ'D	REMARKS
	REG. RODS	TEST RODS			DIAM. U	LENGTH M	ADD FOR ONE UPSET		

Fig. 13. Shop Bill for Upset Screw Rods


Length C.C. pins
Length over-all L
Left thread
Right thread
D
P
M
U
U
M
G1

SHEET NO.___

SALINE BRIDGE COMPANY

BRANCH

ORDER NO.___

CLEVIS RODS FOR

MARK	NO. REQ'D		LENGTH C.TO C. PINS	ROD						LEFT CLEVIS			RIGHT CLEVIS			REMARKS
	REG. RODS	TEST RODS		SIZE	LENGTH OVER-ALL	UPSET			MATERIAL REQ'D	DIAM.	PIN	GRIP	DIAM.	PIN	GRIP	
					L	DIAM. U	LENGTH M	ADD FOR ONE UPSET		D	P	G	D_1	P_1	G_1	

FINISHED SURFACES IN PINHOLES ARE COATED WITH WHITE LEAD AND TALLOW BEFORE SHIPMENT

MADE BY___ 19

CHECKED BY___ 19

IN CHARGE OF___

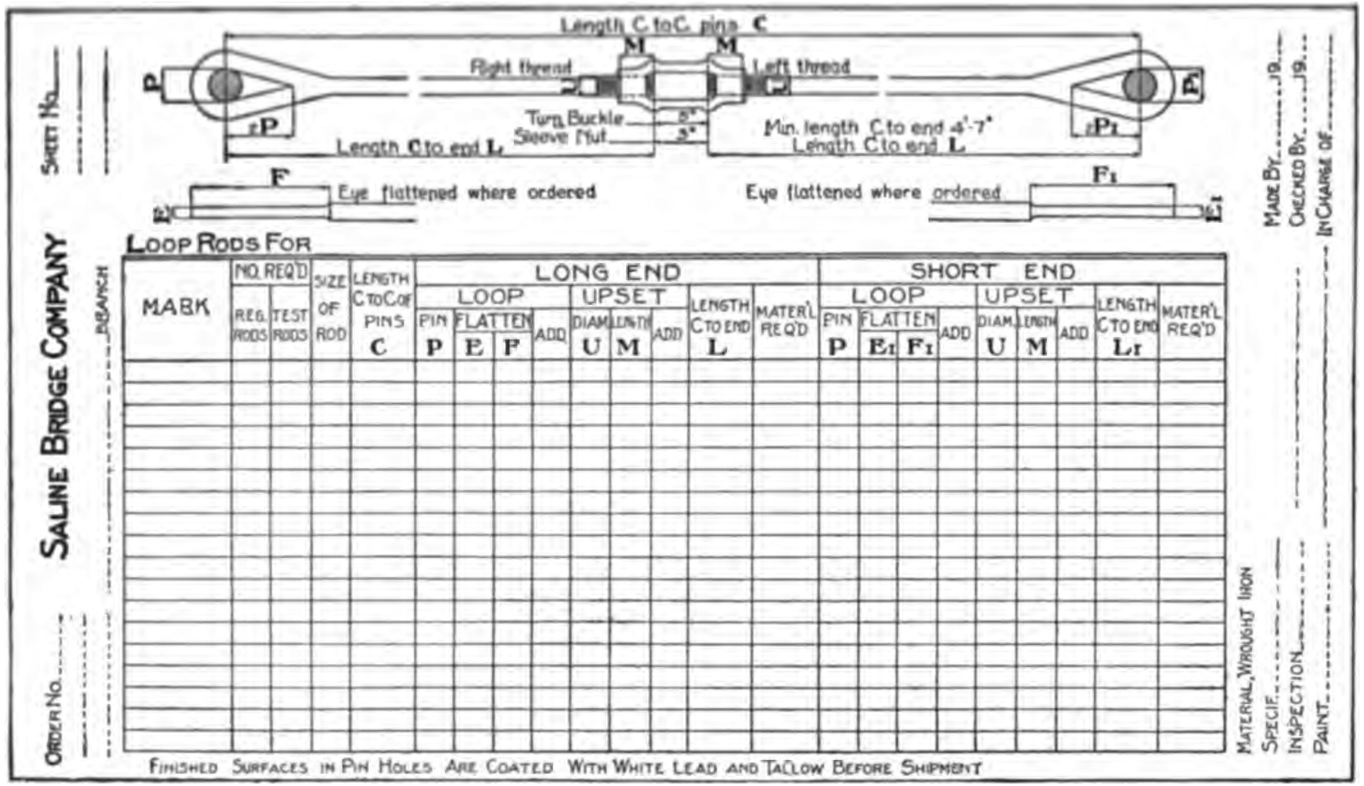
MATERIAL___

SPECIF.___

INSPECTION___

PAINT___

Fig. 14. Shop Bill for Clevis Rods


Length C to C pins C
M
M
Right thread
Left thread
P
P1
2P
2P1
Turn Buckle 5"
Sleeve Nut 3"
Length C to end L1
Min. length C to end 4'-7"
Length C to end L
F
F1
E
E1
Eye flattened where ordered
Eye flattened where ordered

Sheet No.

SALINE BRIDGE COMPANY

Order No.

Branch

LOOP RODS FOR

MARK	NO. REQ'D		SIZE OF ROD	LENGTH C TO C OF PINS	LONG END									SHORT END								
					LOOP				UPSET			LENGTH C TO END	MATER'L REQ'D	LOOP				UPSET			LENGTH C TO END	MATER'L REQ'D
	REG. RODS	TEST RODS			PIN	FLATTEN		ADD	DIAM.	LENGTH	ADD			PIN	FLATTEN		ADD	DIAM.	LENGTH	ADD		
				C	P	E	F		U	M		L		P	E_1	F_1		U	M		L_1	

FINISHED SURFACES IN PIN HOLES ARE COATED WITH WHITE LEAD AND TALLOW BEFORE SHIPMENT

Made By.......... 19....

Checked By.......... 19....

In Charge of..........

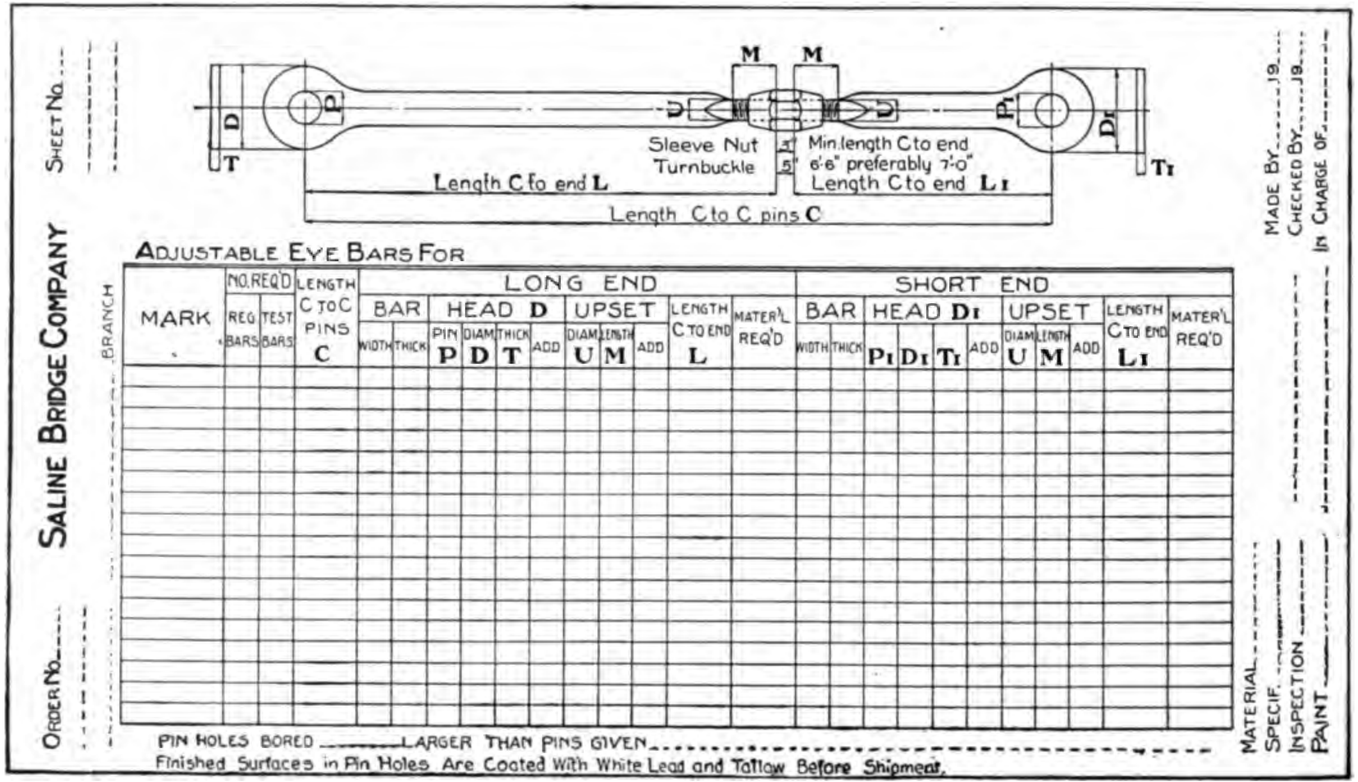
Material, Wrought Iron

Specif..........

Inspection..........

Paint..........

Fig. 15. Shop Bill for Loop Rods


M
M
D
P
U
U
P_1
D_1
T
T_1
Sleeve Nut
Turnbuckle
3"
5"
Min. length C to end 6'6" preferably 7'-0"
Length C to end L
Length C to end L_1
Length C to C pins C

SHEET No. ....

SALINE BRIDGE COMPANY

BRANCH ....

ORDER No. ....

MADE BY .... 19....

CHECKED BY .... 19....

IN CHARGE OF ....

ADJUSTABLE EYE BARS FOR

MARK	NO. REQ'D		LENGTH C TO C PINS C	LONG END											SHORT END										
	REG BARS	TEST BARS		BAR		HEAD D				UPSET			LENGTH C TO END L	MATER'L REQ'D	BAR		HEAD D_1				UPSET			LENGTH C TO END L_1	MATER'L REQ'D
				WIDTH	THICK	PIN P	DIAM D	THICK T	ADD	DIAM U	LENGTH M	ADD			WIDTH	THICK	P_1	D_1	T_1	ADD	DIAM U	LENGTH M	ADD		

PIN HOLES BORED .... LARGER THAN PINS GIVEN ....

Finished Surfaces in Pin Holes Are Coated With White Lead and Tallow Before Shipment.

MATERIAL ....

SPECIF. ....

INSPECTION ....

PAINT ....

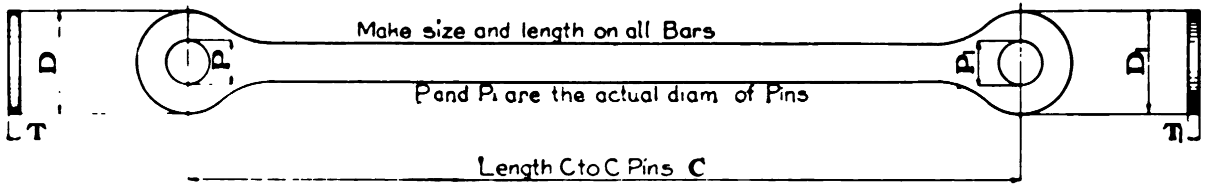
Fig. 16. Shop Bill for Adjustable Eye Bars

ORDER NO..................

SALINE BRIDGE COMPANY

SHEET NO..................

.... BRANCH



Make size and length on all Bars

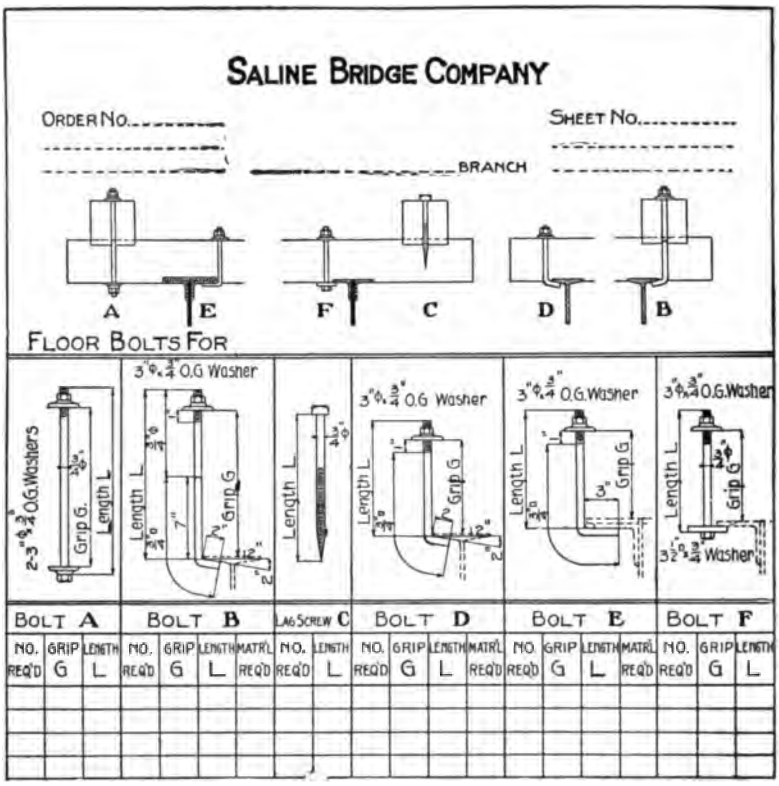
P and P₁ are the actual diam of Pins

Length C to C Pins C

EYE BARS FOR

MARK	NO. REQ'D		SIZE		LENGTH	HEAD D				HEAD D_1				MATERIAL REQUIRED
	REG. BARS	TEST BARS	WIDTH	THICK	C TO C OF PINS **C**	PIN **P**	DIAM. **D**	THICK **T**	ADD	PIN P_1	DIAM. D_1	THICK T_1	ADD	

Fig. 17. Shop Bill for Ordinary Eye Bars



SALINE BRIDGE COMPANY

ORDER NO. .... SHEET NO. ....

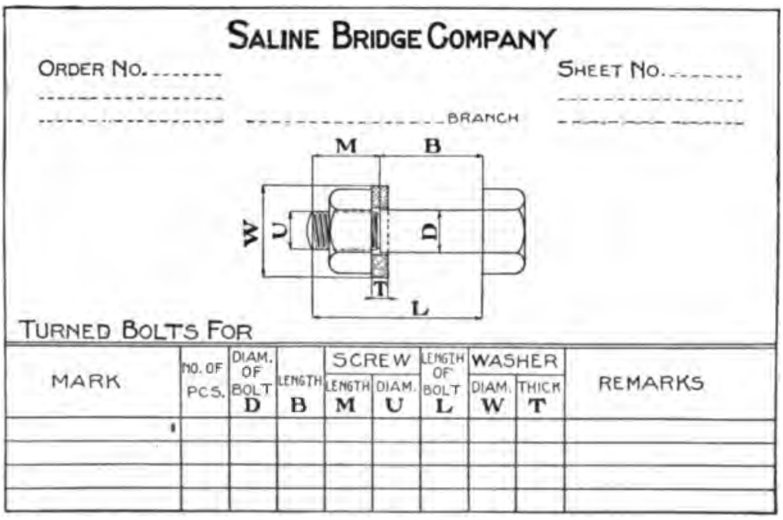
.... BRANCH

FLOOR BOLTS FOR

A
E
F
C
D
B
2-3"φ.¾" O.G. Washers
¾"φ
Grip G.
Length L
3"φ.¾" O.G Washer
Length L
7"
Grip G
12"
2"
¾"φ
3"φ.¾" O.G Washer
3"φ.¾" O.G. Washer
3"
3"φ.¾" O.G. Washer
3½"φ.¾" Washer

BOLT A			BOLT B				LAG SCREW C		BOLT D				BOLT E				BOLT F		
NO. REQ'D	GRIP G	LENGTH L	NO. REQ'D	GRIP G	LENGTH L	MATR'L REQ'D	NO. REQ'D	LENGTH L	NO. REQ'D	GRIP G	LENGTH L	MATR'L REQ'D	NO. REQ'D	GRIP G	LENGTH L	MATR'L REQ'D	NO. REQ'D	GRIP G	LENGTH L

Fig. 18. Shop Bill for Floor Bolts



SALINE BRIDGE COMPANY

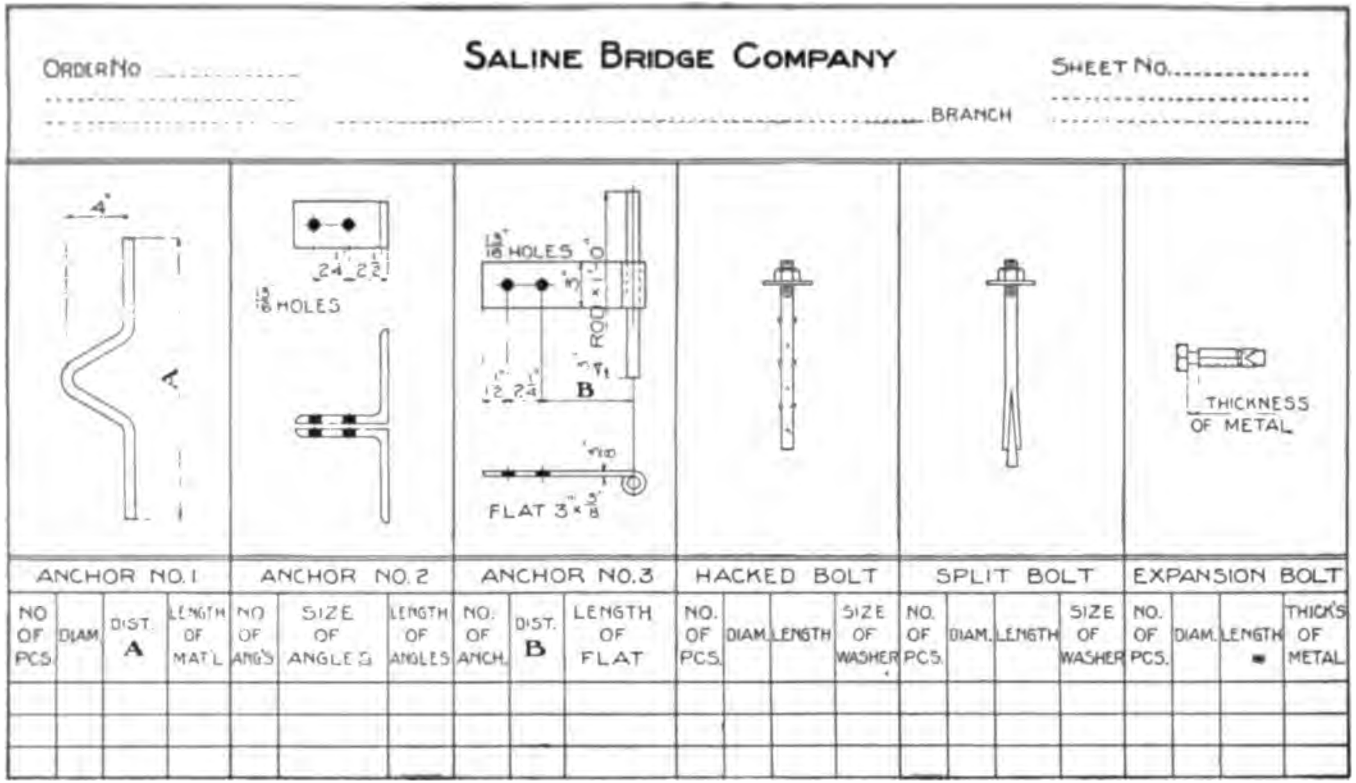
ORDER NO. .... SHEET NO. ....

.... BRANCH

TURNED BOLTS FOR

MARK	NO. OF PCS.	DIAM. OF BOLT D	LENGTH B	SCREW		LENGTH OF BOLT L	WASHER		REMARKS
				LENGTH M	DIAM. U		DIAM. W	THICK T	

Fig. 19. Shop Bill for Turned Bolts


4"
A
2¼" 2½"
13/16" HOLES
13/16" HOLES
1" × 1'-0" ROD
3"
1½" 2¼"
B
3¼"
¼"
FLAT 3" × ⅜"
THICKNESS OF METAL

ORDER NO.

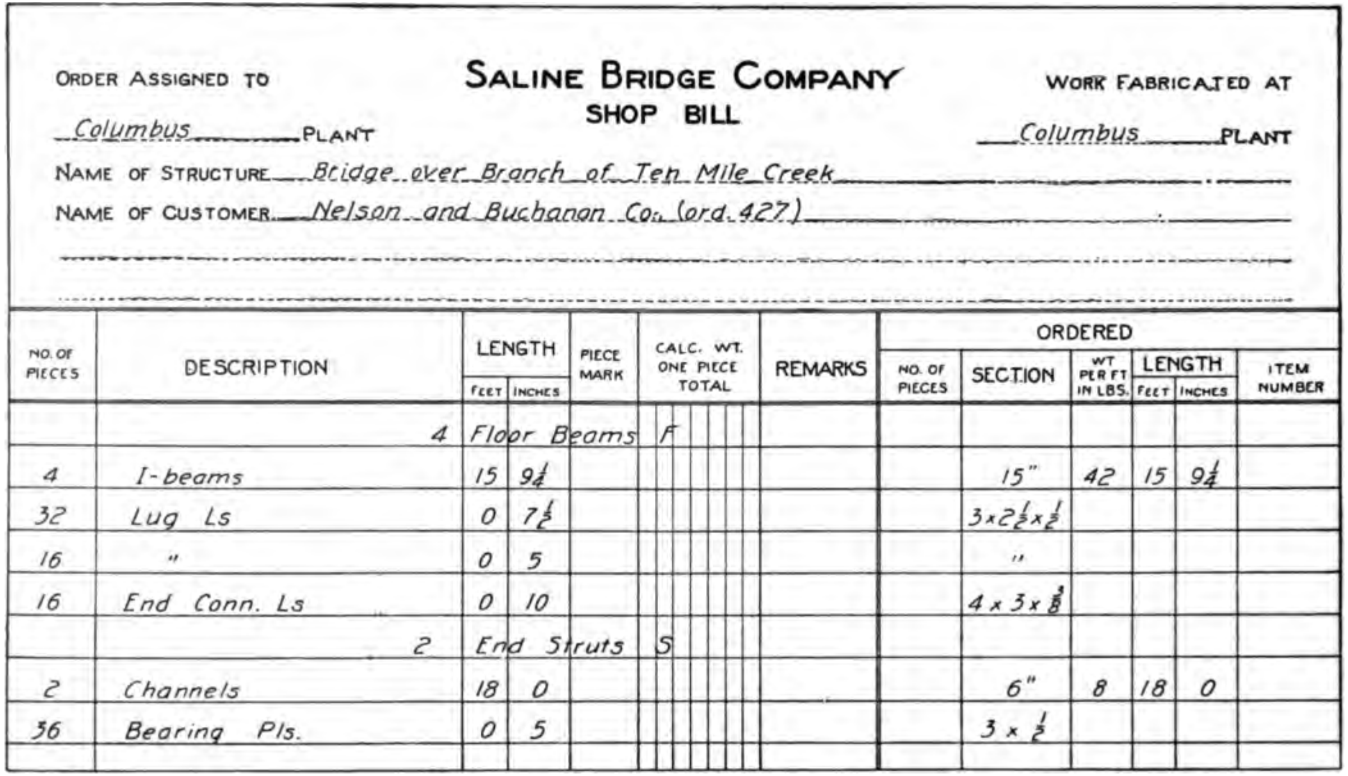
SALINE BRIDGE COMPANY

SHEET NO.

BRANCH

ANCHOR NO. 1				ANCHOR NO. 2			ANCHOR NO. 3			HACKED BOLT				SPLIT BOLT				EXPANSION BOLT			
NO. OF PCS.	DIAM.	DIST. A	LENGTH OF MAT'L	NO. OF ANG'S	SIZE OF ANGLES	LENGTH OF ANGLES	NO. OF ANCH.	DIST. B	LENGTH OF FLAT	NO. OF PCS.	DIAM.	LENGTH	SIZE OF WASHER	NO. OF PCS.	DIAM.	LENGTH	SIZE OF WASHER	NO. OF PCS.	DIAM.	LENGTH	THICK'S OF METAL

Fig. 20. Shop Bill for Anchors and Anchor Bolts



ORDER ASSIGNED TO

Columbus PLANT

SALINE BRIDGE COMPANY
SHOP BILL

WORK FABRICATED AT

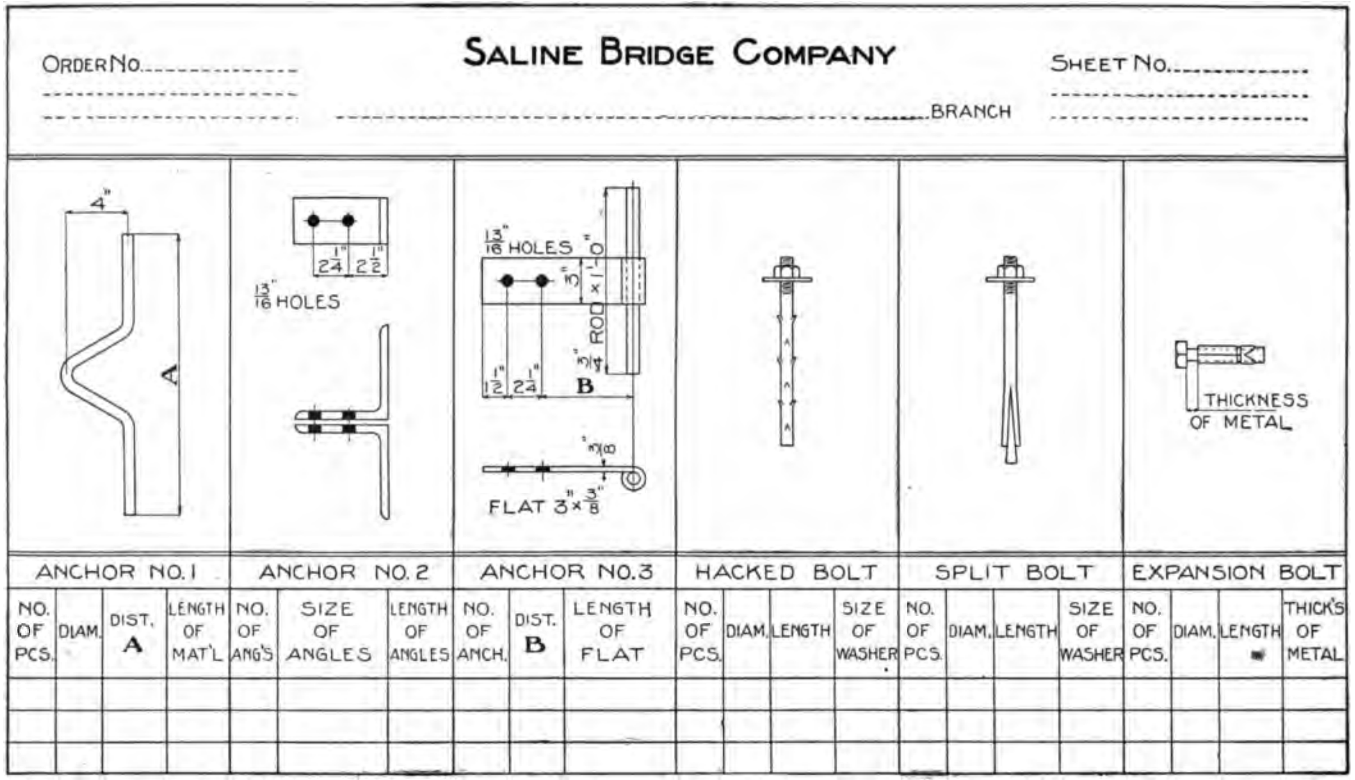
Columbus PLANT

NAME OF STRUCTURE Bridge over Branch of Ten Mile Creek

NAME OF CUSTOMER Nelson and Buchanan Co. (ord. 427)

NO. OF PIECES	DESCRIPTION	LENGTH		PIECE MARK	CALC. WT. ONE PIECE TOTAL	REMARKS	ORDERED					ITEM NUMBER
		FEET	INCHES				NO. OF PIECES	SECTION	WT PER FT IN LBS.	LENGTH FEET	LENGTH INCHES	
	4	Floor Beams		F								
4	I-beams	15	$9\frac{1}{4}$					15"	42	15	$9\frac{1}{4}$	
32	Lug Ls	0	$7\frac{1}{2}$					$3\times2\frac{1}{2}\times\frac{1}{2}$				
16	"	0	5					"				
16	End Conn. Ls	0	10					$4\times3\times\frac{3}{8}$				
	2	End Struts		S								
2	Channels	18	0					6"	8	18	0	
36	Bearing Pls.	0	5					$3\times\frac{1}{2}$				

Fig. 21. Shop Bill for Members of Bridge



ORDER NO.

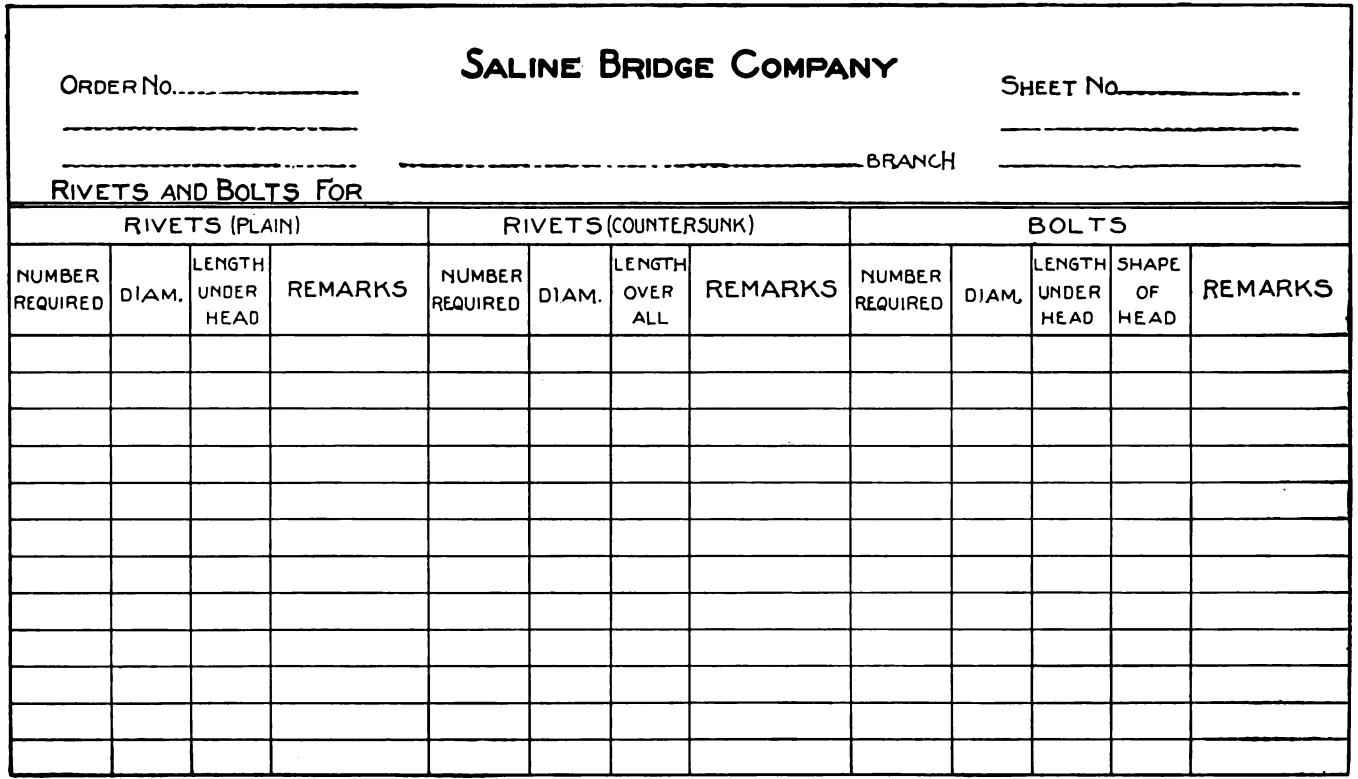
SALINE BRIDGE COMPANY

SHEET NO.

BRANCH

ANCHOR NO. 1				ANCHOR NO. 2			ANCHOR NO. 3			HACKED BOLT				SPLIT BOLT				EXPANSION BOLT			
NO. OF PCS.	DIAM.	DIST. A	LENGTH OF MAT'L	NO. OF ANG'S	SIZE OF ANGLES	LENGTH OF ANGLES	NO. OF ANCH.	DIST. B	LENGTH OF FLAT	NO. OF PCS.	DIAM.	LENGTH	SIZE OF WASHER	NO. OF PCS.	DIAM.	LENGTH	SIZE OF WASHER	NO. OF PCS.	DIAM.	LENGTH	THICK'S OF METAL

Fig. 20. Shop Bill for Anchors and Anchor Bolts



ORDER No.

SALINE BRIDGE COMPANY

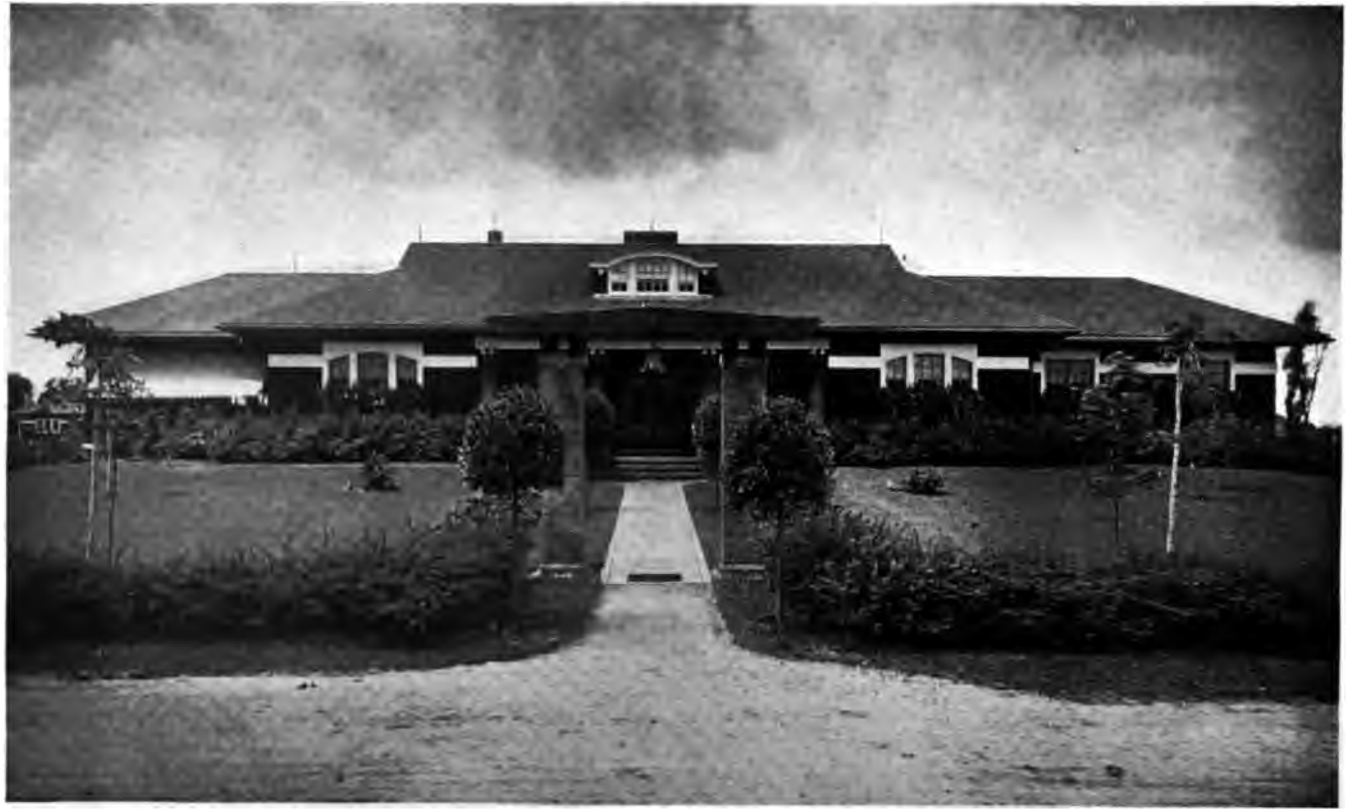
SHEET No.

BRANCH

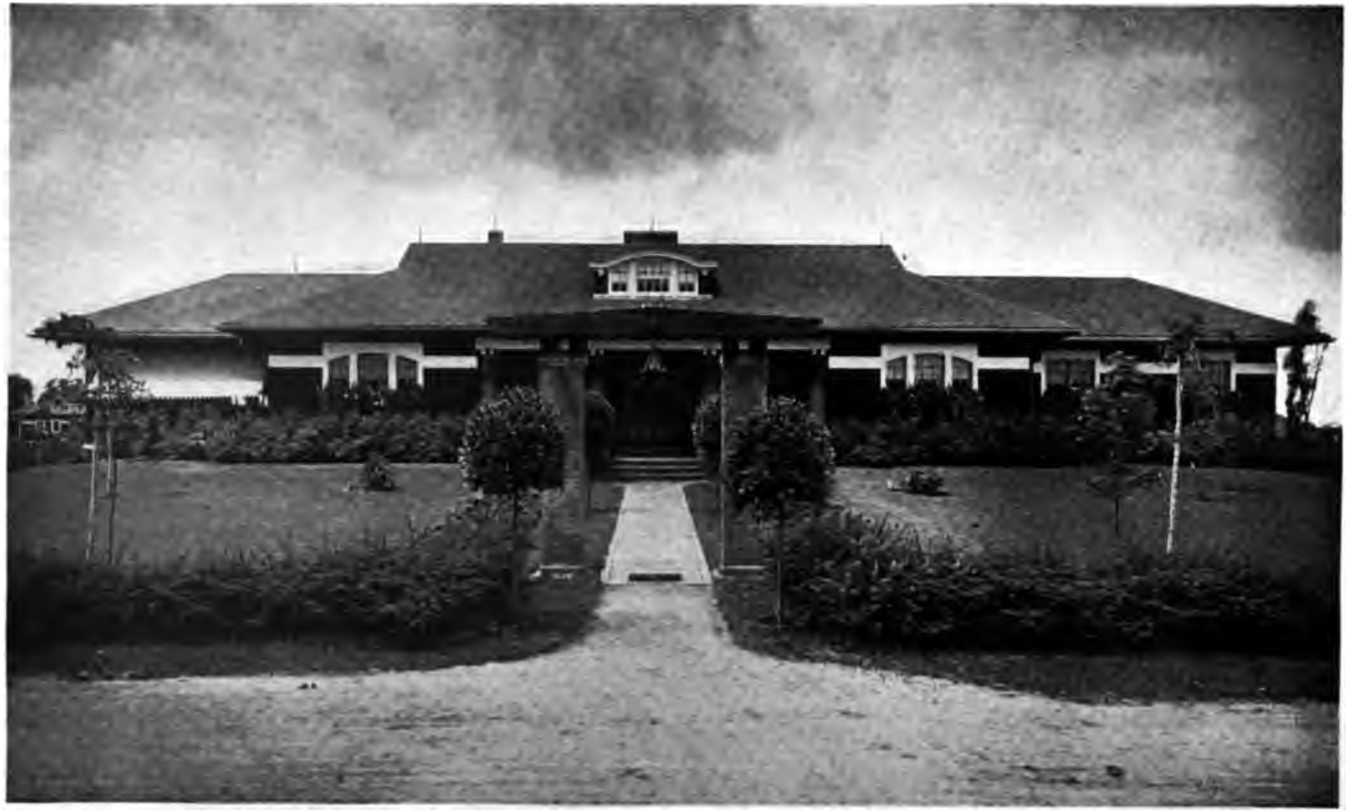
RIVETS AND BOLTS FOR

RIVETS (PLAIN)				RIVETS (COUNTERSUNK)				BOLTS				
NUMBER REQUIRED	DIAM.	LENGTH UNDER HEAD	REMARKS	NUMBER REQUIRED	DIAM.	LENGTH OVER ALL	REMARKS	NUMBER REQUIRED	DIAM.	LENGTH UNDER HEAD	SHAPE OF HEAD	REMARKS

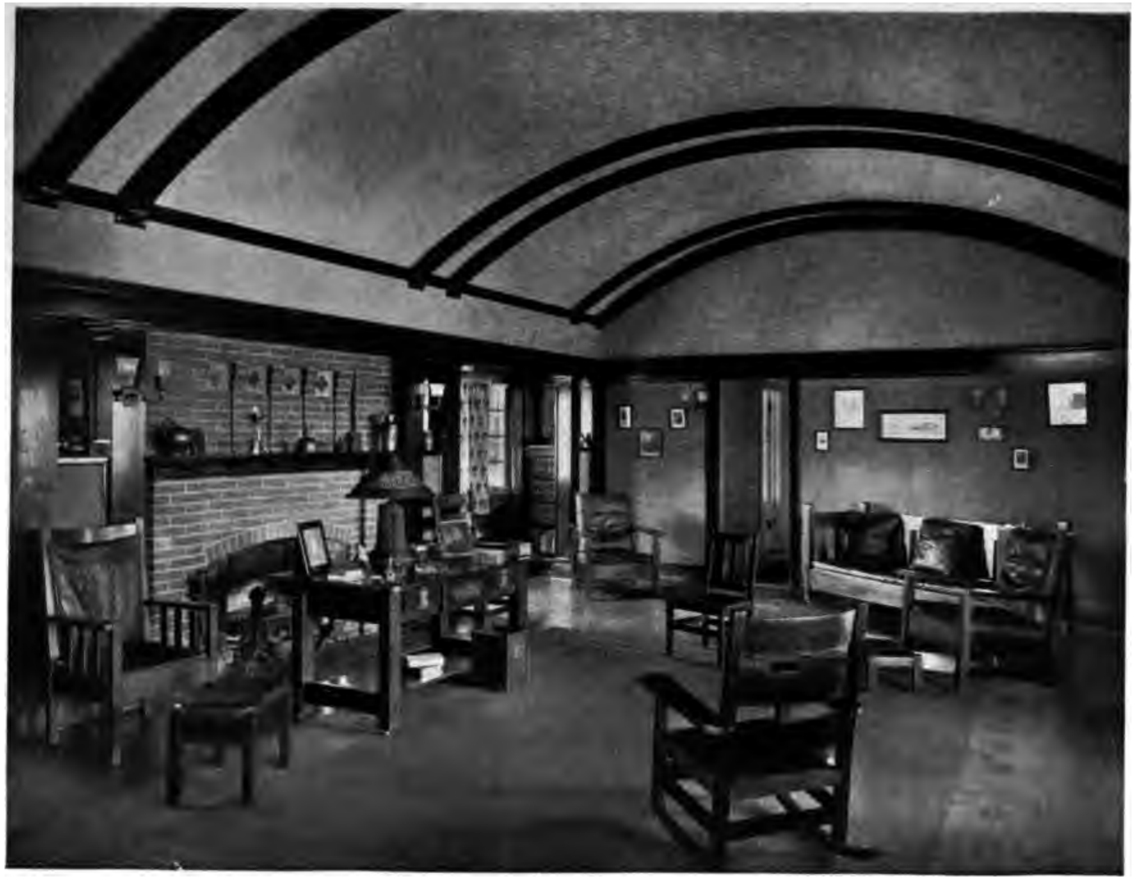
Fig. 23. Shop Bill for Field Rivets and Bolts



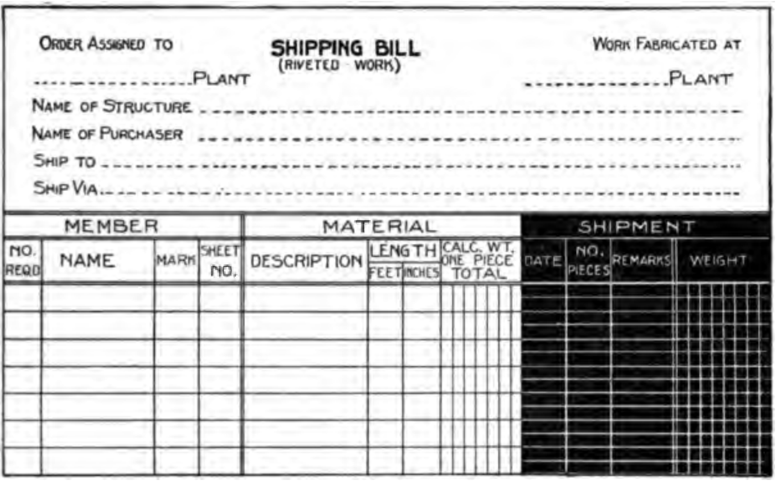
RESIDENCE OF B. J. ROSENTHAL AT HOMEWOOD, ILLINOIS
Alfred S. Alschuler, Architect, Chicago
For interior view see opposite page



RESIDENCE OF B. J. ROSENTHAL AT HOMEWOOD, ILLINOIS
Alfred S. Alschuler, Architect, Chicago
For interior view see opposite page



LIVING ROOM OF B. J. ROSENTHAL RESIDENCE AT HOMEWOOD, ILLINOIS
Alfred S. Alschuler, Architect, Chicago
For exterior view see opposite page



ORDER ASSIGNED TO ....PLANT

SHIPPING BILL
(RIVETED WORK)

WORK FABRICATED AT ....PLANT

NAME OF STRUCTURE ....

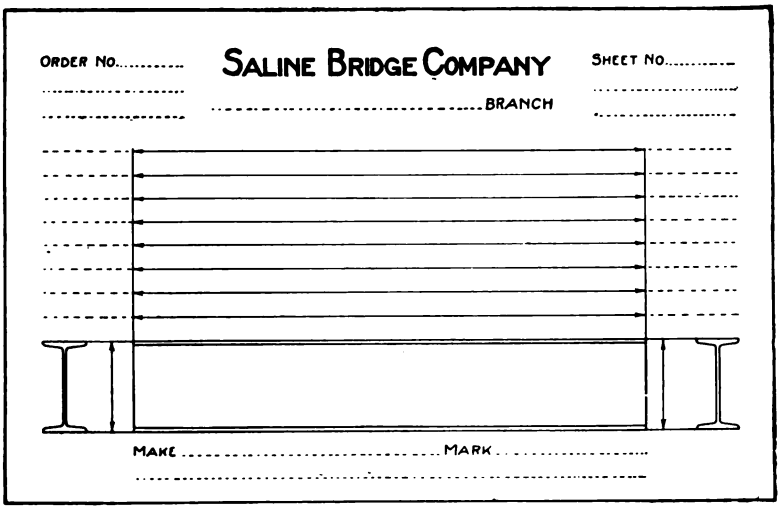
NAME OF PURCHASER ....

SHIP TO ....

SHIP VIA ....

MEMBER				MATERIAL				SHIPMENT			
NO. REQD	NAME	MARK	SHEET NO.	DESCRIPTION	LENGTH		CALC. WT.	DATE	NO. PIECES	REMARKS	WEIGHT
					FEET	INCHES	ONE PIECE TOTAL				

Fig. 24. Shipping Bill



ORDER NO. ....

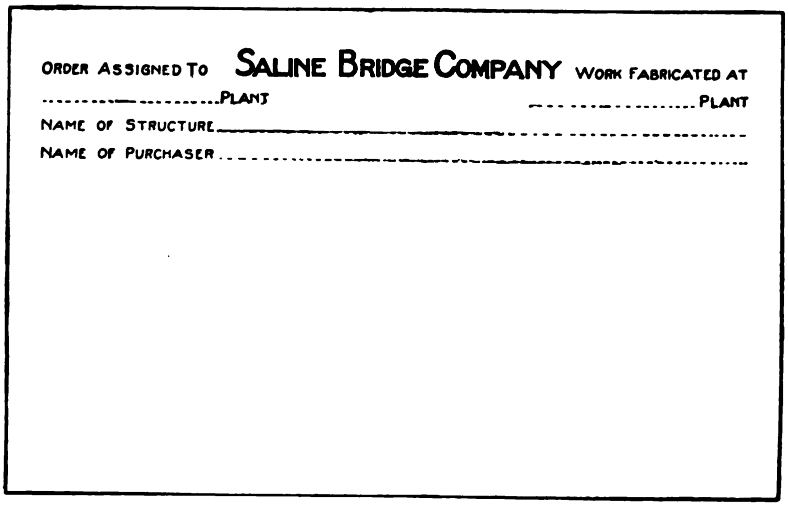
SALINE BRIDGE COMPANY

SHEET NO. ....

................BRANCH

MAKE .... MARK ....

Fig. 25. Shop Bill for I-Beams



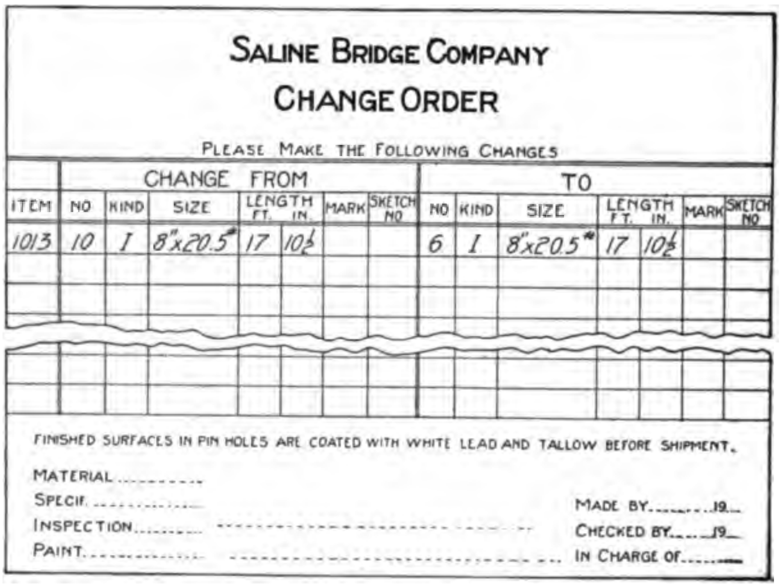
ORDER ASSIGNED TO **SALINE BRIDGE COMPANY** WORK FABRICATED AT

.............................PLANT ....PLANT

NAME OF STRUCTURE.............................

NAME OF PURCHASER.............................

Fig. 26. Shop Bill with Blank Space for Sketch



SALINE BRIDGE COMPANY
CHANGE ORDER

PLEASE MAKE THE FOLLOWING CHANGES

	CHANGE FROM							TO						
ITEM	NO	KIND	SIZE	LENGTH FT.	LENGTH IN.	MARK	SKETCH NO	NO	KIND	SIZE	LENGTH FT.	LENGTH IN.	MARK	SKETCH NO
1013	10	I	8″x20.5#	17	10½			6	I	8″x20.5#	17	10½		

FINISHED SURFACES IN PIN HOLES ARE COATED WITH WHITE LEAD AND TALLOW BEFORE SHIPMENT.

MATERIAL..........

SPECIF..........

INSPECTION..........

PAINT..........

MADE BY..........19...

CHECKED BY..........19...

IN CHARGE OF..........

Fig. 27. Sheet Used for Change of Order

DETAILING—GENERAL INSTRUCTIONS

Lettering. In order that the drawing may give the necessary information and that no mistakes should occur in the reading of the drawing by the shopmen or others, it is necessary that the letters and dimensions upon the drawing be made so that they are exceedingly clear. In order to save time in lettering, an alphabet should be used that can be made quickly and easily. The alphabet which is known as the straight-line alphabet fulfills these conditions. It is made by one of the characters or by a combination of the characters shown in Fig. 28. A study of Fig. 29 will show that the general scheme of this system consists of the oval and the straight line.

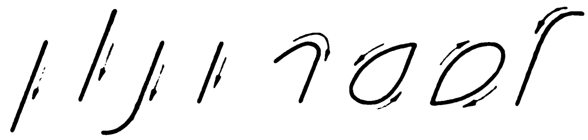


Fig. 28. Method of Constructing Parts of Small Arc-Line Letters

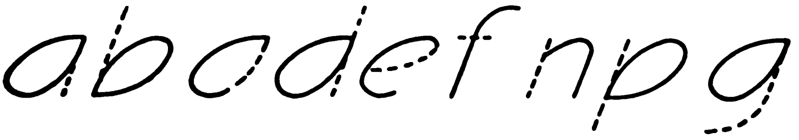


Fig. 29. Method of Constructing Parts of Small Arc-Line Letters

The slant at which these letters are made is a very important factor in a drawing, the proper slant being 3 in 8, as shown in Fig. 30. Even a slight increase, however, will give one the impression that the letters lean too far forward and it will spoil the appearance of a drawing otherwise good.

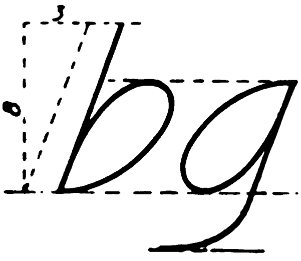


Fig. 30. Method of Constructing Parts of Small Arc-Line Letters

The height of the lower part of the letter should be equal to two-thirds or more of the total height. Figures should be of the same height as the capital letters. The total height of the small letters should not be less than one-tenth of an inch. This makes the capitals three-twentieths of an inch high, not less. The reason for adopting this height of letters is in order that, if necessary, ordinary tracings may be reduced for publication and the letters will then show up clearly. Fig. 31 shows the complete alphabet and

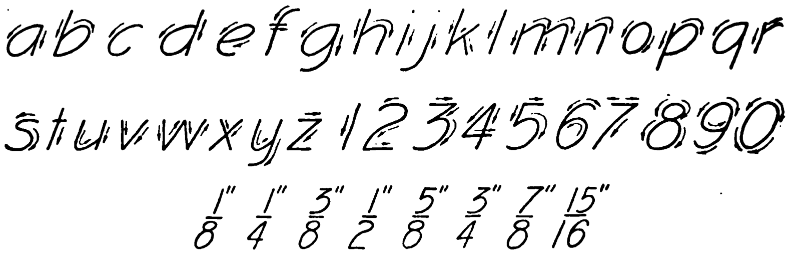


Fig. 31. The Completed Letter, with Arrows Showing Direction of Stroke

the numerals from 1 to 0, also several fractions. The fractions should never be made less than one-tenth of an inch in height for each of the members, and the dividing line should be horizontal, never slanting. Fig. 31 also shows by means of small arrows the direction the stroke should take when making the different letters and figures.

There is a tendency to make several of the letters and figures as shown in Fig. 32. This tendency should be carefully avoided, special attention being called to the turned-up ends of the members of different characters. Care should be taken not to get the upper part of the *s* and the *8* larger than the lower part. If this is done or if the two parts are made equal the upper will appear to be much larger and these characters will look out of proportion.

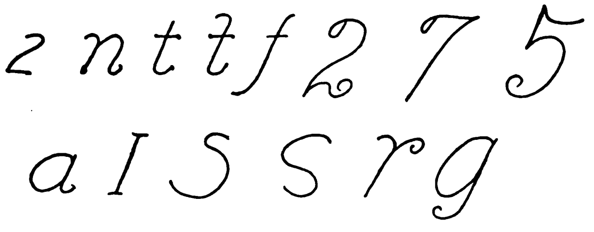


Fig. 32. Example of Poorly Constructed Letters

The capital letters *S*, *G*, *E*, *F*, *P*, and *R*, and the figures 2 and 5, present some difficulties. These characters are shown in Fig. 33, and may briefly be commented on as follows:

Letter S. The letter *S* should begin at the point 1 slightly inside of the circumscribed parallelogram. The line should then be tangent to the top and should come slightly inside of the further side at point 2. It should then cross the center line above the middle height at the point 3 and be tangent at point 4 and point 5 as indicated.

Letter G. The letter *G* should start at the right side of the parallelogram and be tangent to the top, left side, and bottom as well as the right-hand side where it extends upward to a height of one-half of its total height before the horizontal line, which should extend one-half of the distance across the letter, is drawn.

Letter E. The letter *E* presents no difficulties other than a central horizontal line should extend about two-thirds of the distance across the letter and should be at an elevation of two-thirds the height.

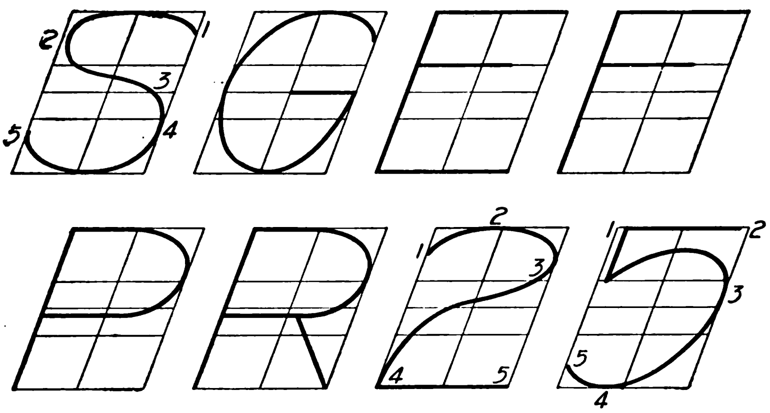


Fig. 33. Proportion and Slant of Capitals

Letter F. The letter *F* is but a part of the letter *E* as indicated.

Letters P and R. *P* and *R* are constructed on the same general principle. The upper part of both letters should be at least one-half or more of the total height, and in the case of *R* the lower right-hand stroke should not extend further than the right-hand side of the circumscribed parallelogram.

Figure 2. The figure *2* is constructed by starting at the left-hand side of the circumscribed parallelogram and continuing tangent as indicated in the figure at points 2, 3, and 4. The lower part 4—5

should in all cases be horizontal and it should never extend further than the right-hand side of the circumscribed parallelogram.

Figure 5. The figure *5* should start at the point 1 and extend downwards one-third of the total height. The lower part of the figure should then be drawn, being tangent at point 3 and 4 and slightly curled up at 5 where it should extend a little further to the left of the upper part. The horizontal part 1—2 should not extend quite up to the right-hand side of the circumscribed parallelogram.

In all cases where inch or foot sizes are employed, they should be made clearly and regularly and should be not less than one-twentieth of an inch in length.

Letters and figures should always be made by beginners by first preparing guide lines drawn with a pencil. Even in case the guide lines have been drawn upon the detailed paper, it is also advisable to draw them upon the tracing cloth, or place under the cloth a sheet similar to Fig. 34, with the lines drawn in ink, to act as a guide. This practice should be continued until enough skill has been acquired to make the letters uniform, without the assistance of more than a line or two.

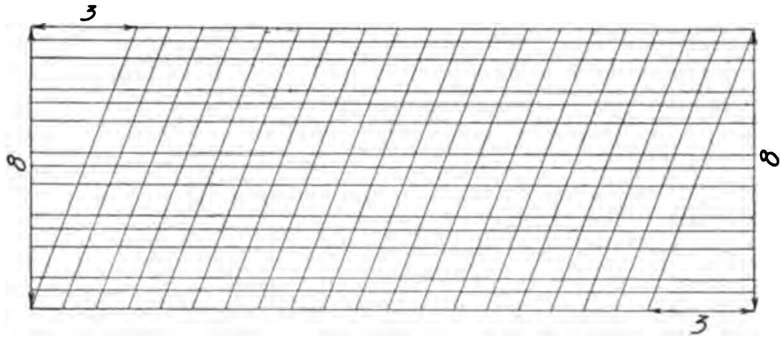


Fig. 34. Guide Sheet for Obtaining Correct Slant in Letters

The only manner in which a person can become proficient in lettering is through practice. A piece of paper ruled up and having the slant of the letter placed upon it as shown in Fig. 34 will be found an excellent thing on which to practice lettering. Letters can not be made nicely and quickly as one would suppose. Care and time are required until the draftsman becomes proficient in this respect.

TABLE IV

Abbreviations

SYMBOL	SIGNIFICANCE
L. or Ls.	*Angle or angles*
[. or [s.	*Channel or channels*
I. or Is.	*I beam or beams*
Z. or Zs.	*Zee bar or bars*
T. or Ts.	*Tee beam or beams*
Pl., Plt., or Pls., Plts.	*Plate or Plates*
@	*at*
Fill.	*Filler*
Stiff.	*Stiffeners*
Fl. or Flg.	*Flange*
r.	*Rivet*
f. r.	*Field rivets*
s.r.	*Shop rivets*
e.	*Eccentricity*
c.l. or ℄	*Center line*
θ or ϕ	*Diameter*
#	*Pound or pounds*
c. to c. or ℄'s	*Center to center*
Latt. or Latts.	*Latticed or lattices*
Lat. or Lats.	*Lateral or laterals*
alt.	*Alternate*
M. Pl.	*Masonry plate*
Spl.	*Splice*

Abbreviations. In the making of drawings certain abbreviations are used in order to save time and for the sake of convenience in many other respects. These abbreviations together with what they signify are given in Table IV. They should be carefully studied and should be written close to the material to which they apply and should at least be one-sixth to one-eighth of an inch from the material. Never write dimensions or letters so close to a line that they will interfere with the line. In writing dimensions at a considerable distance from the piece of material or place to which they apply, an arrow is used to indicate their proper position. In all such cases the arrow head should be at the end of the line which points to the place to which the abbreviation or dimension applies.

Fig. 35 illustrates some cases and also shows the form which the arrow should take in order to present a good appearance on the drawing.

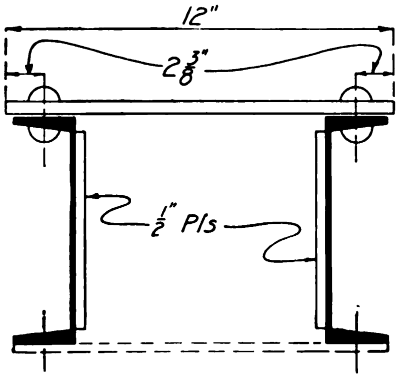


Fig. 35. Correct Use of Arrow and Line in Dimensioning

Dimension and Material Notation. *Proper Placing.* A drawing may be said to have been correctly dimensioned when any desired necessary dimensions may be obtained from it without it being required that any dimensions should be added or subtracted or divided in order to obtain the desired result, and when no unnecessary dimensions are upon the drawing. By necessary dimensions are meant those dimensions which are required in order that the material may be fabricated so that the finished struc-

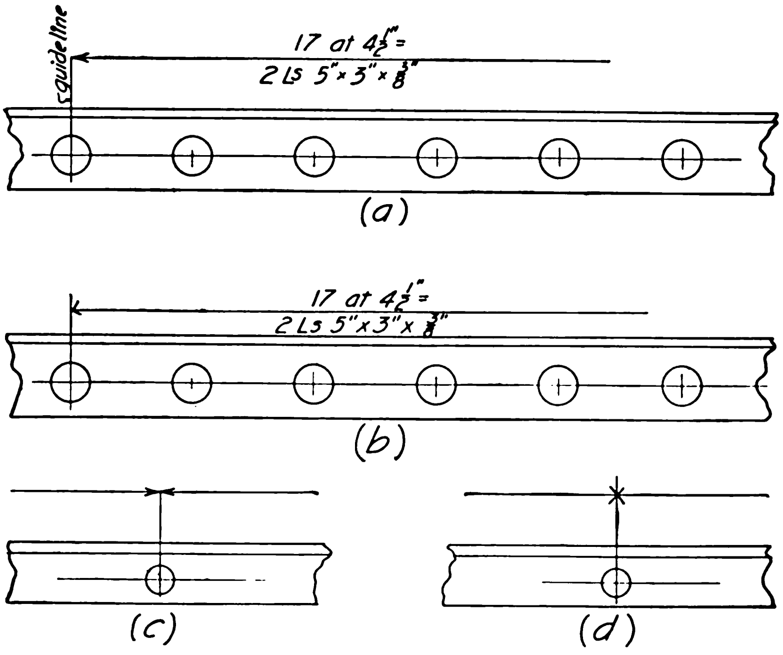


Fig. 36. Examples of Arrow-Head Construction and Proper Location of Dimensions

ture is as desired. Dimension lines should be full, not dotted or dashed; guide lines, which are lines indicating the limits of the dimensions, should not extend beyond the dimension line. The dimensions should be placed where possible above the line and should not, as mentioned before, touch the line at any place. Dimension lines should be far enough from the piece which they dimension in order that the letters and figures indicating the character of the material and its size may be placed between the dimension line and the material itself. Fig. 36a shows good practice and Fig. 36b poor practice.

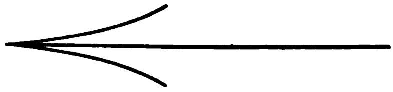


Fig. 37. Correct Arrow Head

Arrow heads are a source of trouble and should be made with care if the drawing is to present a good appearance when finished. They should be made as indicated in Fig. 36a and Fig. 36c, and not as in Figs. 36b and 36d. They should not consist as indicated in the figure showing the wrong construction, of a cross or half cross of a straight or nearly straight line, but should have a gradual slope as in Fig. 37 where it is greatly exaggerated.

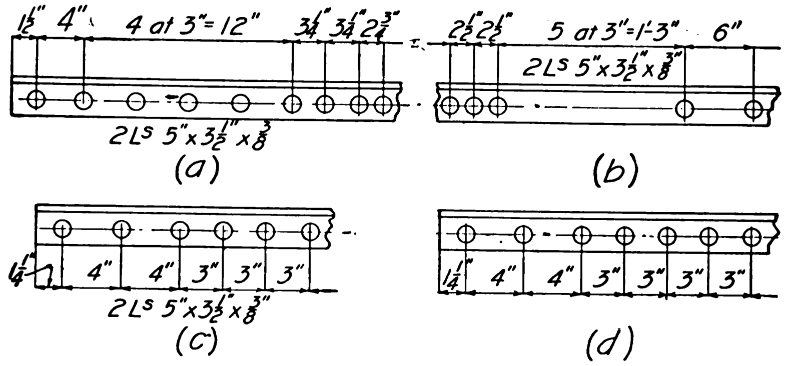


Fig. 38. Correct and Incorrect Placing of Dimensions

Dimensions as mentioned above should be placed above the dimension line where possible and the material should be noted so as not to interfere with the dimensioning. Figs. 38a and 38b show good practice and Figs. 38c and 38d poor practice. Sometimes it is necessary to place the dimensions as in Fig. 38c and 38d but never place the material notation as shown in the same figures. Fig. 38b gives the preferable method.

When several spaces are equal, the matter may be written as so many spaces at so much is equal to so much, or each space may be dimensioned separately as shown in Figs. 38a and 38b. In case the space is too small to write a single dimension in it clearly, the dimension may be put at one side and an arrow used to show where it belongs, Fig. 35.

In writing dimensions the inches should be given as well as the feet, and in case the inches amount to nothing or to a fraction, a cipher should take the place of the inches.

It is not necessary when all rivets are shop rivets to draw in each in such cases to put in the end rivets and to inidicate the spacing and every rivet when the spacing is the same. It is only necessary of those which lie between but which are not shown. Fig. 38b illustrates this. In case of field rivets all rivets must be shown. No departure from this rule should be allowed. Fig. 39 is an example of this. It is noted in this figure that although the spacing of many of the rivets is the same, yet all are shown in their proper place.

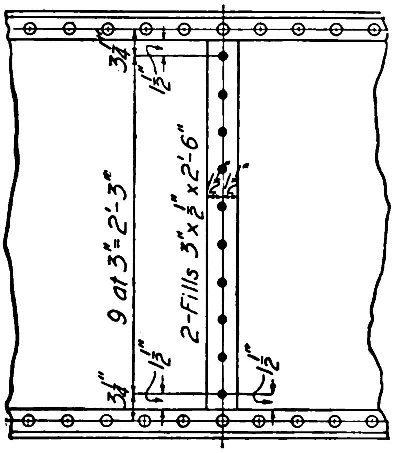
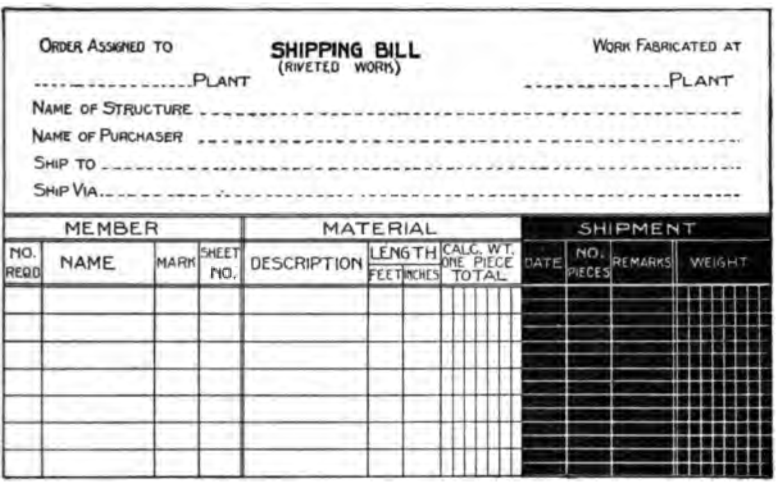


Fig. 39. Correct Method of Indicating Shop Rivets

In placing dimensions where two or more members are detailed together, dimensions for the main member should run straight through from one end to the other. The dimensions of the larger member in so far as they are the same as the dimensions for the smaller may be used for the smaller member and additional or subdimensions be placed in convenient places in order to complete the detailing of the smaller member. As an example of this see Fig. 38a where the edge distances and the method of detailing should be noted, and Fig. 39 also where the edge distances are the subdimensions. In Fig. 39 two dimensions are given at one of the ends. This illustrates two methods of placing the same dimension. The dimension directly under the line of dimensions for the main member is placed in the preferable way. In the placing of subdimensions great care should be taken



ORDER ASSIGNED TO

..................PLANT

SHIPPING BILL
(RIVETED WORK)

WORK FABRICATED AT

..................PLANT

NAME OF STRUCTURE ....

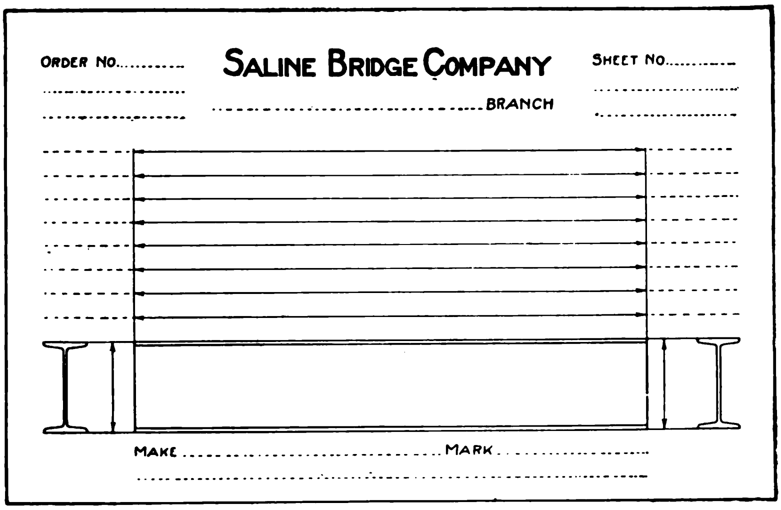
NAME OF PURCHASER ....

SHIP TO ....

SHIP VIA ....

MEMBER				MATERIAL				SHIPMENT			
NO. REQD	NAME	MARK	SHEET NO.	DESCRIPTION	LENGTH		CALC. WT. ONE PIECE TOTAL	DATE	NO. PIECES	REMARKS	WEIGHT
					FEET	INCHES					

Fig. 24. Shipping Bill



ORDER NO..........

SALINE BRIDGE COMPANY

..................BRANCH

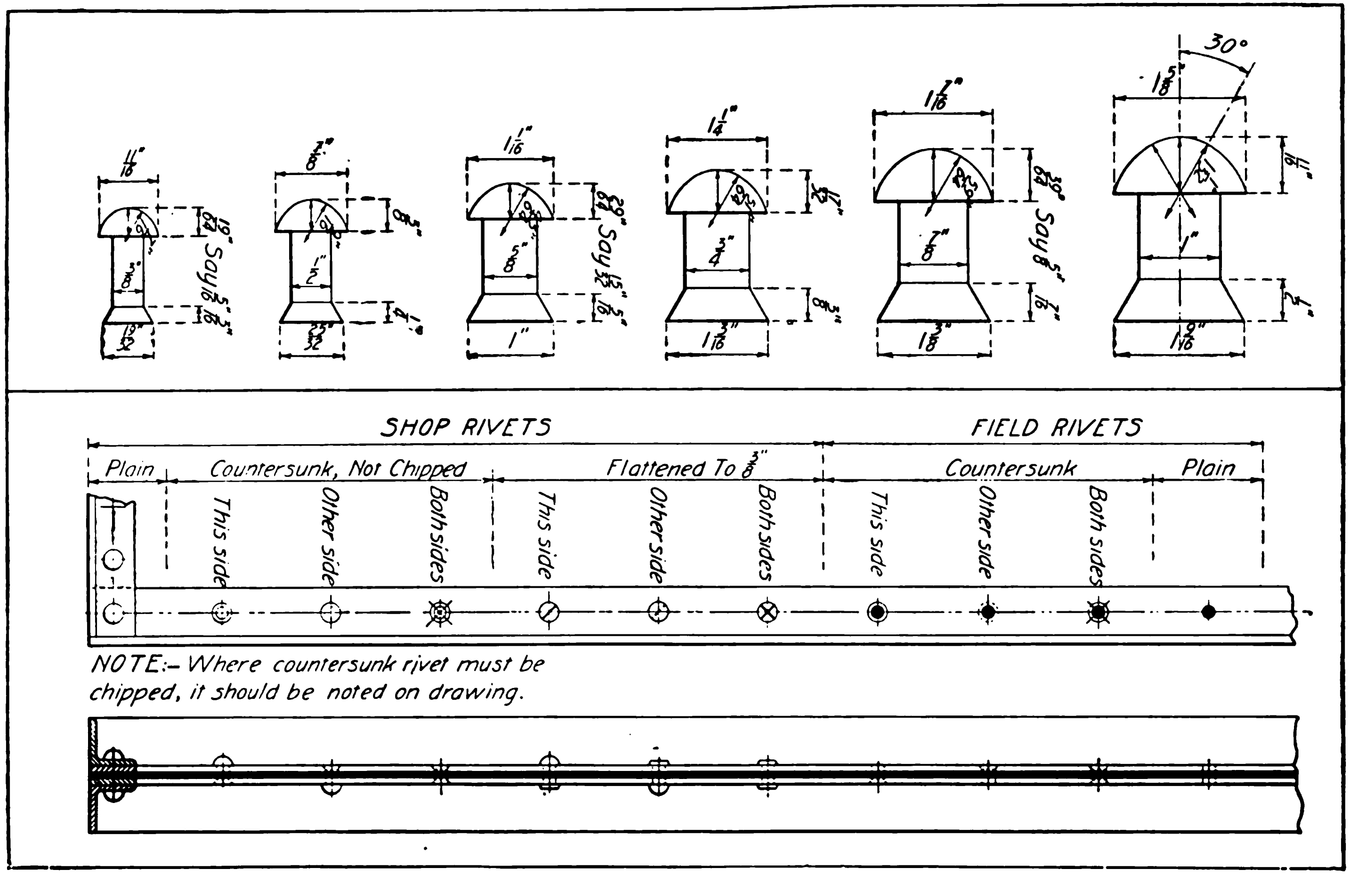
SHEET NO..........

MAKE ....MARK..................

Fig. 25. Shop Bill for I-Beams

TABLE V

Dimensions and Conventional Representation of Rivets



NOTE:– Where countersunk rivet must be chipped, it should be noted on drawing.

For example,

$$3-6'' \times \tfrac{3}{8}'' \text{ Zs} \times 14'-8''$$

Bars are designated by their number, then their size, or diameter, and finally their length in inches.

For example,

$$3-1''\square \times 20'-2''$$
$$1-\tfrac{3}{4}''\,\phi \times 16'-4\tfrac{1}{4}''$$

Rivets and Rivet Spacing. Rivets are made in various sizes and are spoken of according to the diameter of their shank. Thus a $\frac{7}{8}$-inch rivet is one which has its shank $\frac{7}{8}$ inch in diameter. The heads of the rivets are not perfect hemispheres, being less in height than one half the diameter of the head. Table V gives the dimensions of rivets of various diameters and their conventional representation in detail drawings. These dimensions are desirable on the drawings since they are often necessary in order to so figure the work that the material will not strike the heads. Rivets smaller than $\frac{3}{4}$ inch are seldom used except where the size of the material requires it. Rivets larger than $\frac{7}{8}$ inch in diameter are seldom used except in the heaviest work; and the beginner is advised not to use them until he has permission from those above him in charge.

Rivets should not be placed so close that the material between them is unduly injured by pushing or that the driving tool or "dolly" will interfere with one rivet when driving the other; likewise they should not be placed so far apart that the material between them will separate or open up. Unless specified otherwise in the specifications Table VI may be taken as good practice; for $\frac{3}{4}$-inch, $\frac{7}{8}$-inch, and 1-inch rivets, the minimum spacing is seen to be three diameters of the rivet.

TABLE VI

Minimum Rivet Spacing

(All dimensions given in inches)

SIZE of RIVETS	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1
Minimum Spacing Center to Center	1	$1\frac{1}{4}$	$1\frac{3}{4}$	2	$2\frac{1}{4}$	$2\frac{5}{8}$	3

The maximum spacing allowable is usually sixteen times the thickness of the thinnest plate they go through. The minimum and maximum limits placed above are not to be used wherever possible. Few engineers consider it advisable or permit spacings less than 2½ inches and 3 inches, or more than 4 inches and 5 inches for ¾-inch and ⅞-inch rivets, respectively.

The minimum limits above refer to the center to center of rivets, while the maximum values refer to the distance center to center measured along the gauge line or line along which the rivets are placed.

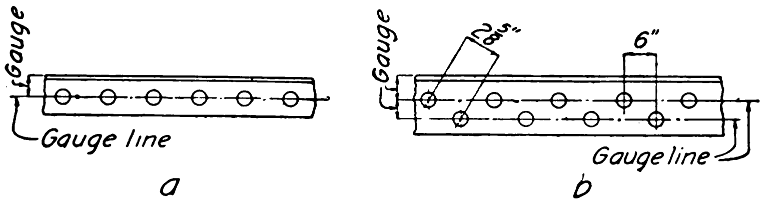


Fig. 40. Angles with Gauge Line

Gauge lines may be single, Fig. 40a, or double as in Fig. 40b. The gauge of a shape is the distance of the gauge line from a certain base. In the angle it is the back, in the channel it is the back, while in the I-beam it is the bisecting line of the web.

The gauges for standard channels and I-beams are given in the handbooks of manufacturers, such as Cambria, Carnegie, etc., which books also give the size of rivet or bolt which can be used in the flange of any certain I-beam or channel. This does not mean that the size of bolt or rivet there given must be used in the web also, in fact, ¾-inch and ⅞-inch should be used in the web, no matter what size is specified for the flange.

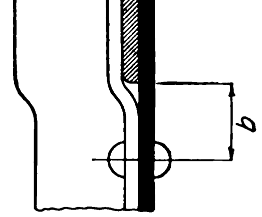


Fig. 41. Section Showing Crimped Angle, Chord Angle, and Web

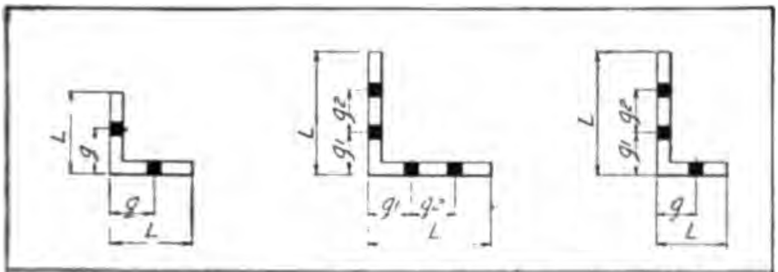
The standard gauges for angles are given in Table VII.

While a double gauge is shown for a 5-inch leg, it is very undesirable to use it. Do not use 5-inch legs with double gauge lines. Likewise, do not use a single-gauge line on an angle with a 6-inch leg or more, unless specially told to do so by those higher in authority.

TABLE VII

Standard Gauges for Angles

(All dimensions given in inches)



L	g	Maximum Rivet or Bolt	L	g	Maximum Rivet or Bolt	L	g	Maximum Rivet or Bolt
8	4½	⅞	3½	2	⅞	2	1⅛	½
7	4	⅞	3	1¾	⅞	1¾	1	½
6	3½	⅞	2¾	1⅝	¾	1½	⅞	⅜
5	3	⅞	2½	1⅜	⅝	1¼	¾	⅜
4	2½	⅞	2¼	1¼	⅝	1	9/16	¼

L	g_1	g_2	L	g_1	g_2
8	3	3	6*	2½	2¼
7	2½	3	5	2	1¾
6	2¼	2½			

*When thickness is ⅝ inch or over.

In the spacing of rivets in crimped angles, the distance "*b*", Fig. 41, should be 1½ inches plus twice the thickness of the chord angles, but never less than 2 inches.

The grip of a rivet is the length under heads after the rivet has been driven. The length of a rivet is the length of the shank before the rivet is driven, Fig. 42, these lengths for various grips being easily found in any manufacturer's handbook.

Care should be taken in case of castings to add ⅛ inch more to those values given.

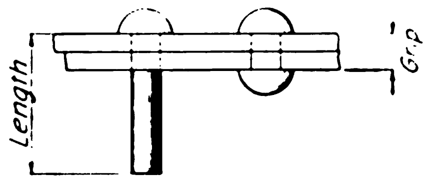


Fig. 42 Rivet Before and After Driving

Rivets may have two full heads or may have one or both heads countersunk or flattened or any combination. Such conditions are signified by certain signs, all of those in common use being listed in the handbooks already referred to, and also shown in Table V.

A rivet can be driven as close to a projection as one-half the diameter of the head plus ¼ inch. This requires a special "dolly". The dolly generally used requires $\frac{2}{3}D+\frac{1}{4}$ inch. This is about 1¼ inches for a ⅞-inch rivet and about 1⅛ inches for a ¾-inch; see Fig. 43 and Table V. In some instances a special gauge, that is, one other than given in Table VII, is used. In such cases care should be taken to see that the distance A, to the fillet, or curve of the angle, is sufficient, otherwise the dolly could not come down evenly and an imperfect head is the result.

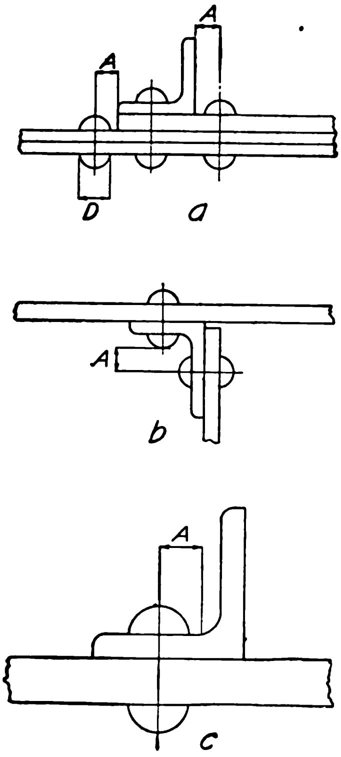
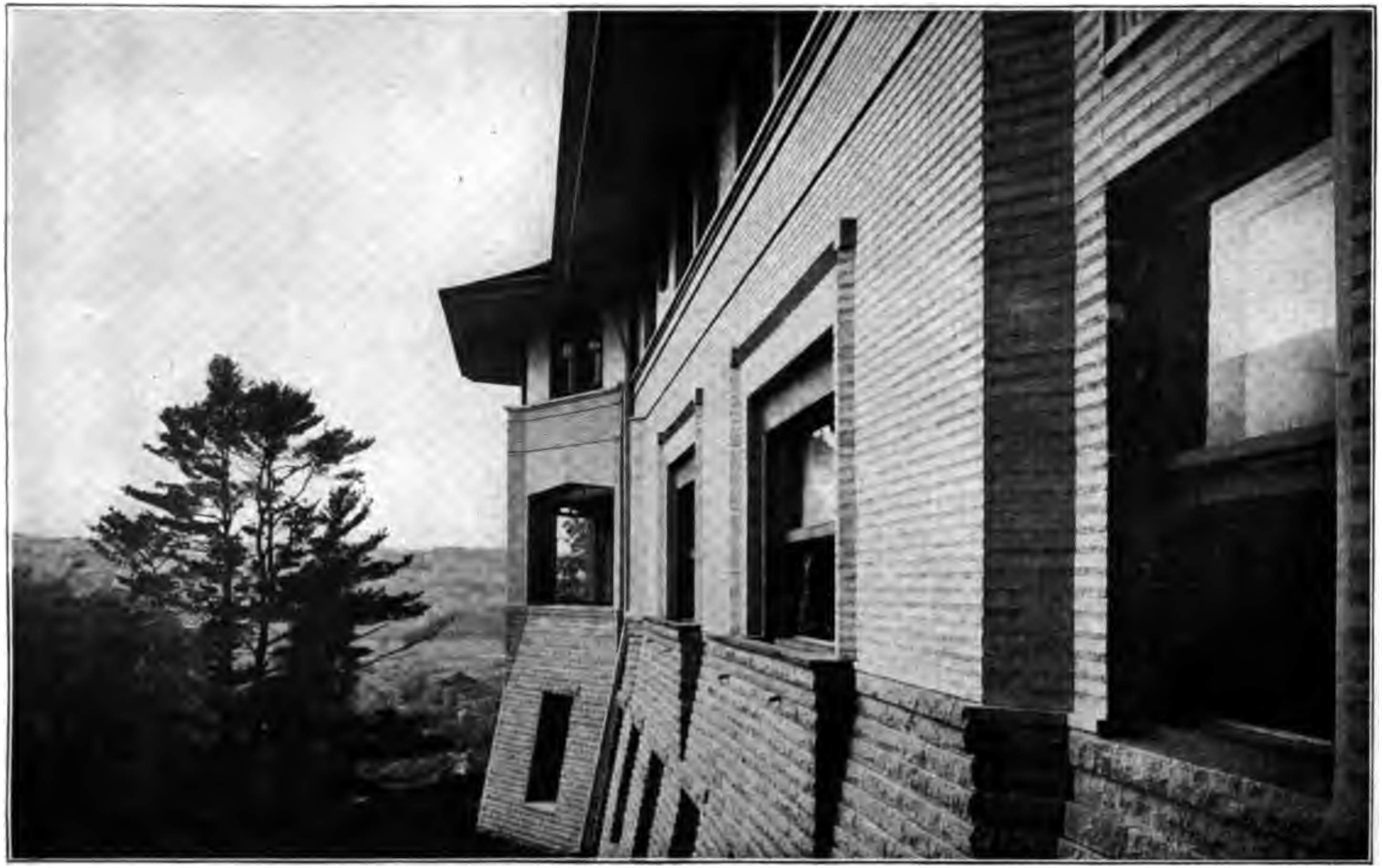


Fig. 43. Diagram for Minimum Rivet Spacing

When rivets are staggered, it is necessary to know how close they may be spaced in order that they may not be less than the minimum allowed distance center to center. Table VIII gives the distances center to center of rivets for given values of the spacing and gauge line. The distances below and to the right of the upper zigzag line are large enough for ¾-inch rivets while those below and to the right of the lower zigzag line are large enough for ⅞-inch rivets. For example, if the gauge "g" was 1¾ inches, the spacing must be at least 2 inches in order that the distance center to center would not be less than 2⅝ inches, the rivets being ⅞ inch. If the rivets were ¾ inch, the spacing must be 1½ inches or more in order to have the distance center to center not less than 2¼ inches. These values are found by going down from the value 1¾ inches in the top row until a value equal to or just greater than the 2⅝ or 2¼ inches is found, and then following across to the first column where required spacing is found.



ALPHA DELTA PHI CHAPTER-HOUSE AT CORNELL UNIVERSITY, ITHACA, NEW YORK

Dean and Dean, Architects, Chicago, Illinois

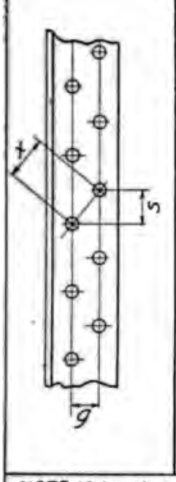
Ground floor split-faced Bedford stone; main floor of Roman brick with ⅜-inch mortar joints; second floor of plaster.

For full exterior and plans, see pages 122 and 139

TABLE VIII

Values Center to Center for Various Spacings

(All dimensions in inches)



VALUES OF x FOR VARYING VALUES OF g AND s

VALUES OF s Inches	VALUES OF g													
	Inches $\frac{7}{8}$	Inches 1	Inches $1\frac{1}{8}$	Inches $1\frac{1}{4}$	Inches $1\frac{3}{8}$	Inches $1\frac{1}{2}$	Inches $1\frac{5}{8}$	Inches $1\frac{3}{4}$	Inches $1\frac{7}{8}$	Inches 2	Inches $2\frac{1}{8}$	Inches $2\frac{1}{4}$	Inches $2\frac{3}{8}$	Inches $2\frac{1}{2}$
$1\frac{1}{8}$	$1\frac{7}{16}$	$1\frac{1}{2}$	$1\frac{9}{16}$	$1\frac{11}{16}$	$1\frac{3}{4}$	$1\frac{7}{8}$	2	$2\frac{1}{16}$	$2\frac{3}{16}$	$2\frac{5}{16}$	$2\frac{3}{8}$	$2\frac{1}{2}$	$2\frac{5}{8}$	$2\frac{3}{4}$
$1\frac{1}{4}$	$1\frac{9}{16}$	$1\frac{5}{8}$	$1\frac{11}{16}$	$1\frac{3}{4}$	$1\frac{7}{8}$	$1\frac{15}{16}$	$2\frac{1}{16}$	$2\frac{1}{8}$	$2\frac{1}{4}$	$2\frac{3}{8}$	$2\frac{7}{16}$	$2\frac{9}{16}$	$2\frac{11}{16}$	$2\frac{13}{16}$
$1\frac{3}{8}$	$1\frac{5}{8}$	$1\frac{11}{16}$	$1\frac{3}{4}$	$1\frac{7}{8}$	$1\frac{15}{16}$	2	$2\frac{1}{8}$	$2\frac{3}{16}$	$2\frac{5}{16}$	$2\frac{7}{16}$	$2\frac{1}{2}$	$2\frac{5}{8}$	$2\frac{3}{4}$	$2\frac{7}{8}$
$1\frac{1}{2}$	$1\frac{3}{4}$	$1\frac{13}{16}$	$1\frac{7}{8}$	$1\frac{15}{16}$	2	$2\frac{1}{8}$	$2\frac{3}{16}$	$2\frac{5}{16}$	$2\frac{3}{8}$	$2\frac{1}{2}$	$2\frac{5}{8}$	$2\frac{11}{16}$	$2\frac{13}{16}$	$2\frac{15}{16}$
$1\frac{5}{8}$	$1\frac{7}{8}$	$1\frac{7}{8}$	2	$2\frac{1}{16}$	$2\frac{1}{8}$	$2\frac{3}{16}$	$2\frac{5}{16}$	$2\frac{3}{8}$	$2\frac{1}{2}$	$2\frac{9}{16}$	$2\frac{11}{16}$	$2\frac{3}{4}$	$2\frac{7}{8}$	3
$1\frac{3}{4}$	$1\frac{15}{16}$	2	$2\frac{1}{16}$	$2\frac{1}{8}$	$2\frac{3}{16}$	$2\frac{5}{16}$	$2\frac{3}{8}$	$2\frac{7}{16}$	$2\frac{9}{16}$	$2\frac{5}{8}$	$2\frac{3}{4}$	$2\frac{7}{8}$	$2\frac{15}{16}$	$3\frac{1}{16}$
$1\frac{7}{8}$	$2\frac{1}{16}$	$2\frac{1}{8}$	$2\frac{3}{16}$	$2\frac{1}{4}$	$2\frac{5}{16}$	$2\frac{3}{8}$	$2\frac{1}{2}$	$2\frac{9}{16}$	$2\frac{5}{8}$	$2\frac{3}{4}$	$2\frac{13}{16}$	$2\frac{15}{16}$	3	$3\frac{1}{8}$
2	$2\frac{3}{16}$	$2\frac{1}{4}$	$2\frac{5}{16}$	$2\frac{3}{8}$	$2\frac{7}{16}$	$2\frac{1}{2}$	$2\frac{9}{16}$	$2\frac{5}{8}$	$2\frac{3}{4}$	$2\frac{13}{16}$	$2\frac{15}{16}$	3	$3\frac{1}{8}$	$3\frac{3}{16}$
$2\frac{1}{8}$	$2\frac{5}{16}$	$2\frac{5}{16}$	$2\frac{3}{8}$	$2\frac{7}{16}$	$2\frac{1}{2}$	$2\frac{5}{8}$	$2\frac{11}{16}$	$2\frac{3}{4}$	$2\frac{13}{16}$	$2\frac{15}{16}$	3	$3\frac{1}{16}$	$3\frac{3}{16}$	$3\frac{1}{4}$
$2\frac{1}{4}$	$2\frac{7}{16}$	$2\frac{7}{16}$	$2\frac{1}{2}$	$2\frac{9}{16}$	$2\frac{5}{8}$	$2\frac{11}{16}$	$2\frac{3}{4}$	$2\frac{7}{8}$	$2\frac{15}{16}$	3	$3\frac{1}{16}$	$3\frac{3}{16}$	$3\frac{1}{4}$	$3\frac{3}{8}$
$2\frac{3}{8}$	$2\frac{1}{2}$	$2\frac{9}{16}$	$2\frac{5}{8}$	$2\frac{11}{16}$	$2\frac{3}{4}$	$2\frac{13}{16}$	$2\frac{7}{8}$	$2\frac{15}{16}$	3	$3\frac{1}{8}$	$3\frac{3}{16}$	$3\frac{1}{4}$	$3\frac{3}{8}$	$3\frac{7}{16}$
$2\frac{1}{2}$	$2\frac{5}{8}$	$2\frac{11}{16}$	$2\frac{3}{4}$	$2\frac{13}{16}$	$2\frac{7}{8}$	$2\frac{15}{16}$	3	$3\frac{1}{16}$	$3\frac{1}{8}$	$3\frac{3}{16}$	$3\frac{1}{4}$	$3\frac{3}{8}$	$3\frac{7}{16}$	$3\frac{9}{16}$

NOTE: Values below or to right of upper zigzag lines are large enough for $\frac{3}{4}''$ rivets
" " " " " " lower " " " " " $\frac{7}{8}''$ "

Care should also be taken that the rivets are not so close that there will not be at least 1″ between the holes in the direction of the line of stress, see Fig. 43d.

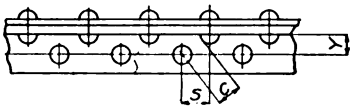
In many cases a row of rivets must be driven below another row and in material which is perpendicular to the material in which the first row is driven. Such a case is in the cover plate of a plate girder, or for that matter in most cases of cover plates. In such cases it is desirable to know what spacing must be used in order that the dolly will not be interfered with by the rivet already driven in the other row. Table IX gives such information. It is to be noted that the value Y is the distance from the inner side of the leg of the angle, and is not the gauge. For example, let it be required to determine the minimum stagger for $\frac{3}{4}$-inch rivets in a $3\frac{1}{2}$-inch leg of a $3\frac{1}{2}''\times3\frac{1}{2}''\times\frac{3}{8}''$ angle. The distance Y is then equal to the gauge of a 3-inch leg less the thickness of the angle, or

$$\begin{aligned} Y &= 2'' - \tfrac{3}{8}'' \\ &= 1\tfrac{5}{8}'' \end{aligned}$$

TABLE IX

Minimum Staggers

c = $1\frac{1}{8}''$ for $\frac{3}{4}''$ rivets
= $1\frac{1}{4}''$ " $\frac{7}{8}''$ "



DIAMETER OF RIVETS	VALUES OF Y										
	$1\frac{1}{8}$	$1\frac{3}{16}$	$1\frac{1}{4}$	$1\frac{5}{16}$	$1\frac{3}{8}$	$1\frac{7}{16}$	$1\frac{1}{2}$	$1\frac{9}{16}$	$1\frac{5}{8}$	$1\frac{11}{16}$	$1\frac{3}{4}$
$\frac{3}{4}$	$1\frac{1}{4}$	$1\frac{3}{16}$	$1\frac{1}{8}$	$1\frac{1}{16}$	$\frac{15}{16}$	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{3}{8}$	0	0
$\frac{7}{8}$	$1\frac{3}{8}$	$1\frac{5}{16}$	$1\frac{1}{4}$	$1\frac{3}{16}$	$1\frac{1}{8}$	1	$\frac{15}{16}$	$\frac{13}{16}$	$\frac{11}{16}$	$\frac{1}{2}$	$\frac{5}{16}$

All dimensions in inches

Looking along the top row the value $1\frac{5}{8}$ inches is found and going downward to the $\frac{3}{4}$-inch line of values, $\frac{3}{8}$ inch is found to be the least distance that the rivet under consideration may be driven from the one in the other leg of the angle.

In some cases it is possible to drive rivets opposite if the proper row is driven first. Thus, in the $5'' \times 3\frac{1}{2}'' \times \frac{3}{8}''$ angle of Fig. 44, if $\frac{3}{4}$-inch rivets in the 5-inch leg were driven first, those in the 3-inch leg must stagger by $\frac{3}{8}$ inch, as figured above, Fig. 44a, but if the rivets in the 3-inch leg were driven first, the distance $Y = 3'' - \frac{3}{8}'' = 2\frac{5}{8}''$, which, being outside the values in Table IX, show that the rivets in the 5-inch leg may be driven with a zero stagger, or just opposite.

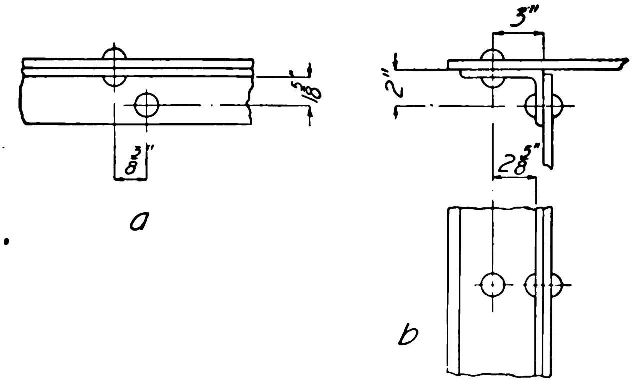


Fig. 44. Rivet Stagger

Certain clauses in most specifications call attention to the fact that rivets must not be used in tension. While it is desirable not to have rivets in tension, and their use to resist tensile stresses should not be encouraged, yet a rivet has a distinct value when used in tension. Also, tests of a confidential nature have come under the author's observation, and they tend to prove that rivets so used show as great an efficiency as a turned bolt of the same diameter.

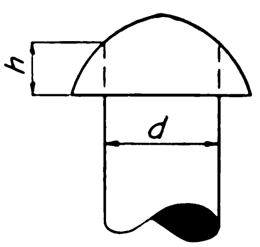


Fig. 45. Drawing of Standard Rivet

However, the strength of such rivets must not be assumed as being equal to a bolt of equal diameter, but must be computed. The head of the rivet must be drawn out to full size, and the distance "*h*," Fig. 45, determined. The value of the rivet in tension is then given by the formula

$$S_t = 3.14\ S_s dh$$

where S_s = the unit shearing stress; d = the diameter of the rivet; and h = the value as determined above.

For a ⅞-inch rivet $h = 0.45$ inch and, therefore, this value of the rivet in tension, S_s being taken at 10,000 pounds per square inch, is

$$S_t = 3.14 \times 10{,}000 \times \tfrac{7}{8} \times 0.45$$
$$= 12{,}360 \text{ pounds}$$

which is seen to be considerable, and which is equal to the body of the rivet being strained up to 20,050 pounds per square inch.

It is thus seen that the head more than develops the strength of the body of the rivet. Therefore, in figuring the amount a rivet should take in tension, one should multiply the area of the cross-section by the allowable unit stress decided upon. Since the specifications do not give this, it will be safe to use the ultimate strength for rivet steel with a factor of safety of 4. Since the ultimate strength of rivet steel should be about 50,000 pounds per square inch, this would make the allowable 12,500 pounds, and a ⅞-inch rivet would have a value of

$$S_t = 12{,}500 \times 0.6013$$
$$= 7{,}510 \text{ pounds}$$

which is less than the amount required to strain its head up to the maximum allowable.

TABLE X

Rivet Spacing Multiplication Table

SPACES	PITCH IN INCHES															SPACES
	1⅛	1¼	1⅜	1½	1⅝	1¾	1⅞	2	2⅛	2¼	2⅜	2½	2⅝	2¾	2⅞	
1																1
2	- 2¼	- 2½	- 2¾	- 3	- 3¼	- 3½	- 3¾	- 4	- 4¼	- 4½	- 4¾	- 5	- 5¼	- 5½	- 5¾	2
3	- 3⅜	- 3¾	- 4⅛	- 4½	- 4⅞	- 5¼	- 5⅝	- 6	- 6⅜	- 6¾	- 7⅛	- 7½	- 7⅞	- 8¼	- 8⅝	3
4	- 4½	- 5	- 5½	- 6	- 6½	- 7	- 7½	- 8	- 8½	- 9	- 9½	-10	-10½	-11	-11½	4
5	- 5⅝	- 6¼	- 6⅞	- 7½	- 8⅛	- 8¾	- 9⅜	-10	-10⅝	-11¼	-11⅞	1- 0½	1- 1⅛	1- 1¾	1- 2⅜	5
6	- 6¾	- 7½	- 8¼	- 9	- 9¾	-10½	-11¼	1- 0	1- 0¾	1- 1½	1- 2¼	1- 3	1- 3¾	1- 4½	1- 5¼	6
7	- 7⅞	- 8¾	- 9⅝	-10½	-11⅜	1- 0¼	1- 1⅛	1- 2	1- 2⅞	1- 3¾	1- 4⅝	1- 5½	1- 6⅜	1- 7¼	1- 8⅛	7
8	- 9	-10	-11	1- 0	1- 1	1- 2	1- 3	1- 4	1- 5	1- 6	1- 7	1- 8	1- 9	1-10	1-11	8
9	-10⅛	-11¼	1- 0⅜	1- 1½	1- 2⅝	1- 3¾	1- 4⅞	1- 6	1- 7⅛	1- 8¼	1- 9⅜	1-10½	1-11⅝	2- 0¾	2- 1⅞	9
10	-11¼	1 0½	1- 1¾	1- 3	1- 4¼	1- 5½	1- 6¾	1- 8	1- 9¼	1-10½	1-11¾	2- 1	2- 2¼	2- 3½	2- 4¾	10
11	1- 0⅜	1- 1¾	1- 3⅛	1- 4½	1- 5⅞	1- 7¼	1- 8⅝	1-10	1-11⅜	2- 0¾	2- 2⅛	2- 3½	2- 4⅞	2- 6¼	2- 7⅝	11
12	1- 1½	1- 3	1- 4½	1- 6	1- 7½	1- 9	1-10½	2- 0	2- 1½	2- 3	2- 4½	2- 6	2- 7½	2- 9	2-10½	12
13	1- 2⅝	1- 4¼	1- 5⅞	1- 7½	1- 9⅛	1-10¾	2- 0⅜	2- 2	2- 3⅝	2- 5¼	2- 6⅞	2- 8½	2-10⅛	2-11¾	3- 1⅜	13
14	1- 3¾	1- 5½	1- 7¼	1- 9	1-10¾	2- 0½	2- 2¼	2- 4	2- 5¾	2- 7½	2- 9¼	2-11	3- 0¾	3- 2½	3- 4¼	14
15	1- 4⅞	1- 6¾	1- 8⅝	1-10½	2- 0⅜	2- 2¼	2- 4⅛	2- 6	2- 7⅞	2- 9¾	2-11⅝	3- 1½	3- 3⅜	3- 5¼	3- 7⅛	15
16	1- 6	1- 8	1-10	2- 0	2- 2	2- 4	2- 6	2- 8	2-10	3- 0	3- 2	3- 4	3- 6	3- 8	3-10	16
17	1- 7⅛	1- 9¼	1-11⅜	2- 1½	2- 3⅝	2- 5¾	2- 7⅞	2-10	3- 0⅛	3- 2¼	3- 4⅜	3- 6½	3- 8⅝	3-10¾	4- 0⅞	17
18	1- 8¼	1-10½	2- 0¾	2- 3	2- 5¼	2- 7½	2- 9¾	3- 0	3- 2¼	3- 4½	3- 6¾	3- 9	3-11¼	4- 1½	4- 3¾	18
19	1- 9⅜	1-11¾	2- 2⅛	2- 4½	2- 6⅞	2- 9¼	2-11⅝	3- 2	3- 4⅜	3- 6¾	3- 9⅛	3-11½	4- 1⅞	4- 4¼	4- 6⅝	19
20	1-10½	2- 1	2- 3½	2- 6	2- 8½	2-11	3- 1½	3- 4	3- 6½	3- 9	3-11½	4- 2	4- 4½	4- 7	4- 9½	20
21	1-11⅝	2- 2¼	2- 4⅞	2- 7½	2-10⅛	3- 0¾	3- 3⅜	3- 6	3- 8⅝	3-11¼	4- 1⅞	4- 4½	4- 7⅛	4- 9¾	5- 0⅜	21
22	2- 0¾	2- 3½	2- 6¼	2- 9	2-11¾	3- 2½	3- 5¼	3- 8	3-10¾	4- 1½	4- 4¼	4- 7	4- 9¾	5- 0½	5- 3¼	22
23	2- 1⅞	2- 4¾	2- 7⅝	2-10½	3- 1⅜	3- 4¼	3- 7⅛	3-10	4- 0⅞	4- 3¾	4- 6⅝	4- 9½	5- 0⅜	5- 3¼	5- 6⅛	23
24	2- 3	2- 6	2- 9	3- 0	3- 3	3- 6	3- 9	4- 0	4- 3	4- 6	4- 9	5- 0	5- 3	5- 6	5- 9	24
25	2- 4⅛	2- 7¼	2-10⅜	3- 1½	3- 4⅝	3- 7¾	3-10⅞	4- 2	4- 5⅛	4- 8¼	4-11⅜	5- 2½	5- 5⅝	5- 8¾	5-11⅞	25
26	2- 5¼	2- 8½	2-11¾	3- 3	3- 6¼	3- 9½	4- 0¾	4- 4	4- 7¼	4-10½	5- 1¾	5- 5	5- 8¼	5-11½	6- 2¾	26
27	2- 6⅜	2- 9¾	3- 1⅛	3- 4½	3- 7⅞	3-11¼	4- 2⅝	4- 6	4- 9⅜	5- 0¾	5- 4⅛	5- 7½	5-10⅞	6- 2¼	6- 5⅝	27
28	2- 7½	2-11	3- 2½	3- 6	3- 9½	4- 1	4- 4½	4- 8	4-11½	5- 3	5- 6½	5-10	6- 1½	6- 5	6- 8½	28
29	2- 8⅝	3- 0¼	3- 3⅞	3- 7½	3-11⅛	4- 2¾	4- 6⅜	4-10	5- 1⅝	5- 5¼	5- 8⅞	6- 0½	6- 4⅛	6- 7¾	6-11⅜	29
30	2- 9¾	3- 1½	3- 5¼	3- 9	4- 0¾	4- 4½	4- 8¼	5- 0	5- 3¾	5- 7½	5-11¼	6- 3	6- 6¾	6-10½	7- 2¼	30
SPACES	1⅛	1¼	1⅜	1½	1⅝	1¾	1⅞	2	2⅛	2¼	2⅜	2½	2⅝	2¾	2⅞	SPACES
	PITCH IN INCHES															

TABLE X (Continued)

Rivet Spacing Multiplication Table

SPACES	PITCH IN INCHES															SPACES
	3	3⅛	3¼	3⅜	3½	3¾	4	4¼	4½	4¾	5	5¼	5½	5¾	6	
1																1
2	-6	-6¼	-6½	-6¾	-7	-7½	-8	-8½	-9	-9½	-10	-10½	-11	-11½	1-0	2
3	-9	-9⅜	-9¾	-10⅛	-10½	-11¼	1-0	1-0¾	1-1½	1-2¼	1-3	1-3¾	1-4½	1-5¼	1-6	3
4	1-0	1-0½	1-1	1-1½	1-2	1-3	1-4	1-5	1-6	1-7	1-8	1-9	1-10	1-11	2-0	4
5	1-3	1-3⅝	1-4¼	1-4⅞	1-5½	1-6¾	1-8	1-9¼	1-10½	1-11¾	2-1	2-2¼	2-3½	2-4¾	2-6	5
6	1-6	1-6¾	1-7½	1-8¼	1-9	1-10½	2-0	2-1½	2-3	2-4½	2-6	2-7½	2-9	2-10½	3-0	6
7	1-9	1-9⅞	1-10¾	1-11⅝	2-0½	2-2¼	2-4	2-5¾	2-7½	2-9¼	2-11	3-0¾	3-2½	3-4¼	3-6	7
8	2-0	2-1	2-2	2-3	2-4	2-6	2-8	2-10	3-0	3-2	3-4	3-6	3-8	3-10	4-0	8
9	2-3	2-4⅛	2-5¼	2-6⅜	2-7½	2-9¾	3-0	3-2¼	3-4½	3-6¾	3-9	3-11¼	4-1½	4-3¾	4-6	9
10	2-6	2-7¼	2-8½	2-9¾	2-11	3-1½	3-4	3-6½	3-9	3-11½	4-2	4-4½	4-7	4-9½	5-0	10
11	2-9	2-10⅜	2-11¾	3-1⅛	3-2½	3-5¼	3-8	3-10¾	4-1½	4-4¼	4-7	4-9¾	5-0½	5-3¼	5-6	11
12	3-0	3-1½	3-3	3-4½	3-6	3-9	4-0	4-3	4-6	4-9	5-0	5-3	5-6	5-9	6-0	12
13	3-3	3-4⅝	3-6¼	3-7⅞	3-9½	4-0¾	4-4	4-7¼	4-10½	5-1¾	5-5	5-8¼	5-11½	6-2¾	6-6	13
14	3-6	3-7¾	3-9½	3-11¼	4-1	4-4½	4-8	4-11½	5-3	5-6½	5-10	6-1½	6-5	6-8½	7-0	14
15	3-9	3-10⅞	4-0¾	4-2⅝	4-4½	4-8¼	5-0	5-3¾	5-7½	5-11¼	6-3	6-6¾	6-10½	7-2¼	7-6	15
16	4-0	4-2	4-4	4-6	4-8	5-0	5-4	5-8	6-0	6-4	6-8	7-0	7-4	7-8	8-0	16
17	4-3	4-5⅛	4-7¼	4-9⅜	4-11½	5-3¾	5-8	6-0¼	6-4½	6-8¾	7-1	7-5¼	7-9½	8-1¾	8-6	17
18	4-6	4-8¼	4-10½	5-0¾	5-3	5-7½	6-0	6-4½	6-9	7-1½	7-6	7-10½	8-3	8-7½	9-0	18
19	4-9	4-11⅜	5-1¾	5-4⅛	5-6½	5-11¼	6-4	6-8¾	7-1½	7-6¼	7-11	8-3¾	8-8½	9-1¼	9-6	19
20	5-0	5-2½	5-5	5-7½	5-10	6-3	6-8	7-1	7-6	7-11	8-4	8-9	9-2	9-7	10-0	20
21	5-3	5-5⅝	5-8¼	5-10⅞	6-1½	6-6¾	7-0	7-5¼	7-10½	8-3¾	8-9	9-2¼	9-7½	10-0¾	10-6	21
22	5-6	5-8¾	5-11½	6-2¼	6-5	6-10½	7-4	7-9½	8-3	8-8½	9-2	9-7½	10-1	10-6½	11-0	22
23	5-9	5-11⅞	6-2¾	6-5⅝	6-8½	7-2¼	7-8	8-1¾	8-7½	9-1¼	9-7	10-0¾	10-6½	11-0¼	11-6	23
24	6-0	6-3	6-6	6-9	7-0	7-6	8-0	8-6	9-0	9-6	10-0	10-6	11-0	11-6	12-0	24
25	6-3	6-6⅛	6-9¼	7-0⅜	7-3½	7-9¾	8-4	8-10¼	9-4½	9-10¾	10-5	10-11¼	11-5½	11-11¾	12-6	25
26	6-6	6-9¼	7-0½	7-3¾	7-7	8-1½	8-8	9-2½	9-9	10-3½	10-10	11-4½	11-11	12-5½	13-0	26
27	6-9	7-0⅜	7-3¾	7-7⅛	7-10½	8-5¼	9-0	9-6¾	10-1½	10-8¼	11-3	11-9¾	12-4½	12-11¼	13-6	27
28	7-0	7-3½	7-7	7-10½	8-2	8-9	9-4	9-11	10-6	11-1	11-8	12-3	12-10	13-5	14-0	28
29	7-3	7-6⅝	7-10¼	8-1⅞	8-5½	9-0¾	9-8	10-3¼	10-10½	11-5¾	12-1	12-8¼	13-3½	13-10¾	14-6	29
30	7-6	7-9¾	8-1½	8-5¼	8-9	9-4½	10-0	10-7½	11-3	11-10½	12-6	13-1½	13-9	14-4½	15-0	30
SPACES	3	3⅛	3¼	3⅜	3½	3¾	4	4¼	4½	4¾	5	5¼	5½	5¾	6	SPACES
	PITCH IN INCHES															

On account of the fact that riveted heads are not driven symmetrically the value of the rivet in tension is not certain, and their use in tension is not to be advised. Use turned bolts.

The bearing and shearing values of rivets may be found in the handbooks of the various manufacturers. The values for all values of allowable stresses are not usually given, but by a little trouble almost any values may be obtained by dividing those values there given, or by taking a multiple of them. For example, the bearing value of a ⅞-inch rivet in a $\frac{7}{16}$-inch plate, unit allowable bearing stress 18,000 pounds, may be obtained by taking 1½ times the value given in the 12,000-pound table, giving 6,885 pounds.

In cases of the webs of channels or I-beams, or other thicknesses of metal which are not in even sixteenths of an inch, but are given in decimal fractions, the values may, with the help of the slide rule, be obtained from the tables. For example, let it be required to find the value of a ⅞-inch rivet in bearing in the web of a 15″×33# channel, the unit-bearing stress allowed being 15,000 pounds. From Cambria, the thickness is seen to be 0.4, and the bearing of a ⅞-inch rivet in a ½-inch plate is found to be 6,563 pounds. Therefore, the value sought will be

$$V=\frac{6{,}563}{0.5}\times 0.4=5{,}250 \text{ pounds}$$

For convenience in rivet spacing, Table X will be found convenient, the value of any number of spaces of a given length being determined at a glance.

Bolts, Nuts, and Washers. Bolts are made by forming a head on one end and cutting a thread on the other end of an iron rod. In such cases the body of the bolt does not represent the strength, but the area at the root of the threads. In the handbooks is given the diameter of the screw thread for any bar or bolt of given diameter, and from this the strength of a bolt may be calculated, once the allowable unit tensile stress is determined. The diameter given for the rod or bolt is the diameter of the upset screw end. The strength of the bolt is then obtained by multiplying the diameter of the screw at root of thread by itself, by 0.7854,* and by the allowable unit stress, thus

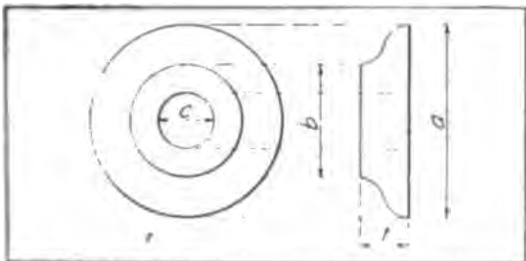
$$\text{Strength of Bolt}=0.7854\ d_1^2\times S$$

---

*In some books the area at root of thread is given direct.

TABLE XI

Standard Cast O. G. Washers



DIMENSIONS

$a = 4d + \frac{1}{2}''$

$b = 2d + \frac{1}{4}''$

$t = d$

$c = d + \frac{1}{8}''$

DIAMETER OF BOLT d	½	⅝	¾	⅞	1	1⅛	1¼	1⅜	1½	1¾	2
DIAMETER a	2½	2¾	3½	3¾	4¼	4¾	5¼	5¾	6¼	7	8½
WT OF 100 WASHERS IN Lbs	21	43	70	115	175	236	352	455	610	865	[illegible]

All dimensions in inches

For example, let it be required to determine the strength of a 1¼-inch bolt, the unit allowable stress in tension being 18,000 pounds per square inch. It is

$$\text{Strength of Bolt} = 0.7854 \times 1.284^2 \times 18{,}000$$
$$= 23{,}350 \text{ pounds}$$

while if the area of the body of the bolt was used the strength would be 31,800, from which it is seen that in determining the strength of bolts care must be taken to use the diameter at the root of the thread.

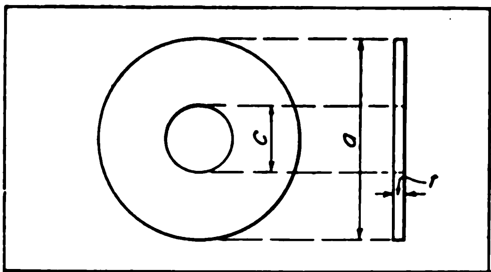
Information regarding bolts and nuts in general is given in the handbooks. Here the exact dimensions of the heads and nuts are given. In detailing it will be sufficiently accurate to assume the side of a square head or nut or the short diameter of a hexagonal head or nut as twice the diameter of the bolt, the thickness of each being equal to the diameter of the bolt.

When the nut is screwed up, the bolt should extend from ⅛ inch to ¼ inch above the nut.

Washers are of two kinds, cast and cut. The former are designated as O. G. (pronounced Oh Gee) washers on account of the curve given to their side. The sizes and weights of O. G. washers are given in Table XI.

Cut washers are made by stamping them out of sheet metal, and are principally used as separators where two angles are bolted together, or under the heads and nuts of small bolts which bolt timber in place. General information regarding them is given in Table XII.

TABLE XII
Standard Cut Washers
(In 200-pound kegs)



$a = 2d + \frac{1}{2}''$
$c = d + \frac{1}{16}''$ up to $d = 1''$
$= d + \frac{1}{8}''$ when d is greater than $1''$

SIZE OF BOLT OR UPSET = d	DIAMETER a	DIAMETER OF HOLE c	THICKNESS t	NO IN 100 POUNDS
3/16	9/16	1/4	3/64	36 200
1/4	3/4	5/16	1/16	4 900
5/16	7/8	3/8	"	11 100
3/8	1	7/16	5/64	6 700
7/16	1 1/4	1/2	"	4 100
1/2	1 3/8	9/16	7/64	2 600
9/16	1 1/2	5/8	"	2 700
5/8	1 3/4	11/16	9/64	1 300
3/4	2	13/16	"	1 000
7/8	2 1/4	15/16	5/32	728
1	2 1/2	1 1/16	5/32	595
1 1/8	2 3/4	1 1/4	"	507
1 1/4	3	1 3/8	"	428
1 3/8	3 1/4	1 1/2	11/64	328
1 1/2	3 1/2	1 5/8	"	284
1 5/8	3 3/4	1 3/4	11/64	248
1 3/4	4	1 7/8	"	218
1 7/8	4 1/4	2	"	194
2	4 1/2	2 1/8	"	173

All dimensions in inches

Tension Members. These may consist of square, round, or rectangular bars, or they may be of shapes riveted together. The latter class will be considered under the detailing of tension members.

When the bar is square or circular in section, it may be formed into loops at its ends, or upset and nuts put on, in order to attach it to other parts of the structure in which it is used. In the former case it is called a loop bar. In case it is rectangular in section it may be formed into a loop bar, or may have its ends forged out into a somewhat circular shape, see Fig. 46, and a hole bored in them in order to connect them to the rest of the structure. In this case it is called an eye bar.

In order to be assured that the eye bar will not break in the head, the distances *a* are made such that $2a$ is greater than w, usually between $1.3w$ and $1.4w$. If not required by the specifications, it is usually left to the manufacturers with the stipulation that the eye bars must break in the body of the bar, not in the head.

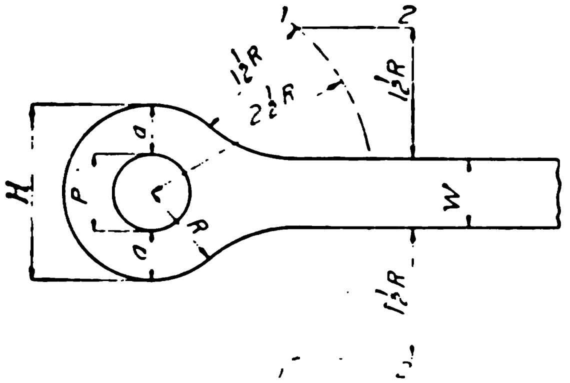
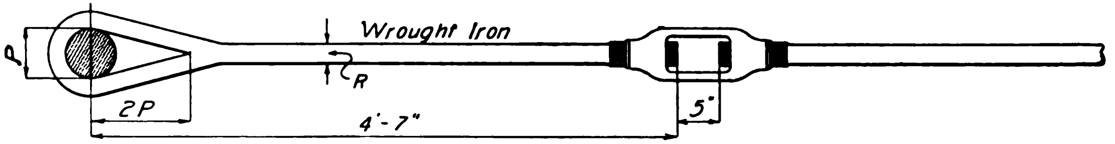


Fig 46 Dimensions Required in Eye-Bar Design

The dimensions of eye bars are given in the handbooks. In Cambria the excess through the pin hole for the 2-inch bar is $(4\frac{1}{2} - 1\frac{7}{8}) \div 2 = 1\frac{11}{16}$, an excess of 33 per cent.

Care should be taken to note that the values here given are the minimum thicknesses. Bars thinner than these are liable to upset so imperfectly as to be unsafe in the heads. An eye bar should not, as a rule, be less in thickness than one-sixth of the depth. The pins given in the tables in the handbooks are maximum pins. The

TABLE XIII
Loop Bars. Allowance for One Loop. American Bridge Company Standards
(All dimensions in inches)



Length in inches beyond pin center to form one eye equals 3.7(P+R)+5"

DIAM. OF PINS	DIAMETER OR SIDE OF BARS																			DIAM. OF PINS
	¾	⅞	1	1⅛	1¼	1⅜	1½	1⅝	1¾	1⅞	2	2⅛	2¼	2⅜	2½	2⅝	2¾	2⅞	3	
1	11½	12	12½																	1
1¼	12½	12⅞	13⅜	13⅞	14¼															1¼
1½	13⅜	13⅞	14¼	14¾	15¼	15⅝	16⅛													1½
1¾	14¼	14¾	15¼	15⅝	16⅛	16⅝	17	17½	18											1¾
2	15¼	15⅝	16⅛	16⅝	17	17½	18	18⅜	18⅞	19⅜	19¾									1¾
2¼	16⅛	16⅝	17	17½	18	18⅜	18⅞	19⅜	19¾	20¼	20¾	21⅛	21⅝							2
2½	17	17½	18	18⅜	18⅞	19⅜	19¾	20¼	20¾	21⅛	21⅝	22⅛	22⅝	23	23½					2¼
2¾	18	18⅜	18⅞	19⅜	19¾	20¼	20¾	21⅛	21⅝	22⅛	22⅝	23	23½	24	24⅜	24⅞	25⅜			2½
3	18⅞	19⅜	19¾	20¼	20¾	21⅛	21⅝	22⅛	22⅝	23	23½	24	24⅜	24⅞	25⅜	25⅞	26¼	26¾	27¼	2¾
3¼	19¾	20¼	20¾	21⅛	21⅝	22⅛	22⅝	23	23½	24	24⅜	24⅞	25⅜	25⅞	26¼	26¾	27¼	27⅝	28⅛	3
3½	20¾	21⅛	21⅝	22⅛	22⅝	23	23½	24	24⅜	24⅞	25⅜	25⅞	26¼	26¾	27¼	27⅝	28⅛	28⅝	29	3¼
3¾	21⅝	22⅛	22⅝	23	23½	24	24⅜	24⅞	25⅜	25⅞	26¼	26¾	27¼	27⅝	28⅛	28⅝	29	29½	30	3½
4	22⅝	23	23½	24	24⅜	24⅞	25⅜	25⅞	26¼	26¾	27¼	27⅝	28⅛	28⅝	29	29½	30	30½	31	3¾
4¼	23½	24	24⅜	24⅞	25⅜	25⅞	26¼	26¾	27¼	27⅝	28⅛	28⅝	29	29½	30	30½	31	31⅜	31⅞	4
4½	24⅜	24⅞	25⅜	25⅞	26¼	26¾	27¼	27⅝	28⅛	28⅝	29	29½	30	30½	31	31⅜	31⅞	32¼	32¾	4¼
4¾	25⅜	25⅞	26¼	26¾	27¼	27⅝	28⅛	28⅝	29	29½	30	30½	31	31⅜	31⅞	32¼	32¾	33¼	33⅝	4½
5	26¼	26¾	27¼	27⅝	28⅛	28⅝	29	29½	30	30½	31	31⅜	31⅞	32¼	32¾	33¼	33⅝	34⅛	34⅝	4¾
5¼	27¼	27⅝	28⅛	28⅝	29	29½	30	30½	31	31⅜	31⅞	32¼	32¾	33¼	33⅝	34⅛	34⅝	35	35½	5
5½	28⅛	28⅝	29	29½	30	30½	31	31⅜	31⅞	32¼	32¾	33¼	33⅝	34⅛	34⅝	35	35½	36	36½	5¼
5¾	29	29½	30	30½	31	31⅜	31⅞	32¼	32¾	33¼	33⅝	34⅛	34⅝	35	35	36	36½	37	37⅜	5½
6	30	30½	31	31⅜	31⅞	32¼	32¾	33¼	33⅝	34⅛	34⅝	35	35½	36	36½	37	37⅜	37⅞	38¼	6

NOTE: Maximum shipping length should not exceed 35 feet   All dimensions in inches

American Bridge Company practice requires the smallest pin to be not less than three-fourths the width of the eye bar.

Bars of a square or circular section could, as in the case of bolts, have a screw thread cut on their ends and by means of nuts be connected to the other part of the structure, but such an operation would be costly since the bars are long and much of the section would be wasted for a great length. In such cases the bars are ordered 6 inches longer than required and this 6 inches is, after heating to a welding heat, upset or pushed in 6 inches, thus increasing the diameter of the bar at the end so that the diameter at the bottom of the screw threads will be greater than the diameter of the original bar. This is done so that the bar will break in the body, and not at the joint.

The sizes of upsets for bars of various sizes are given in the handbooks. Let it be required to determine the size hole through which a 1⅛-inch bar with upset end would pass and the nut required. We find opposite the 1⅛ the value 1½, showing that the upset will be 1½ inches. In another table opposite 1½ is given the size and weight of a square nut, viz, 1½ inches thick, 3 inches on the side, and weight 3.175 pounds. The use of square nuts is not to be encouraged, the hexagonal form being the better, on account of their lighter weight.

Instead of the rods being fitted with nuts and threads at their ends, they may, as mentioned above, be made into loop bars. Loop bars are welded, and for this reason are not to be desired since welds are never as strong as the original. However, the loop bar has 100 per cent excess through the pin, and in order to have an efficiency of 100 per cent it must have a weld with an efficiency of 50 per cent. Since such a weld is well within the limits of possibility, it is permissible to use loop bars in highway bridges or other structures where the impact is not great, and in counters, since here the pins are usually of such a diameter that they would be too great for an eye bar of the section of the counter. Table XIII gives information regarding loop bars. They must be made of wrought iron since steel does not weld well.

Clearances. It is very important that each member of a structure fit together well in the field; and it is equally important that the draftsman should so detail his work that the various parts of any particular member should, without further cutting than the first, fit together. Also the rivets should be so spaced and placed that they can be driven.

The rivet clearances have been mentioned under "Rivets and Rivet Spacing" and will not be taken up here. It is sufficient to say that on the rivet clearances is where the novice makes the most of his mistakes.

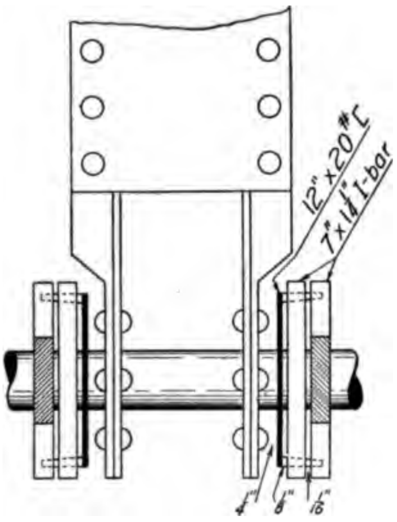


Fig. 47. Joint Showing Clearance between Members

Where the distance between the outer faces of several members placed together is to be computed, it is necessary, on account of the liability of plates to exceed their nominal thicknesses, and rivet heads their nominal height, to make certain allowances. The usual practice is:

(1) Between eye (or loop) bars allow 1/16 inch.

(2) Between an eye(or loop) bar and a built-up member ⅛ inch.

(3) Between two built-up members ¼ inch.

For example, suppose it was required to compute the distance out to out of the members shown in Fig. 47. The clearance would be as indicated, and the distance D would be:

$$D=2\left(\frac{10}{2}+0.4+\tfrac{5}{8}+\tfrac{1}{4}+0.28+\tfrac{1}{8}+1\tfrac{1}{4}+\frac{1}{16}+1\tfrac{1}{4}\right)$$
$$=18.485=18\tfrac{1}{2}\text{ inches}$$

This value would be the grip of the pin which was used at this joint. The 0.4 inch and 0.28 inch in the above are the thicknesses of the channel webs, and the ⅝ inch is the height of a ⅞ inch rivet head.

In the use of eye bars, it is essential to see that their heads as well as their bodies clear. In order to determine the dimension of a section for the necessary clearance, the size of the head must be ascertained. This is best done by drawing up the head to a large scale. The method of procedure is as follows: (1) Draw the circle representing the pinhole; (2) for the width of eye bar under consideration, subtract the radius of the largest pinhole in Cambria for that bar from the radius of the given head and add the result to one-half the pinhole diameter in your particular case, thus giving you *R*, Fig. 46; (3) with the radius *R* describe a full circle; (4) with the center of the pin as a center and a radius equal to 2½ *R* describe a couple of arcs 1, 1; (5) parallel to the bar and at a distance 1½ *R* from it, draw two lines, 2, 2, intersecting the arcs 1, 1; and (6) with these intersections as centers and a radius equal to 1½ *R* describe the small arcs completing the head, see Fig. 46.

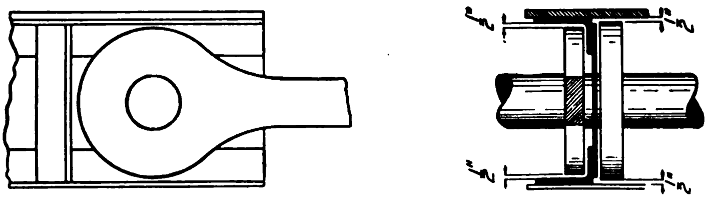


Fig. 48. Eye-Bar and Built-Up Member Showing Clearance Allowed

No material should be closer to the edge of the eye-bar head than ½ inch. This clearance should always be given, see Fig. 48, although the clearance of ⅛ or $\frac{1}{16}$ on the side should be allowed as usual in case it was against a built-up member or another eye bar.

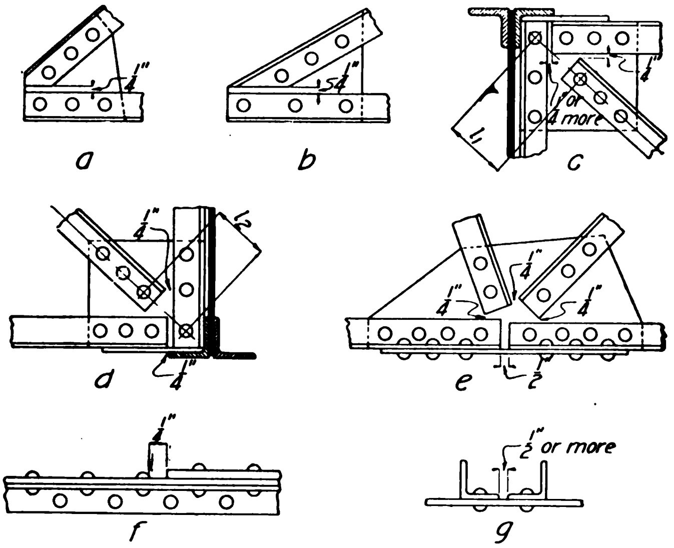


Fig. 49. Riveted Joints Showing Clearance Allowed

In case the head is on the interior of a channel or so as to come near the fillet of an angle, the ½ inch must be measured from the curve of the fillet. This ½ inch does not apply to the body of the bar, the clearance there being ¼ inch in accordance with what follows.

Wherever several pieces of metal are riveted to the same side of a plate or other member and could, theoretically, come close against each other, ¼-inch clearance is allowed for each case where the ends are not planed. This allows for the slight variations in length liable to occur when the surfaces are sheared. The members will then be sufficiently close together for all practical purposes. In order that no errors occur, the joint should be drawn up on a separate sheet to a scale of at least 1½ or 2 inches to the foot in case the pieces meet at an angle. In case the pieces meet at right angles, the distances may be computed. Fig. 49 gives a few of the most common cases.

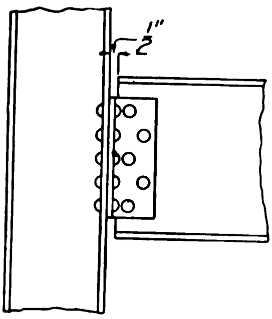


Fig. 50. Column and Beam Connection Showing Clearance

As in the case of Fig. 49c and 49d the clearances at one end will be ¼ inch and at the other end may be more, and should be, in order that the distances l_1 and l_2 shall be the same. (The distance from the first rivet to the end of the angle is usually 1¼ or 1½, generally the latter.) It must not be understood that the clearance is exactly ¼ inch; it must be at least ¼ inch, and may be more, up to ⅜ inch or ½ inch in order that the distance from the rivet to some other point or rivet may be in an even $\frac{1}{16}$ inch or ⅛ inch.

When I-beams or channels are placed as mentioned above, ½-inch clearance or more instead of the ¼-inch is required, one of the most common cases where such clearance is required being shown in Fig. 50. For other clearances in beams see "The Detailing of Beams," page 72.

Wherever bolts, rods, upsets, or rolled bars pass through a hole or slot, the aperture should be ⅛ inch greater in diameter or ¼ inch greater in dimensions in case there is a slot. The above is in case the material is rolled steel or iron. In case of a casting, ¼ inch should be added to the dimensions of the member which is to pass through the opening.

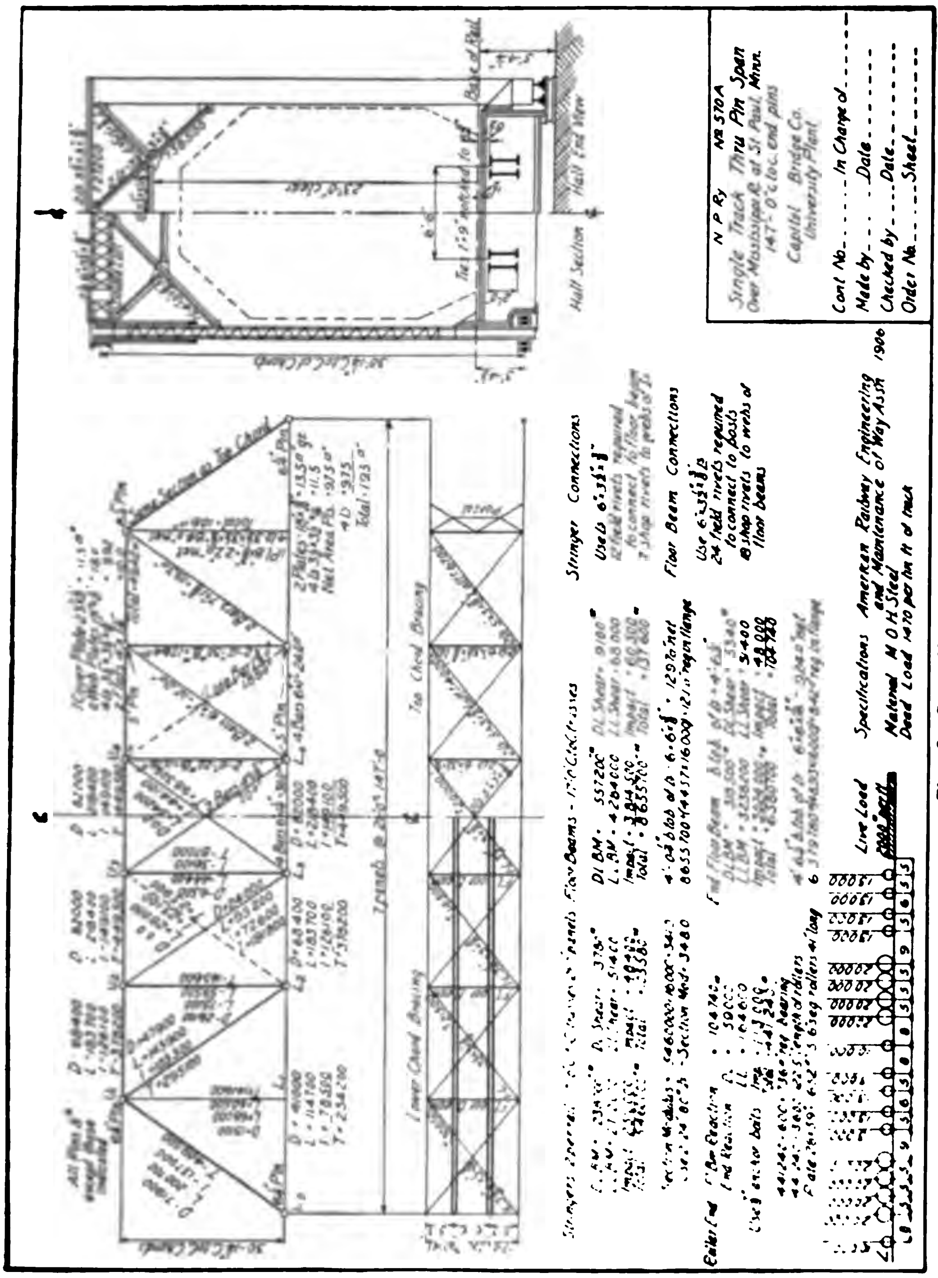
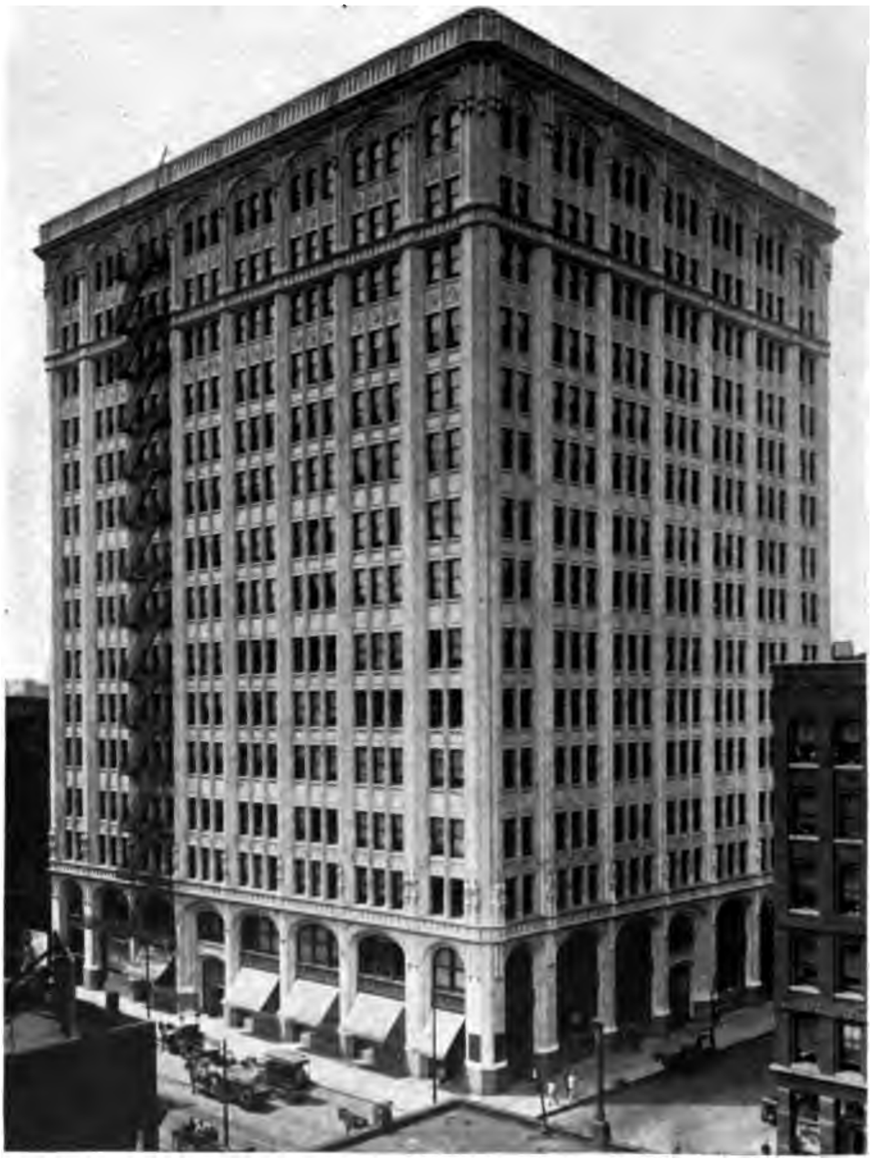


Plate I. Stress Sheet of Truss Bridge



BURLINGTON BUILDING, CORNER CLINTON AND JACKSON STREETS, CHICAGO
Marshall and Fox, Architects

STRUCTURAL DRAFTING

PART II

DETAILING METHODS

Detailing of Angles. The line upon which the rivets are spaced is the gauge line. The standard gauges given in Table VII should not be departed from unless instructions are given otherwise or unless it is impossible to make the detail without doing so, in which cases a "special" gauge is used. In deciding upon a special gauge care must be taken to see that the gauge line is not less than the standard edge and clearance distances for the rivet used. In Fig. 51 is shown the minimum values of these distances.

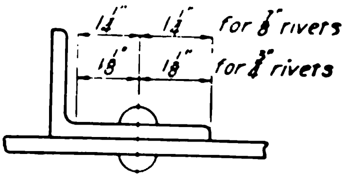


Fig. 51. Minimum Clearance Distances in Angle Detailing

In some cases, as in the flange of plate girders, the rivet spacing is determinate, but in the majority of cases this is not so. In the former case the spacing between certain limits should be made equal to that at the lower limit. It is unwise and not economical to change the spacing every few feet. When the spacing is not determined, the rivets may be placed as desired, the only limitations being (1) that they can be driven, and (2) that they do not take out too much section, providing that the angle is in tension. Of course the limitations as to maximum and minimum spacings apply here, the spacing, being used from 3 inches to 4 or 4½ inches for ¾-inch or ⅞-inch rivets, the lower limit being used if possible in order to keep down the size of the connection plates. It is economical to make all the spaces equal. The spacing may be governed by the desire to have the connection plate symmetrical. The distance from the end of the angle to the first rivet is usually 1¼ inches for ¾-inch rivets, but 2 inches is sometimes, though seldom, used. In case two angles are used as tension members, they may be riveted together at distances not greater than 12 to 18 inches.

The gauge line in single gauge angles is used as the working line, and passes through the working points. In the case of a double-gauge line, the inner gauge line should be used as the line of reference since by so doing the stresses in the angle will be less than if the reference line was taken midway between the two lines.

Angles in either tension or compression should be connected by both legs, otherwise the stresses due to eccentricity will cause the total stresses to be far above the average stress, as a usual case 100 per cent. This should be done by a "clip" angle, and as many rivets should go from the angle into the clip as go from the clip into the connection plate. As a usual thing it is not necessary to detail the vertical leg of the clip, as the shopmen will attend to it; only show the heads of the rivets. Figs. 50 and 52 illustrate the principles mentioned above.

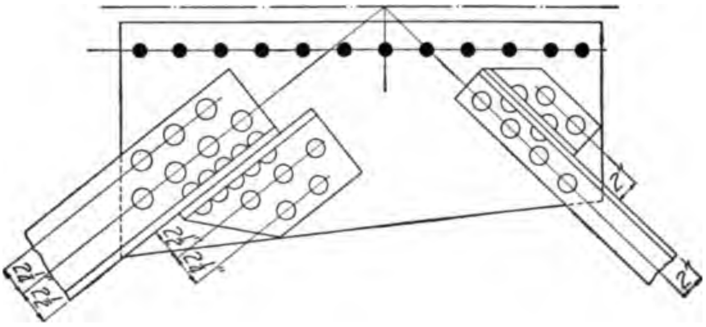


Fig. 52. Angle Detailing—Vertical Leg Not Shown

The gauge should always be placed on the angle whether it is standard or special, but do not give the distances from the gauge to the edge of the leg.

In detailing diagonals, the end distances to the working points should be so chosen that the length of the angle will end in an eighth of an inch. If anything, make the angle a little short center to center of end holes to accomplish this. The member can easily be drawn up in place by the moderate use of drift pins, this making it taut when riveted up in place.

Detailing of Plates. The governing features of the detailing of plates are: (1) To keep the plate as small as can consistently be done; (2) to cut it as few times as possible in getting it into the final shape; (3) to have it symmetrical if possible; (4) to keep it an even number of inches in width; (5) to make it thick enough so that the number of rivets will be small and, therefore, the plate also; and (6) to detail it so that the rivet centers may be determined quickly and with certainty.

The plate is kept as small as possible for economical reasons, and for the same reason it should be cut as few times as possible, two cuts being the maximum and the desired number. It is usually more economical to leave the material on the plate than to trim it up. Therefore, it is important that the rivet spacing be so arranged that this can be done. In such cases the company is not only saved the labor of trimming the plate into some irregular shape, but it gets paid for the extra weight left on. Fig. 53a illustrates a plate poorly detailed and Fig. 53b, one well detailed. In the latter the plate, although somewhat larger, is of rectangular shape, and most engineers would prefer it to the other.

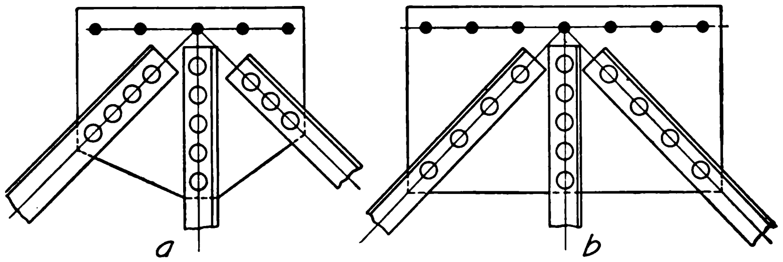


Fig. 53. Methods of Detailing Angles. (a) Poor Detailing. (b) Good Detailing

It is not always possible to have a plate symmetrical. When a plate is symmetrical, the templet work, and, therefore, the cost, is much reduced. If they cannot be made symmetrical, the next best thing is to have as many as possible alike or of the same width. With a little thought along these lines a draftsman can save his week's wages for his company many a day.

For economy's sake the plate should be in an even number of inches in width. Plates are not rolled in fractions of an inch except below 6 inches. If a plate is so detailed as to require a fraction of an inch in width, the next wider plate must be ordered and cut down to the required size, thus causing expense due to metal wasted and to labor required to get it cut to size. The length may be any dimension, but it is best to have it in even eighths of an inch, thus 2′-8½″ or 3′-5⅞″, etc. As mentioned before, the width of the plate must be stated in inches, not feet and inches. A plate is noted thus 1-Pl. 18″×⅜″×2′-5⅛″ or 1-Pl. 25″×⅜″×3′-8¼″.

If the stress in a member is great, the number of rivets will necessarily be large. In such cases the thickness of the plate may be made thicker than ⅜ inch in railroad, or 5/16 inch in highway or building work, and the member so arranged as to bring the rivets in double shear. Unless the rivets can be brought in double shear it is unnecessary to increase the thickness of the plate, for shear governs in case of a ⅜-inch plate, and in case of a 5/16-inch plate the change to a ⅜-inch plate does not reduce the number of rivets sufficiently to warrant it.

A plate must also be of sufficient section to transmit the stress from one member to the other. Since the area between the rivets is greater than that bearing upon the rivet, this is automatically attended to. In the case of shear along planes between members a different condition obtains. Here the computations are more or less involved, but the draftsman need not consider this phase of the design since in such cases the experienced designer will design the plate and it will come to him with the correct thickness marked upon it.

The rectangular detailing of plates is, of course, simple. The rivets are, except in such cases as in the webs of plate girders, spaced after the manner of the spacing in angles, the same conditions governing. The lines on which the rivets are placed are in such cases parallel to one of the sides of the plate and the spacing is readily laid off.

When a diagonal row of rivets is on a plate, it may be detailed in two ways: (1) By rectangular co-ordinates; and (2) by spacing along a line located by a bevel. Only in exceptional cases is the first method, shown in Fig. 54a, to be used.

The better way, and indeed it might be said the standard way, is shown in Fig. 54b. This way is easier for all concerned, and, in case of diagonals, lends itself especially well, since the distance between working points of the plates at the ends can be computed, and the rivet spacing being measured along this line gives the distance between the last holes on the plates by simple subtraction.

A plate should be detailed from one edge (the working edge) and the working point. The various distances to the rivet holes are measured from these places of reference. The distance from the last rivet hole to the far or side edges is not given. The plate of course being originally laid out to a large scale—a layout—care is taken that if the distances measured out from the working edge or point are used, the last rivet will not come closer to or farther from the far side of the plate than is allowed by specifications. In the case of the $\frac{7}{8}$-inch rivet, these limits are $1\frac{1}{2}$ inches for the smaller and 2 inches for the greater. Specifications govern this distance by making it a function of the thickness of the plate or of the diameter of the rivet; one specification requires 2 diameters of the rivet for the least and 8 times the thickness of the plate for the maximum, but not to exceed 6 inches. As a usual thing, engineers desire the distance to be $1\frac{1}{2}$ inches for $\frac{7}{8}$-inch rivets, both limits being the same, and $1\frac{1}{4}$ inches for $\frac{3}{4}$-inch rivets.

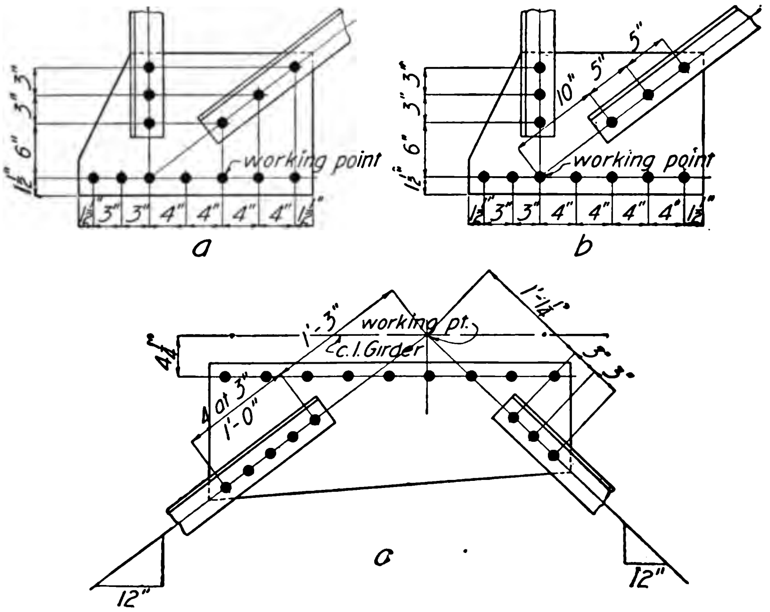


Fig. 54. Methods of Detailing Diagonal Row of Rivets. (a) Poor. (b) Good. (c) Example Where Working Point is Not on the Plate

The working point may or may not be on the plate. In many cases it is not. However, the distances must be measured from the working point. A familiar example of this is seen in the connection plates of the lateral systems of plate girders, see Fig. 54c.

In indicating the bevel, two methods are used. One is to state the actual rectangular dimensions, and the other is to reduce them so that the larger is 12 inches and the other a proportional part. For example, in Fig. 55 are given four working points and the lines connecting them. The plates are shown in outline. The bevel may be represented by the full dimensions or by taking the longest side as 12 inches and the other as 9.41 inches, or $9\frac{7}{16}$ inches, since it is unnecessary to get the bevel closer than the nearest sixteenth. The value 9.41 is computed as follows:

$$\frac{Y}{12''} = \frac{6'\text{-}6''}{8'\text{-}3\frac{1}{2}''}$$
$$Y = 9.41''$$

The method of indicating the bevel in feet and inches is much used, but indicating it in inches is preferable, since it is suitable to bench work. With the foot-and-inch method the floor of the templet shop has to be used in order to lay it out. The smaller values are advised for all work which can be worked on a bench.

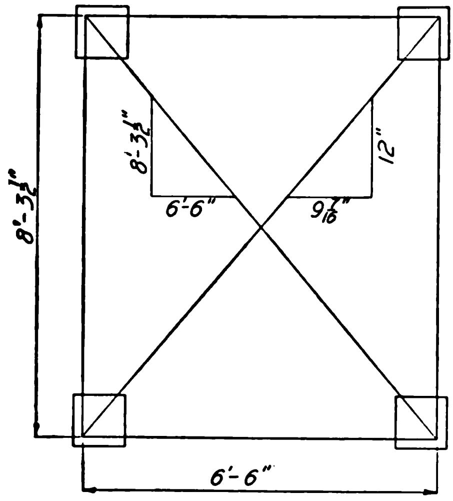


Fig. 55. Method of Indicating the Bevel

The number of rivets required in the connection plate in any direction must be sufficient to withstand the component of the main member attached to the plate. This can be easily determined by projecting the number of rivets in the diagonal against the line where the required number is to be placed. For example, let it be required to determine the number of rivets in the top of the plate in Fig. 56a, there being as shown 4 rivets in one diagonal and 3 in the other. Draw a diagram, as Fig. 56b, making *o*-1 and *o*-2 the same in slant as the diagonals above in 56a. Now with any scale whatsoever lay off four divisions from *o* towards 1 and three divisions from *o* towards 2 and project a line up from the last division mark to the horizontal line. Now measure *o-a* and *o-b* to the same scale which was used to lay off the divisions on the diagonal lines. There results 3.1 and 2.3 which means that 3.1 rivets are required for S_1 and 2.3 for S_2, or a total of 5.4 or 6 rivets for both.

It may be that the problem is as is shown in Fig. 56c. In this case the method of procedure is similar. Here, after drawing *o*-1 to the same slant as the member above, seven spaces to any scale are laid off and the projection made to the top and side. The results show that 6 rivets are required at the top and 3.2, or 4, are required at the side.

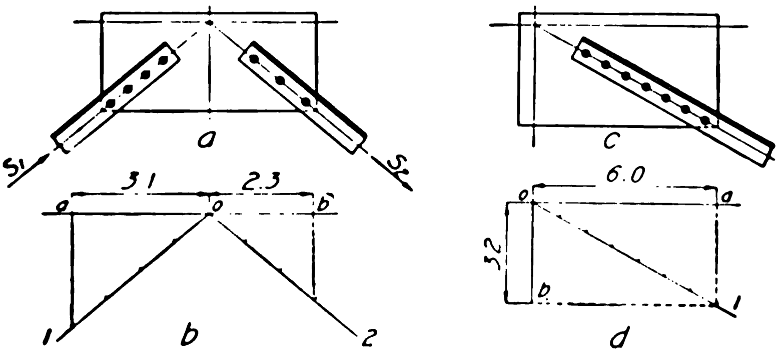


Fig. 56. Methods of Determining Number of Rivets in Connection Plate from the Diagonal

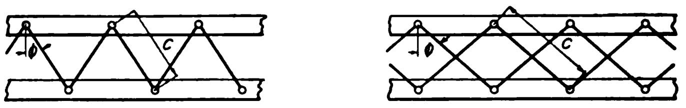
Other problems may be solved in a similar manner. The rivet spacing in the sides and tops is so arranged as to be equal and to fill out the plate, allowing the required edge distance. The plates should be kept rectangular as far as possible.

In many cases, as in roof truss or wind bracing work, the computed number of rivets will be two or less. In such cases three rivets should be put in in order to have a satisfactory joint which will not loosen under vibrations which are liable to occur.

Detailing of Combinations of Structural Shapes. The general methods to be followed are the same as those which have been given together with those which are exemplified in the discussions which

TABLE XIV

Thickness of Lacing Bars



Single Lacing ($t = \frac{c}{40}$, $\phi = 30°$)		Double Lacing ($t = \frac{c}{60}$; $\phi = 45°$)	
t	c	t	c
1/4"	0'-10"	1/4"	1'-3"
5/16"	1'-0½"	5/16"	1'-6¾"
3/8"	1'-3"	3/8"	1'-10½"
7/16"	1'-5½"	7/16"	2'-2¼"
1/2"	1'-8"	1/2"	2'-6"
9/16"	1'-10½"	9/16"	2'-9¾"
5/8"	2'-1"	5/8"	3'-1½"

follow. In general, the combinations consist of plates or other shapes held together by angles, lacing bars, or tie plates, the size and section of the angles being determined in the design since they are part of the section of the member itself, while the lattice bars and tie or batten plates are chosen in accordance with the specifications employed. The specifications for lacing bars make their size a function of the distance between rivets. Table XIV gives the thickness of lacing bars for any distance between rivets.

Detailing of Beams. This is for the most part done on "Beam Sheets". These sheets are the size of the shop bills, 8½×14 inches, and have a printed heading and footing as on the shop bills. Between the heading and the footing are printed elevations and cross-sections of I-beams, as in Fig. 57, the number on a sheet varying with the number of dimension lines above and material below, *i. e.*, from two to four. In some cases, those blank sketches are printed lengthwise of the sheet and then two only are placed upon a sheet. In case a channel is to be indicated, the draftsman blocks out one half of the section or end view, see Fig. 58, lower cut.

On these blank sketches the draftsman notes the rivets and rivet holes, puts on the connection and other angles, and shows all other information necessary for the complete fabrication of the beam ready for the structure of which it is a part. Figs. 58, 59, 60, and 61 are beam sheets which have been filled in, and illustrate very nicely the general principles.

The general rules regarding beam sketches are given in the following:

In all possible cases the holes in the end connections to the webs should be according to the standards given in the handbooks. If the connection is standard for that beam, no mention need be made of the fact, and if it is in the center of the web, no dimension is required, see Fig. 58, first view, left end.

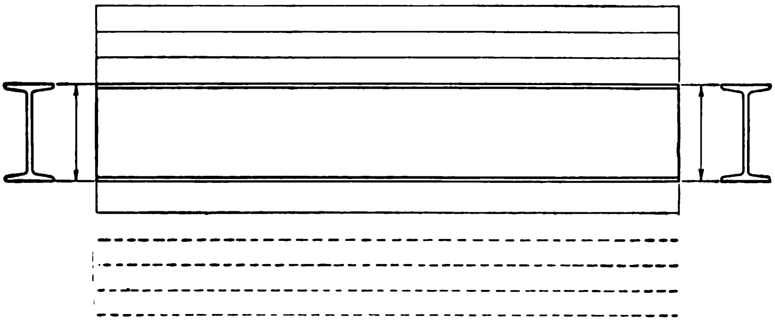
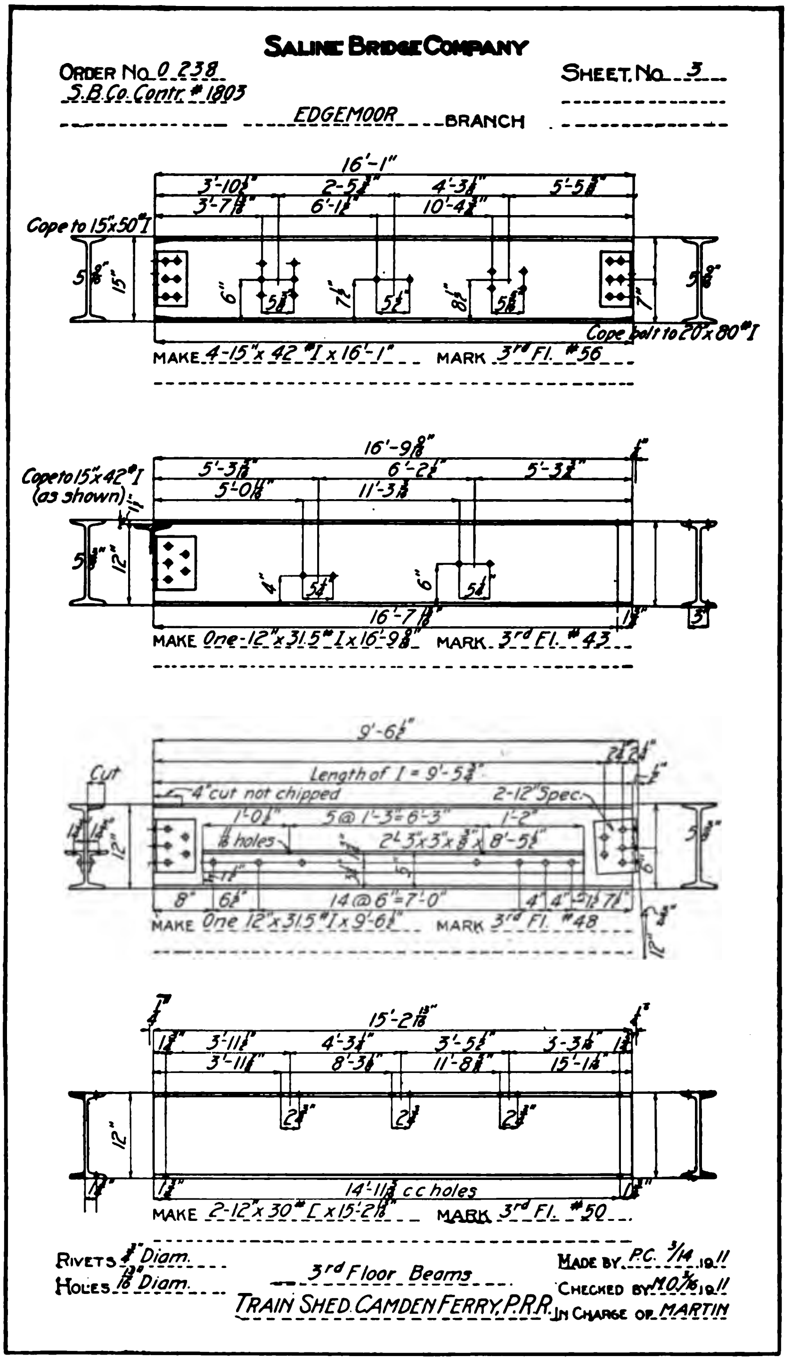


Fig. 57. Method of Detailing an I-Beam on Beam Sheets

If the connection is not in the middle of the web, but it is standard, the location of its center from the bottom should be given, see Fig. 58, first view, right end.

If the connection is not standard, it must be noted and detailed as in Fig. 59, second view, and if it were not in the center of the web, its distance from the bottom should be given as in the case of the standard connection. In case there are holes in the outstanding leg, they should be shown as in Fig. 62. Where the leg against the web is standard and the outstanding leg is of the same punching, no dimensions need be shown, but the outstanding leg must be shown and the material notation of the angle put on as in Fig. 59, first view, right end.



SALINE BRIDGE COMPANY

ORDER No. 0 238

S.B.Co. Contr. # 1803

SHEET No. 3

EDGEMOOR BRANCH

MAKE 4-15"x 42#I x 16'-1" MARK 3rd Fl. #56

MAKE One-12"x 31.5#I x 16'-9 3/8" MARK 3rd Fl. #43

MAKE One 12"x 31.5#I x 9'-6 1/2" MARK 3rd Fl. #48

MAKE 2-12"x 30# [x 15'-2 13/16" MARK 3rd Fl. #50

RIVETS 3/4" Diam.

HOLES 13/16" Diam.

3rd Floor Beams

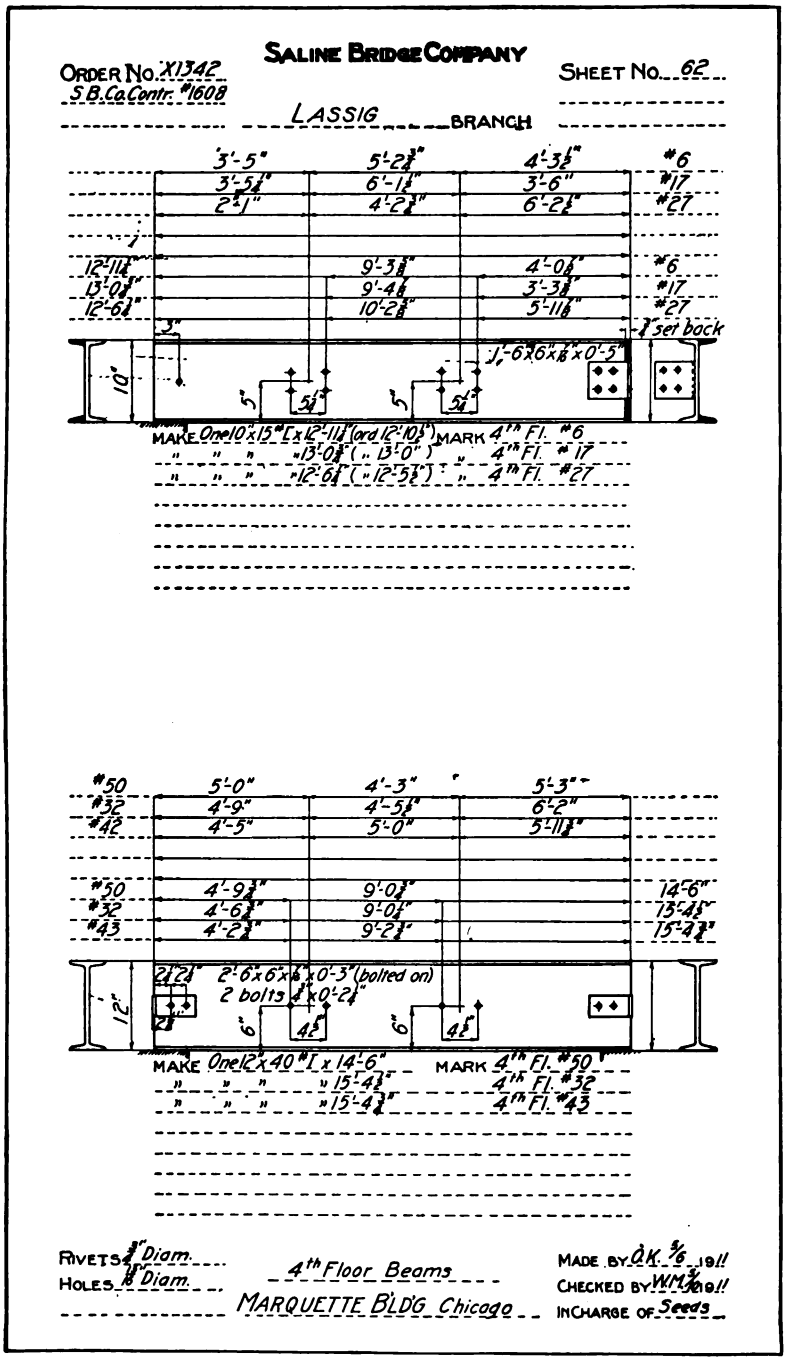
TRAIN SHED. CAMDEN FERRY, P.R.R.

MADE BY P.C. 3/14 1911

CHECKED BY M.O. 3/16 1911

IN CHARGE OF MARTIN

Fig. 58. Typical Beam Sheet Showing Dimensions Filled in According to Specifications



SALINE BRIDGE COMPANY

ORDER NO. X1342

S.B.Co.Contr. #1608

SHEET NO. 62

LASSIG BRANCH

MAKE One 10"×15#[×12'-11¼" (ord 12'-10½") MARK 4th Fl. #6
" " " "13'-0⅞" (" 13'-0") " 4th Fl. #17
" " " "12'-6¼" (" 12'-5½") " 4th Fl. #27

MAKE One 12"×40#I × 14'-6" MARK 4th Fl. #50
" " " " 15'-4½" 4th Fl. #32
" " " " 15'-4¾" 4th Fl. #43

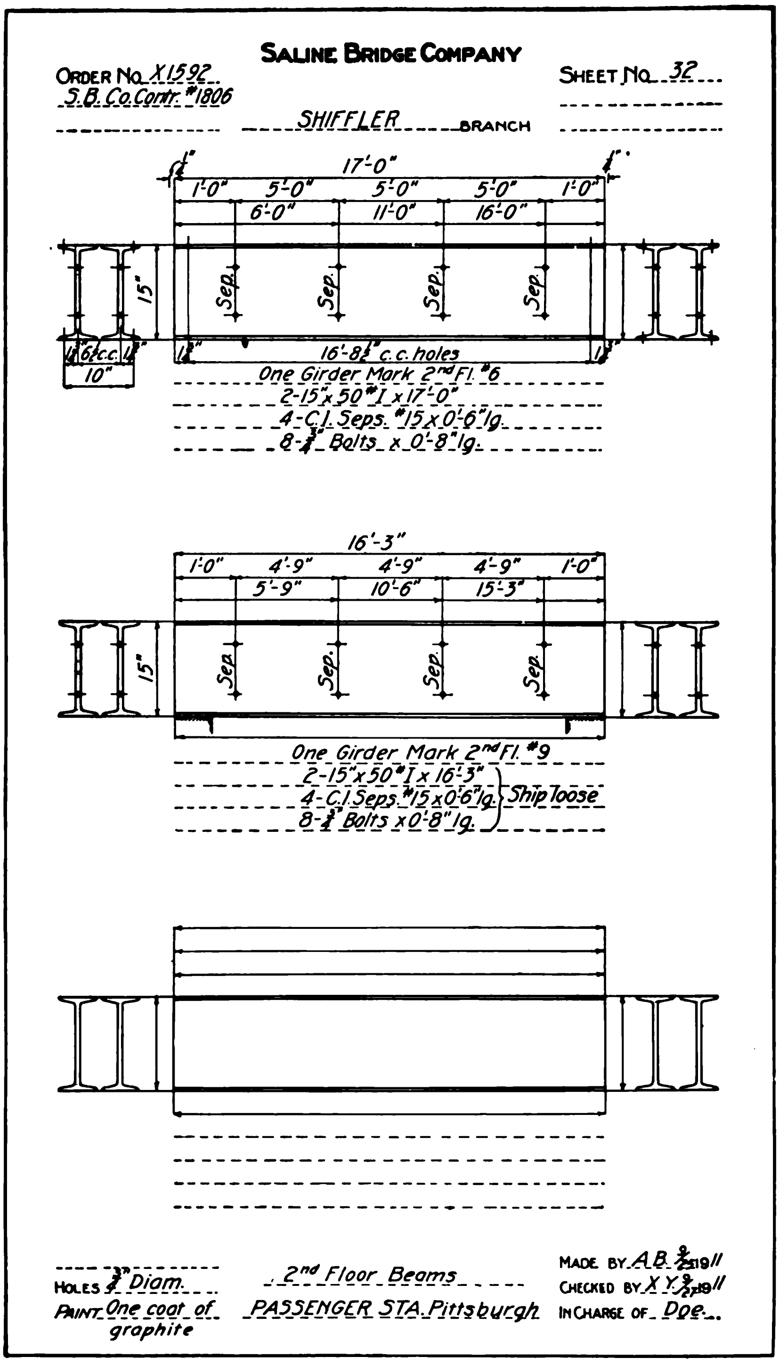
RIVETS ¾" Diam.
HOLES 13/16" Diam.

4th Floor Beams

MARQUETTE B'LD'G Chicago

MADE BY O.K. 5/6 1911
CHECKED BY W.M. 5/6 1911
IN CHARGE OF Seeds

Fig. 59. Typical Beam Sheet Showing Dimensions Filled in According to Specifications



SALINE BRIDGE COMPANY

ORDER No. X1592
S.B. Co. Contr. #1806

SHEET No. 32

SHIFFLER BRANCH

One Girder Mark 2nd Fl. #6
2-15"x50# I x 17'-0"
4-C.I. Seps. #15 x 0'-6" lg.
8-¾" Bolts x 0'-8" lg.

One Girder Mark 2nd Fl. #9
2-15"x50# I x 16'-3"
4-C.I. Seps. #15 x 0'-6" lg.
8-¾" Bolts x 0'-8" lg.
Ship loose

HOLES ¾" Diam.
PAINT One coat of graphite

2nd Floor Beams
PASSENGER STA. Pittsburgh

MADE BY A.B. 9/25 1911
CHECKED BY X.Y. 9/27 1911
IN CHARGE OF Doe.

Fig. 60. Typical Beam Sheet Showing Dimensions Filled in According to Specifications

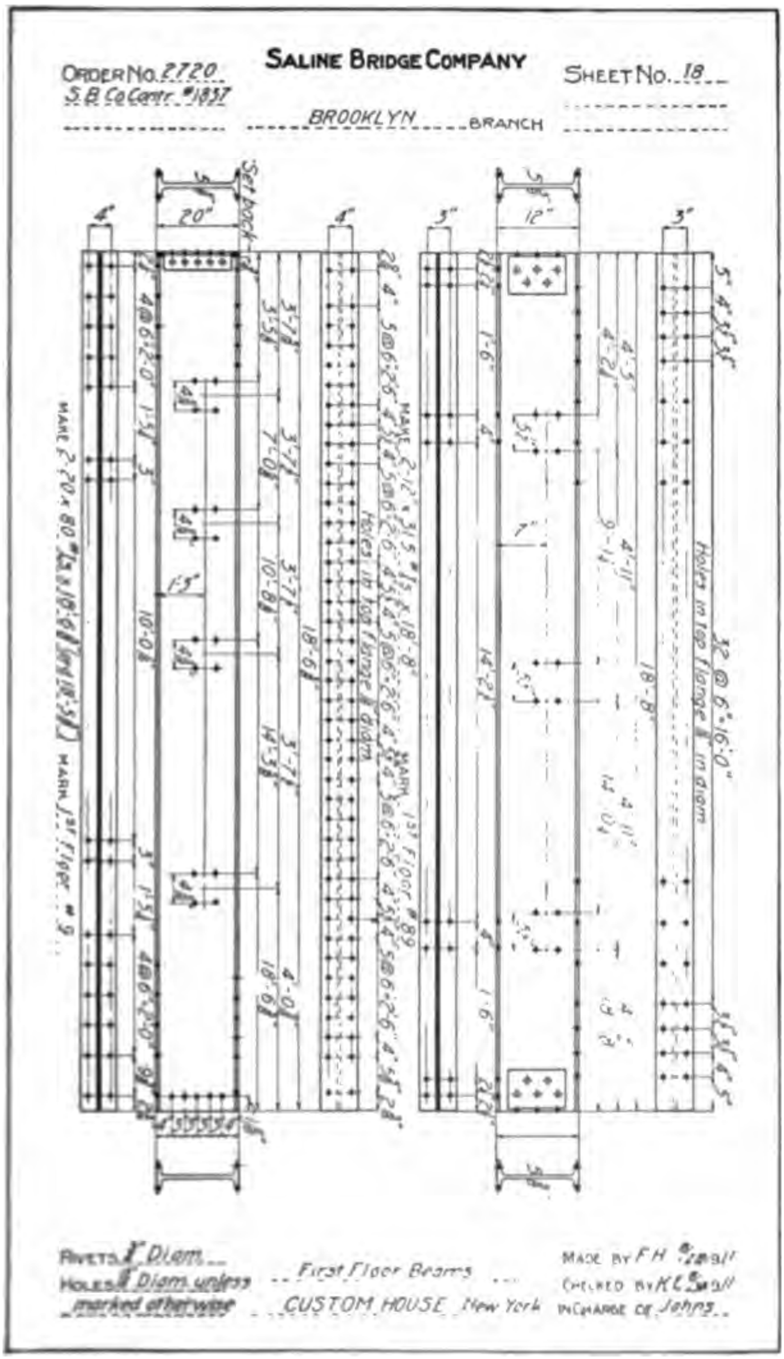


Fig. 61. Typical Beam Sheet Showing Dimensions Filled in According to Specifications

When beams are on a slight bevel, it is desirable to have the bevel taken up in the connection angles and the holes in the web of the beam at right angles to the center line. The bevel should be indicated, see third view, Fig. 58, right end.

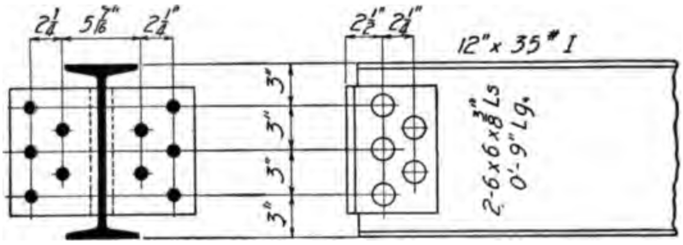


Fig. 62. Detailing Connection P ate When There Are Holes in the Outstanding Leg

In case field connections to the web are made, as in cases where other beams are riveted to it, it is unnecessary to give the vertical spacing of the holes if the connection is standard. The horizontal distances and their number will designate which connection is required.

For example, in the first view, Fig. 58, the six holes 5⅜-inch centers show this to be a standard for 12-inch beams, while the four holes 5$\frac{7}{16}$-inch centers indicate the standard connection for a 7-inch, 8-inch, 9-inch, or 10-inch beam. In all cases the vertical spacing will be 2½ inches. It should be noted that in all cases of standard connections of 8 holes or less in a vertical row the rivet spacing is 2½ inches, while all over 8 have a spacing of 3 inches.

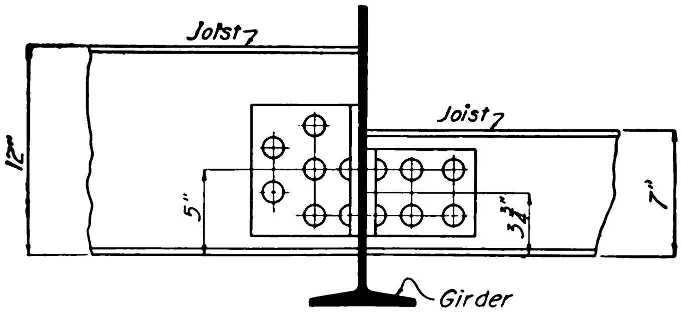


Fig. 63. Method of Bringing Beams to the Same Level on Main Girders

The centers of all groups of field holes above the bottom of the beam should be given.

Tie rods are put in in case no beams are riveted to the webs, to keep the beams from lateral motion. The holes for these are 4½ inches apart, and they are referenced as in Fig. 59, second view.

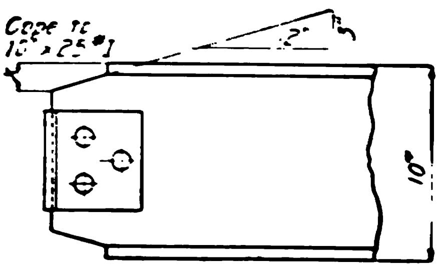


Fig. 64. Method of Coping a Beam Top and Bottom

Where two beams are placed close together, they should be connected by "separators" to prevent lateral motion. When such is the case the holes are indicated as shown in Fig. 60. The various kinds of wall anchors are shown in the handbooks and in Fig. 20. Care should be taken to provide for their connection to the beams when required.

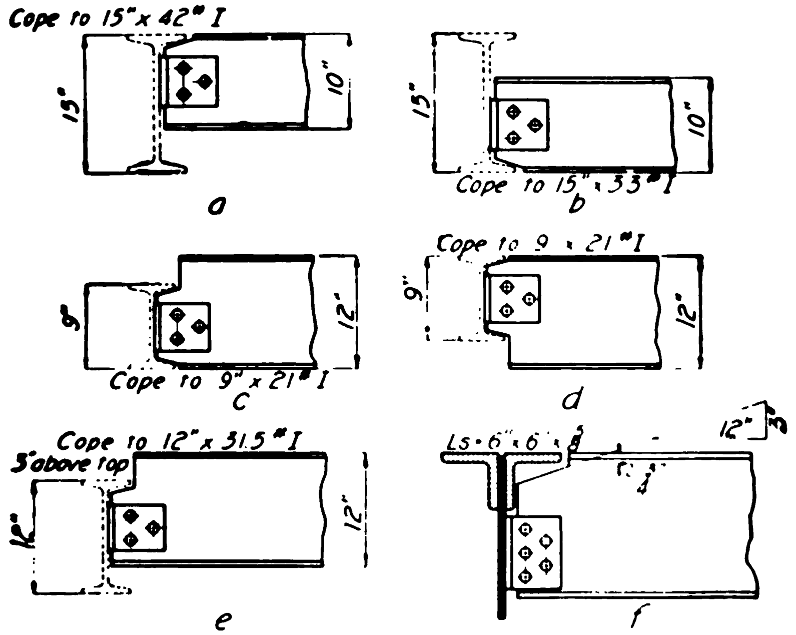
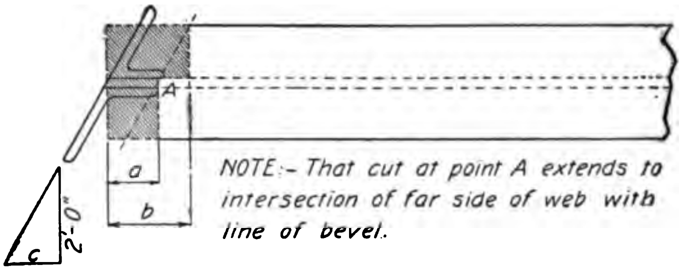


Fig. 65 Methods of Coping Beams to Fit Beams of Various Heights

When beams are used in building work, it is usually required that either the upper or the lower flanges of part or all of the beams be at the same elevation. When the girder or main beams are deep enough, the joist top or bottom flanges may be brought to the same elevation as shown in Fig. 63 which shows a 12-inch and a 7-inch beam. The connection *angles are in all cases arranged so that the rivets through the girder web and the smaller connection angles go through the connection of the larger beam also.*

In case it is desirable to have beams so as to have all their tops or bottoms at the same elevation, it may be accomplished by an operation known as "coping" the beam. By coping is meant that the flange is cut back for a certain distance depending on the size of the beam which is to join the beam under consideration and the web is then cut down a distance X and sloped back on a bevel of 3 inches in 12 inches, see Fig. 64.



NOTE:- That cut at point A extends to intersection of far side of web with line of bevel.

Fig. 66. Method of Cutting Flanges When a Beam is Coped on a Bevel

Fig. 64 shows a beam coped top and bottom to fit into another beam of its own depth. A beam may be coped on top only, Fig. 65a, or on bottom only, Fig. 65b. Other conditions of coping are shown in Fig. 65c—f, together with the ways of indicating them. Fig. 58 shows some indicated in the beam sketches.

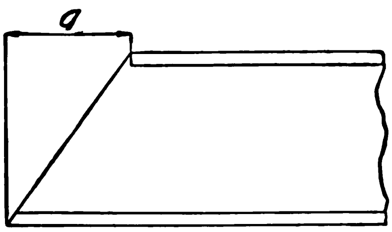
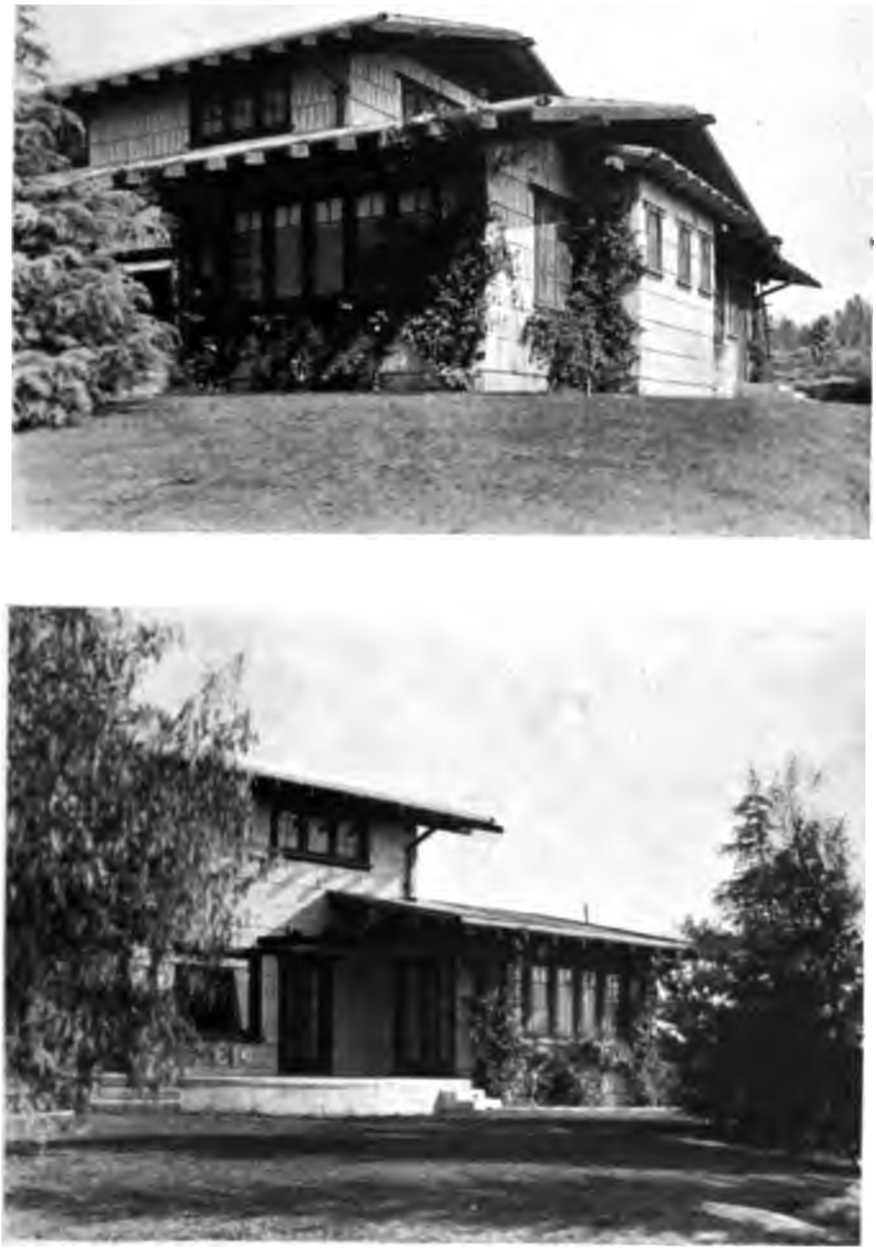


Fig. 67. Method of Cutting an I-Beam or Channel to a Bevel

When a beam is to be coped on a bevel, the flanges are not cut to a bevel, but are cut as in Fig. 66. The distances a and b should be given allowing a $\frac{1}{4}$-inch clearance, and the portion of the beam coped is to be shown cross-hatched. This method of cutting to a bevel should be used whenever possible, whether the beam is coped to fit another or is simply cut to a bevel.

When a single beam or a girder formed of two beams having a cover plate riveted thereto is cut to a bevel, the cover plate should be sheared to the line of bevel and the beam should be cut as shown in Fig. 66.



TWO EXTERIOR VIEWS OF W. A. CHILDS RESIDENCE AT RIVERSIDE, CALIFORNIA
George A. Clark, Architect, Pasadena, California
For floor plans and interiors see page 1[illegible]

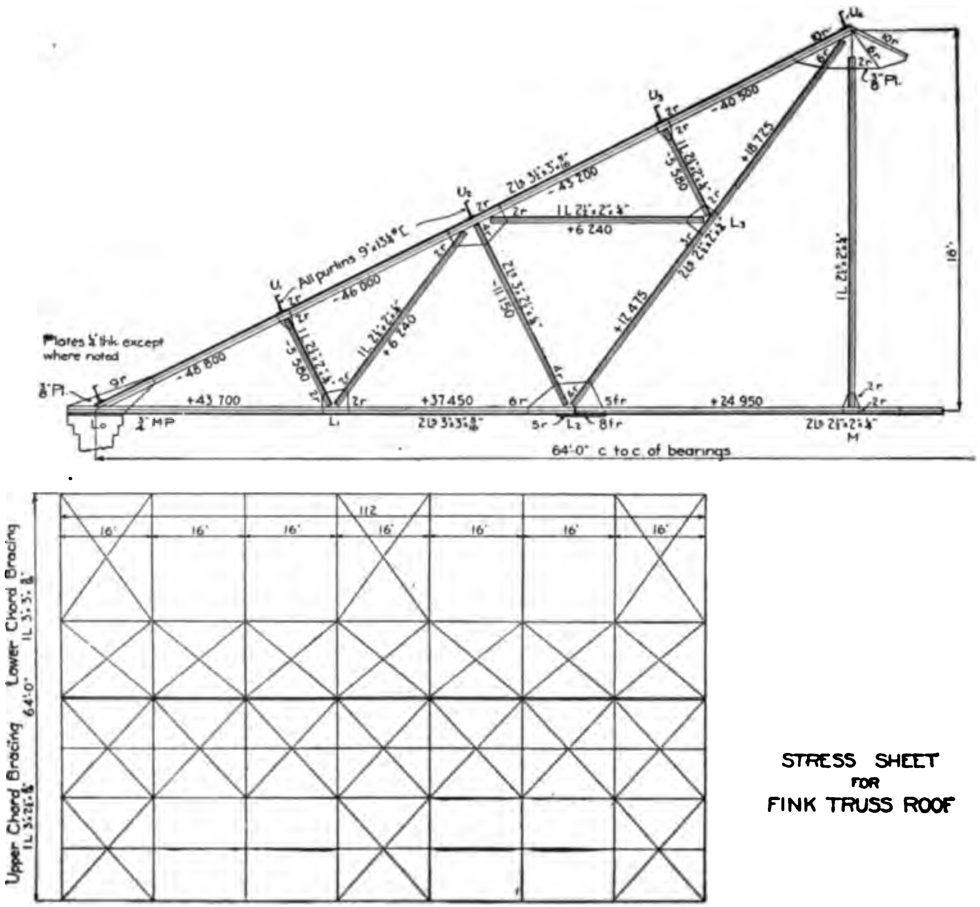
When an I-beam of a channel is cut to a bevel across the depth, the cut should be made as shown in Fig. 67, and the distance "a" should be given.

Detailing of Roof Trusses. The first thing to determine in this respect is the outline of the outer line of the roof and the end, and the center depths. The chords should now be located by center lines corresponding to the gauge lines of the angles, or the center of gravity lines of the pieces, as the case may be. The above mentioned determinations may be obtained from the architect's drawing and from the stress sheet; and in many, if not most all cases, the center lines of the chords are shown on the stress sheet. The stress sheet may be an outline with the stresses and the sections on it, or it may and in fact should be as shown on Plate I. Here the designer, who is an experienced man, has shown the general details. It now remains for the draftsman to draw this up so that the shopmen can make it. After he has finished, the results will be as shown on Plates II and III, which will now be discussed in detail.

After the center lines of the chords are drawn in, the angles themselves should be drawn on by laying of the gauge lines on one side and then the other edge of the leg on the other side of the gauge line. After this the top chord should be divided into a certain number of equal parts at each of which a purlin is to be placed. This done, lines from these points should be drawn perpendicular to the top chord and their points of intersection with the bottom chord should be noted. From the intersection of the center one with the bottom chord to the apex or top, a line is now drawn, and this is the center line of the main interior tie, or tension member. The member itself should now be drawn on this gauge line. After this the other members should be drawn in as shown.

In order to proceed, the distances between the various points of intersection must be carefully computed, thus giving the remaining data necessary to compute the bevels, which should now be done.

In order to determine the length of the members and the sizes of the plates, it is now necessary to take each point of intersection where any members meet at any other than a right angle and make a layout of that joint to some large scale, say 1½ to 2 inches to the foot. The customary ¼-inch clearance should be allowed where there is any liability of pieces touching and, after the ends of the



PLAN OF ROOF

Plate II. Stress Sheet for a Fink Truss Roof

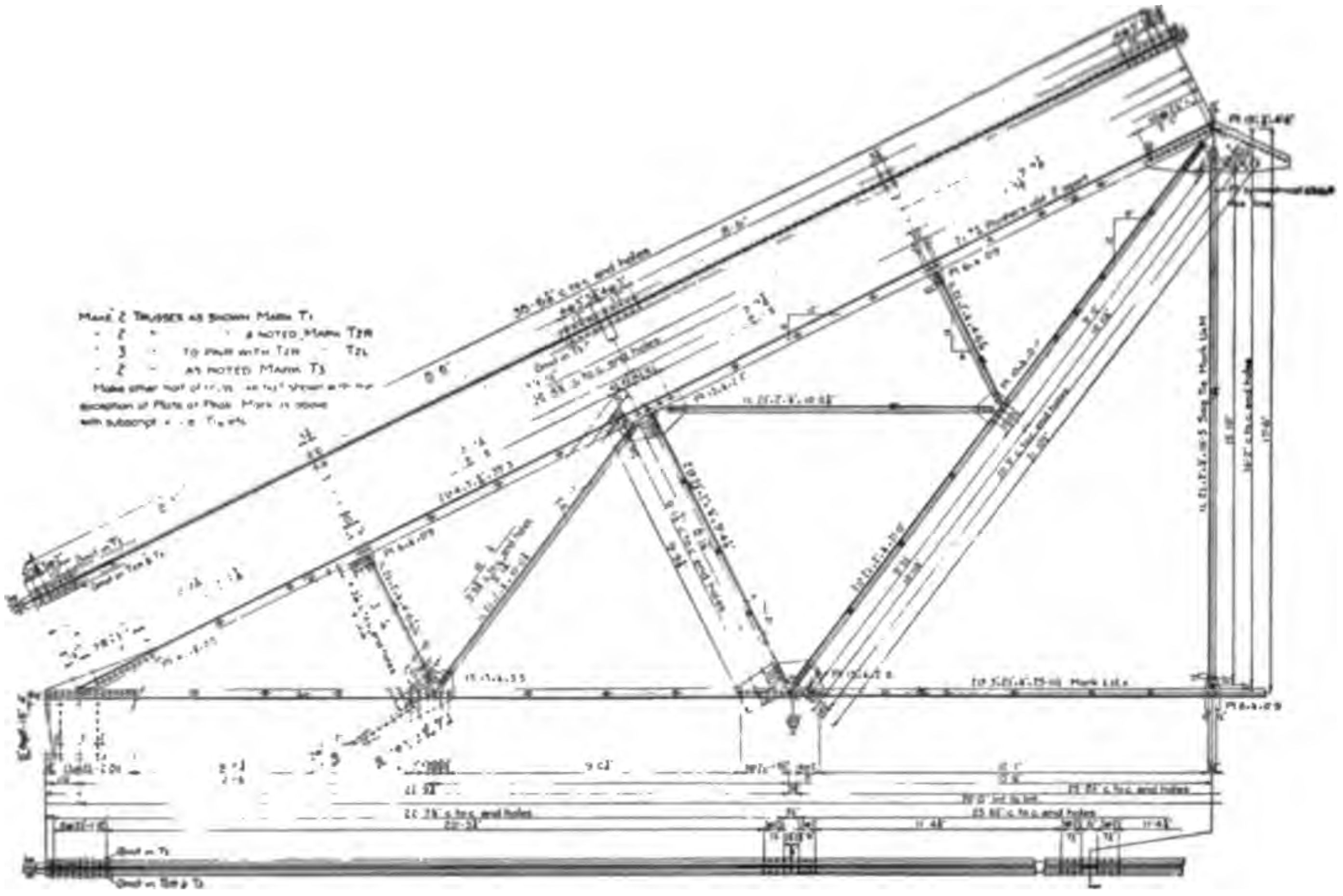

Make 2 Trusses as shown Mark T1
" 2 " & noted Mark T2R
" 3 " to pair with T2R " T2L
" 2 " as noted Mark T3
Make other half of truss as half shown with the
exception of Plate of Peak. Mark is above
with subscript

Plate III. Work-Shop Drawing of a Roof Truss

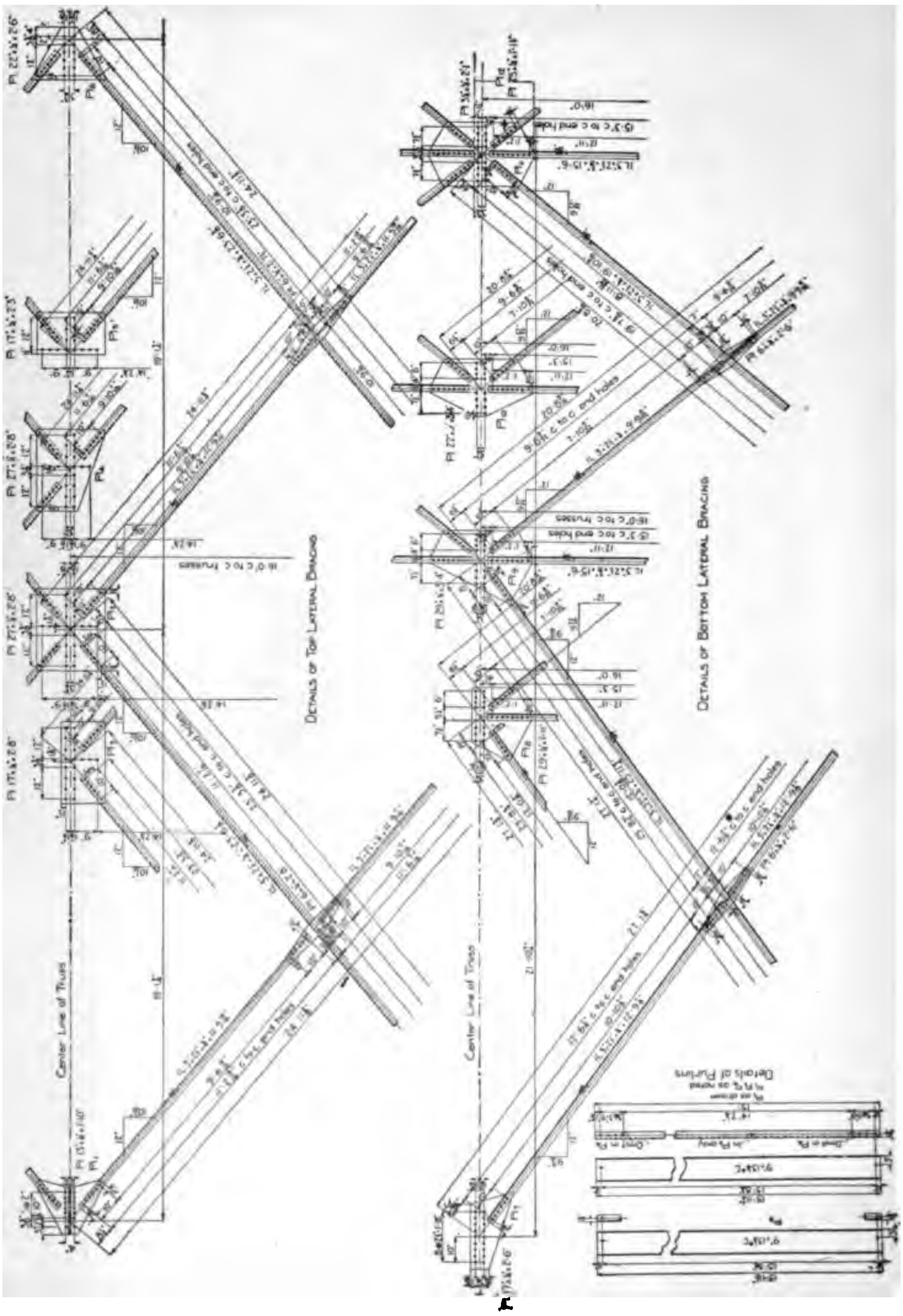


Plate IV. Work-Shop Drawing Showing Detail of Top and Bottom Lateral Bracing of a Roof Truss

various angles are drawn in, the first rivet is set back $1\frac{1}{8}$, $1\frac{1}{4}$, or $1\frac{1}{2}$ inches as the sizes of the angle and of the rivet allow, and the other spacing is so arranged as to make the size and shape of the plate advantageous and economical. The distance from the first rivet to the intersection is measured off and noted. After the layout for each joint has been made and the necessary dimensions of the plates and the distance from each intersection to the first rivet has been determined, the length of each member may be computed. This is equal to the length, intersection to intersection, plus the sum of the distances from the first rivet at the ends to the end of the member, minus the sum of the distances from the first rivet to the nearest intersection. For example, in the main interior tie U_4 L_2, Plate I, the length, intersection to intersection, is $21'\text{-}10\frac{1}{2}''$, the distance from each first rivet to the end of the angle is $1\frac{1}{2}$ inches, and the sum of the distances from each first rivet to the nearest intersection is $(4\frac{1}{2}+9) = 13\frac{1}{2}$ inches, which is $1'\text{-}1\frac{1}{2}''$. The length of the member is:

$$21'\text{-}10\tfrac{1}{2}''+2\ (1\tfrac{1}{2}'')-1'\text{-}1\tfrac{1}{2}''=21'\text{-}0''$$

At the point L_2, Plate I, a field connection must be made as well as at U_4 on account of the fact that the truss must be shipped in part in case the span is larger than 30 feet, the length of an ordinary gondola freight car. At L_2 both legs of the angle should be connected, the horizontal leg connection being by a plate. In case of riveted lateral bracing such as is used here, the connection plate may also be used as a splice plate, see Pls. 8, 9, and 10 in Plate III.

At point U_4 as many shop rivets are put in as there are field rivets required. This will keep the plate symmetrical, and will allow the same templets to be used for the top chord and main interior tie on both sides of the truss. This more than overbalances the cost of driving the few additional shop rivets.

At L_0 in this case the truss has been designed so that the rivets are symmetrical about the point of intersection and, therefore, only a sufficient number are required to take up the direct stress in the top and bottom chords. In many cases the end of a roof truss is as shown in Fig. 68, in which case the number of rivets L_0 L_2 may be calculated from the equation:

$$n^2v - Rn = \frac{6\,Re}{p}$$

in which n = number of rivets required; v = allowable stress on one rivet; R = the vertical reaction; p = the rivet spacing in inches; and e = distance shown in Fig. 68.

The number of rivets in $L_0 U_1$ may be determined from the equation:

$$n^2v - Sn = \frac{6\,S\,e_1}{p}$$

in which S is the stress in $L_0 U_1$, and e_1 the distance shown in Fig. 68. These formulas allow for the stress due to eccentricity. The rivet spacing p is usually taken as 3 inches, although it may be taken as any value permissible by the specifications.

In the detailing of the lateral systems, Plate III, the same method of procedure as above mentioned should be followed. Care should be exercised in making the layouts for the lateral plates so that sufficient clearances are allowed, both in regards to clearances between members and clearances in rivet driving.

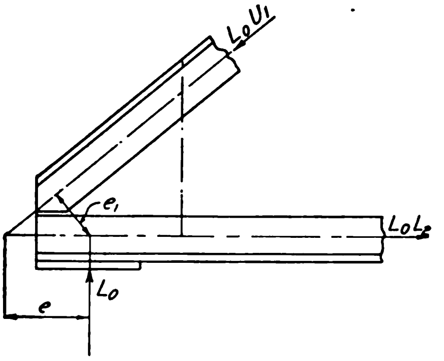


Fig. 68. Typical Detail for the End of a Roof Truss

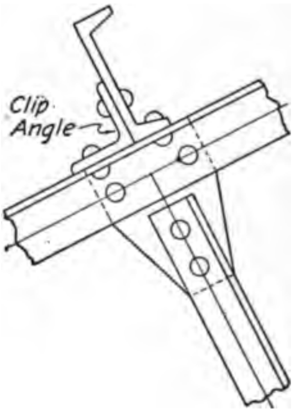


Fig. 69. Method of Riveting Clip Angles for Carrying Purlins

The purlins, or rafters, may be detailed directly upon the main sheet with the bracing or truss, or upon a beam sheet, preferably the latter. In Plate III they are upon the lateral sheet. These purlins should be riveted, not bolted to the chords of the trusses. In order to facilitate erection, clip angles should be riveted to the top chord as shown in Fig. 69 so that the purlin may be put in place and riveted up without having to hold it in place with ropes or chains.

Also by this method the purlin may be put in place and used as support for erection apparatus. In Plate III, the additional pair of holes at panel points of the top chord are for these clip angles.

After the draftsman has finished his drawing he should carefully check up all dimensions and bevels and inspect the drawing for errors in rivet clearances. The passing in of accurate detail drawings will soon result in a promotion to checker, a more pleasant position, but one with greater responsibilities attached.

Detailing of Plate Girder Spans. The information which the draftsman has to start with is in the form of the stress sheet. This may be as Plate V which is the latest and most approved form, or it will be like Plate VI. In both cases the number of rivets for the lateral connections are given, but on Plate V the rivet curve for the spacing in the flanges is given and also the curve of the total and dead load shears and moments.

As soon as a plate-girder stress sheet is turned over to the draftsman, he should lay it out at once and determine the exact location of the web splices, the stiffeners, and the cover plates and their lengths (if not given), should decide upon the lengths of the panels of the lateral bracing, and should also make layouts of the lateral plates, if possible, so that the material can be ordered at once if necessary. In making the above layout the following should be observed:

(1) Be careful in locating splices to see that they come at a panel point of the lateral system.

(2) Locate all splices and stiffeners with a view of keeping the rivet spacing as regular as possible.

(3) Have the panels of the lateral systems equal if possible. If not, have a smaller one at the ends of the girder, the remainder being of equal length.

(4) Stiffeners to which cross-frames are attached should have fillers.

(5) The outstanding leg of stiffener angles should have a gauge of $2\frac{3}{4}$ inches or more. This will enable the cross-frames or floor beams to be swung in during erection without spreading the girders.

(6) It is always best to use as few sizes as possible for stiffeners, connection plates, etc., and avoid all unnecessary cutting of plates and angles.

(7) Locate the end holes for laterals and diagonals so that they can be sheared by a single operation, see Fig. 70. This will, as a rule, throw the end rivet further back from the working point, and may increase the size of the connection plate, but it is desirable.

(8) It is preferable to have an even number of panels in the lateral system since the girders can in most cases then be made symmetrical or nearly so about the center.

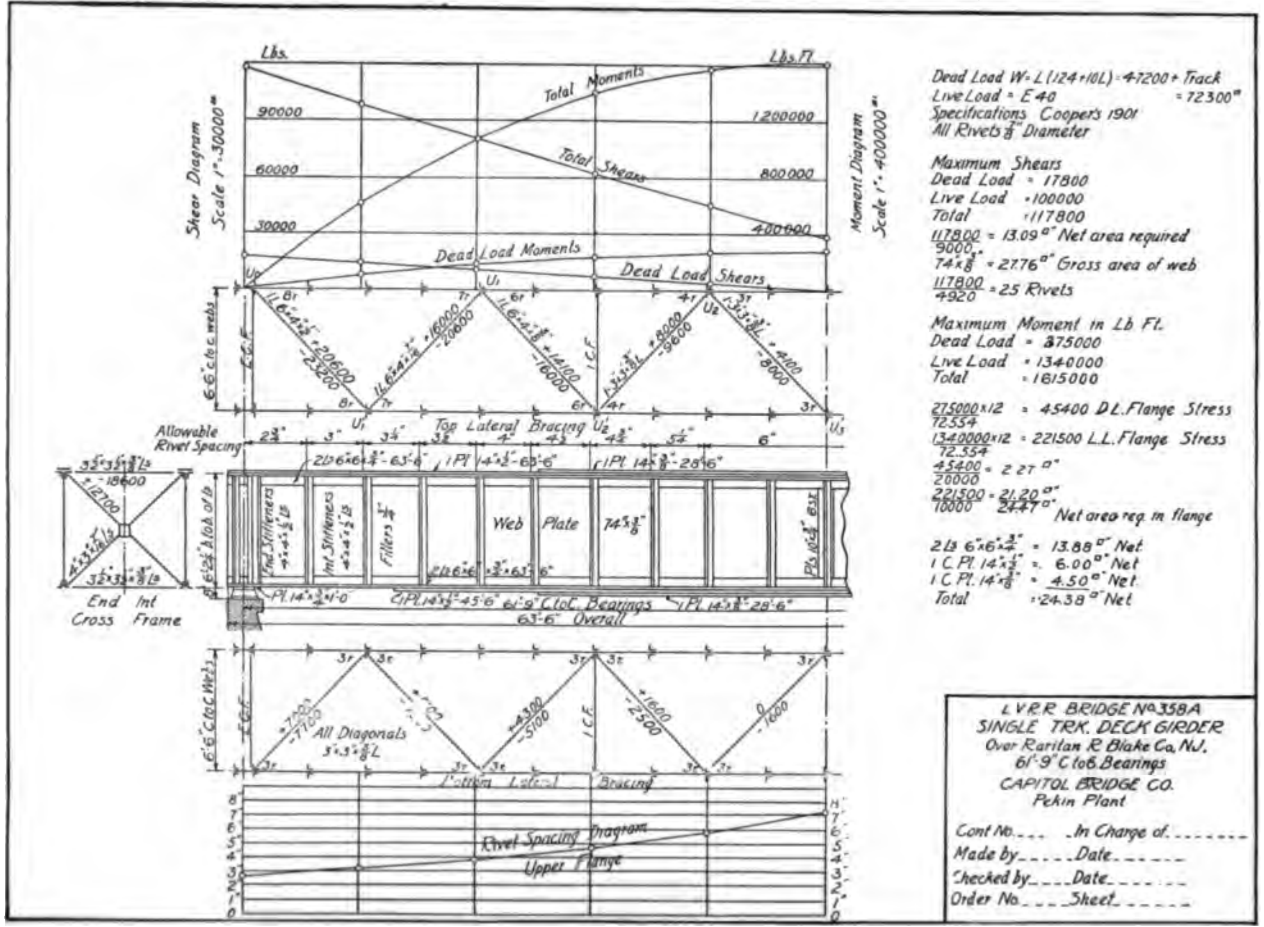


Plate V. Stress Sheet of a Single-Track Deck-Girder Span

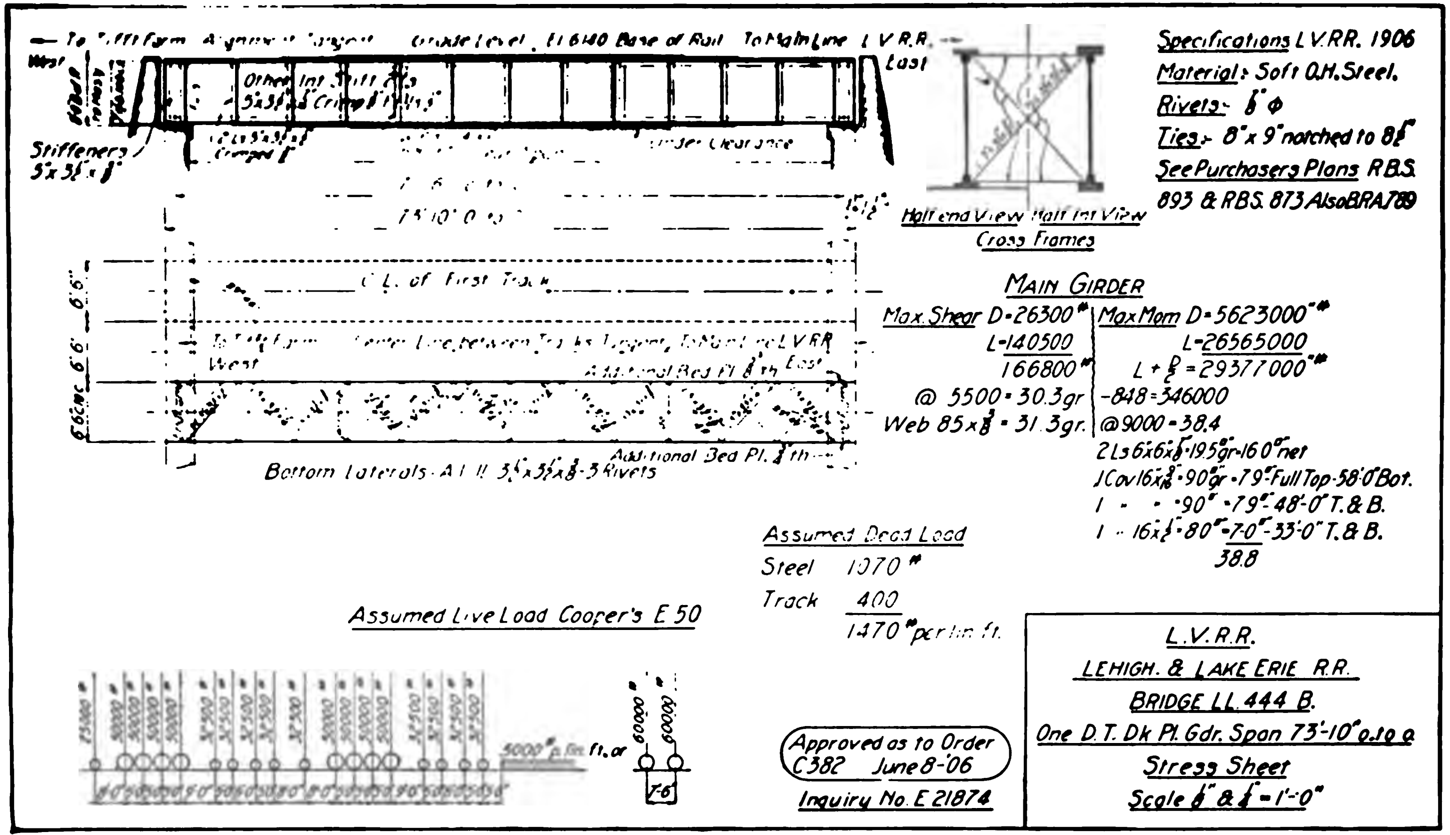


Plate VI. Stress Sheet for Plate-Girder Span

(9) The rivet spacing curve should be constructed if it is not given on the stress sheet.

In addition to the above the following rules which apply directly to the detailing should be followed. They are:

(1) The second pair of stiffeners over the ends of the bed plate shall be so placed that the plate will extend not less than 1 inch beyond the outstanding leg.
(2) If spans are on a grade, unless otherwise specified, put the bevel in the bed plate or masonry plate and not in the base or sole plate, sometimes called the bearing plate.
(3) In short spans, 50 feet or less, put slotted holes for anchor bolts in both ends of the girder. This will usually be covered by a clause in the specifications.
(4) In square spans show only one-half, but give main dimensions such as "overall" and "center to center" and lengths of cover plates for the entire span.

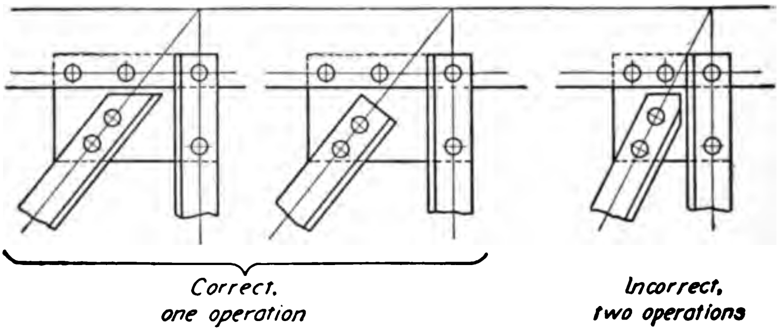


Fig. 70. Method of Locating End Holes for Laterals and Diagonals so that They May be Sheared by a Single Operation

(5) The girder detailed is always the far girder and is looked at from the inside.
(6) If a span has no lower lateral bracing, only sufficient of the ends of the girder are to be shown in order that the detail of the base plate and its connection to the flange may be shown.
(7) If the fillers become 12 to 15 inches wide, they become too heavy to be slipped in in the field and they should be riveted in place in the shop with at least two countersunk rivets.
(8) When the ends of two girders meet on the same pier the masonry plate should be made continuous, that is, one plate to extend under both spans. Never make the base plates continuous since they could not be riveted up in the field.
(9) Detail the bed (masonry) plate separately, never show it in connection with the base plate.

At least two sheets are necessary to complete the detail drawings of any girder span, viz, (1) the Floor, Masonry, and Erection Plan, Plate VII, and (2) the Girders and Bracing, Plate VIII, although in many cases the information on this sheet is put on two sheets, the girder on one and the bracing on the other. The first sheet should show:

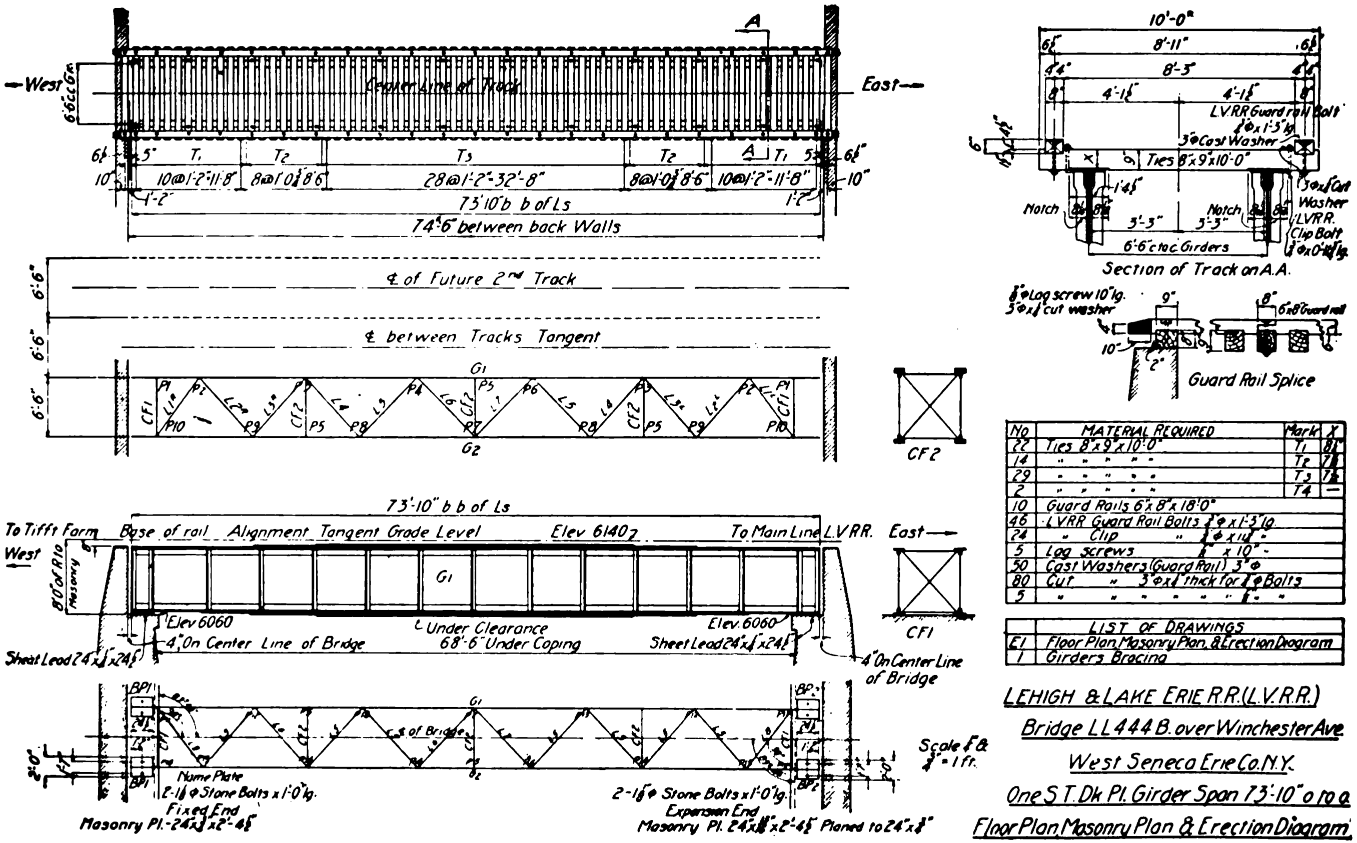
(1) A cross-section of the floor.
(2) A longitudinal view of the floor.
(3) A side elevation of the floor.
(4) The angle of skew and the width of the bridge seat.
(5) The elevation of the bridge seats and the grade of base of rail.
(6) The marking diagram.
(7) All clearances.
(8) Other essential information.

In the marking diagram all members which are entirely alike should be given the same mark. It may be, and usually is a fact, that all marks can not be put on the marking diagram until the detail drawing is done since then and only then is it possible, especially with the plates, to determine all members which are alike. Only those members which are shipped loose are given a mark. Thus it is seen that while each connection plate has a mark, only the entire cross-frames are given one mark since the members which compose them are all shop-riveted together. "Other essential information" is seldom required. In this special case there is shown another track which it is proposed will be put in in the future. Another case is where each end of the span has a different height from the base of rail to the masonry. In such cases this should be shown.

On this sheet should be shown the masonry plates, and if the ends are supported on cast-steel bases the height of these and also the dimensions of the base should be given.

The following general rules apply to the second sheet, Plate VIII.

(1) At the top of the sheet show a top view of the span with cross-frames, laterals, and their connections complete, the girders being placed at their proper distances apart.
(2) Below this show the elevation of the far girder from the inside, with all field holes in the flanges and stiffeners indicated and blackened in.
(3) If the span has lower lateral bracing, show below the elevation a horizontal section of the span just above the tops of the lower flange angles. On this drawing show the lower lateral bracing.
(4) Cross-frames shall, whenever possible, be detailed on the right hand of the sheet in line with the elevation. The frame shall be of such a depth as



No	MATERIAL REQUIRED	Mark	X
22	Ties 8"x9"x10'-0"	T1	8½
14	" " " " "	T2	7½
29	" " " " "	T3	7½
2	" " " " "	T4	—
10	Guard Rails 6"x8"x18'-0"		
46	LVRR Guard Rail Bolts ⅞"φ x 1'-5" lg.		
24	" Clip " ⅞"φ x 14" "		
5	Log screws ⅞" x 10" "		
50	Cast Washers (Guard Rail) 3"φ		
80	Cut " 3"φ x ¼" thick for ⅞"φ Bolts		
5	" " " " " " ⅞" "		

	LIST OF DRAWINGS
E1	Floor Plan, Masonry Plan, & Erection Diagram
1	Girders Bracing

LEHIGH & LAKE ERIE R.R. (L.V.R.R.)

Bridge LL 444 B. over Winchester Ave.

West Seneca Erie Co. N.Y.

One S.T. Dk. Pl. Girder Span 73'-10" o to o

Floor Plan, Masonry Plan & Erection Diagram

Plate VII. Floor, Masonry, and Erection Plan for Plate-Girder Span

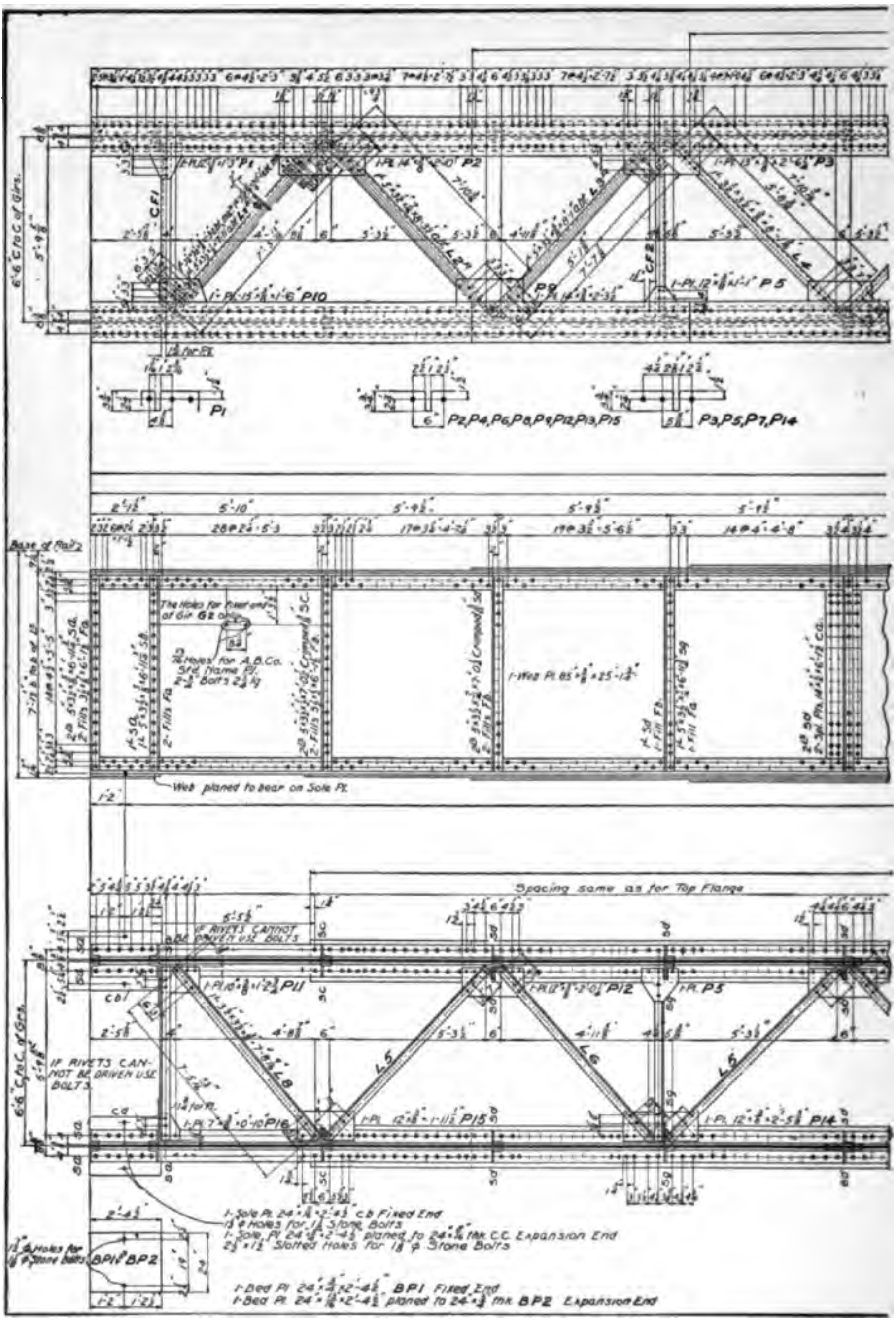
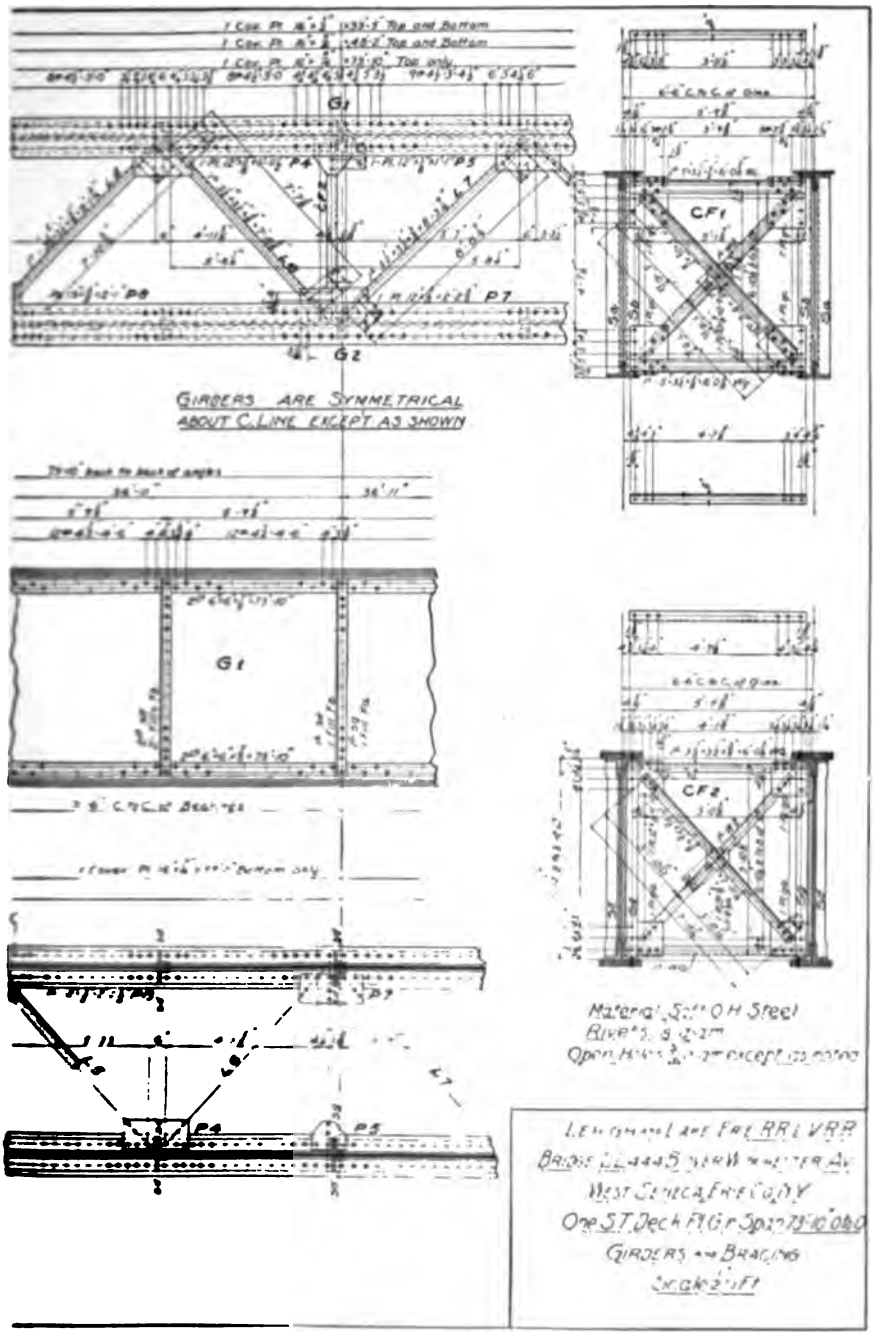
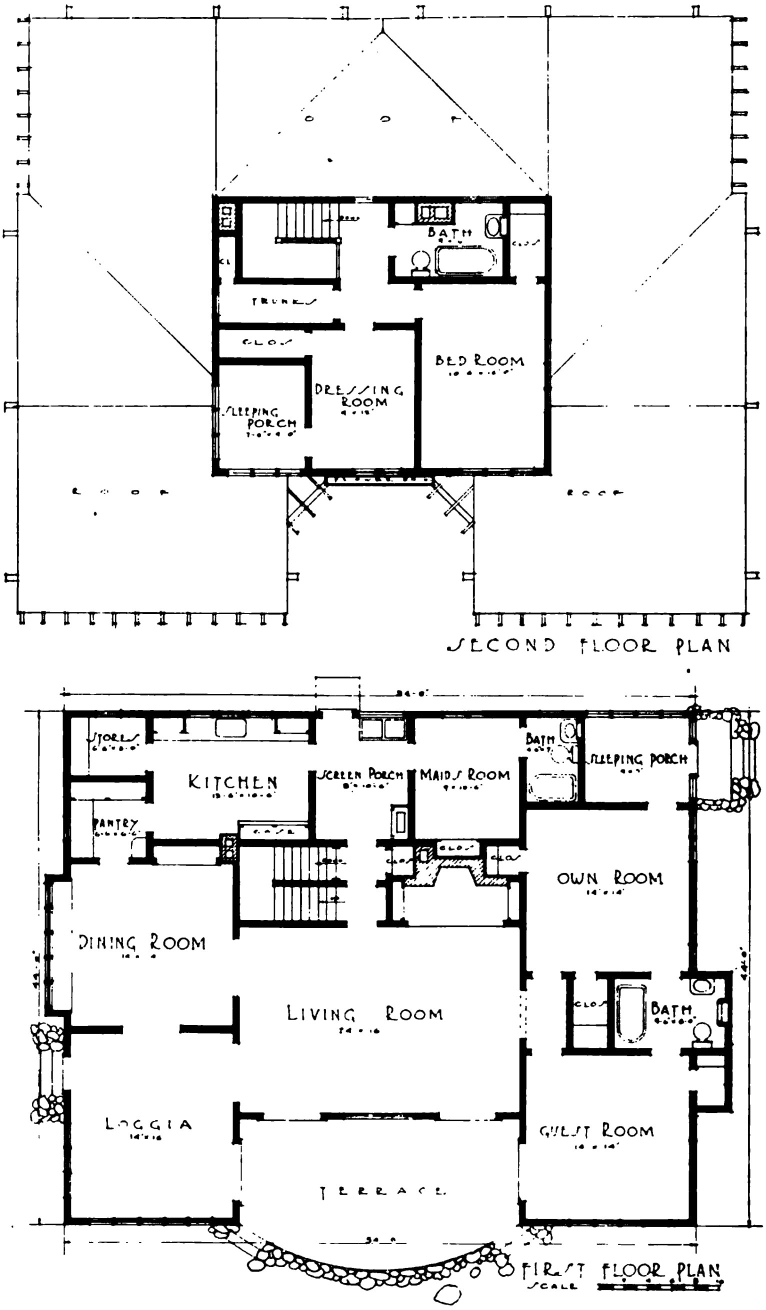
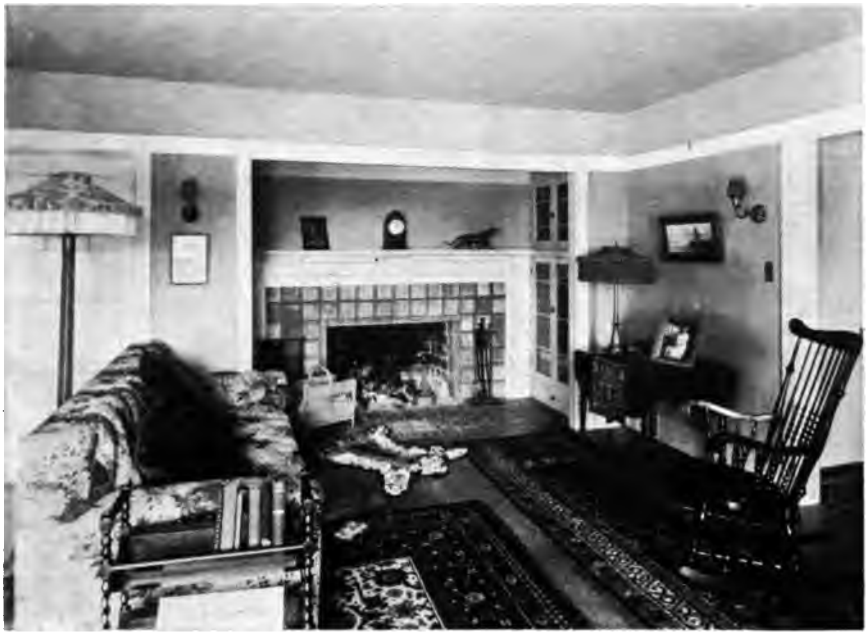

6'-6" C. to C. of Grs.
P1
P2
P3
P5
P10
CF1
CF2
P2, P4, P6, P8, P9, P12, P13, P15
P3, P5, P7, P14
Base of Rail
Web planed to bear on Sole Pl.
Spacing same as for Top Flange
IF RIVETS CANNOT BE DRIVEN USE BOLTS
P11
P12
P14
P15
P16
BP1
BP2
1-Sole Pl. 24 × ⅞ × 2'-4½ c.b. Fixed End
1-Bed Pl. 24 × ⅞ × 2'-4½ BP1 Fixed End
1-Bed Pl. 24 × ⅞ × 2'-4½ planed to 24 × ¾ mk. BP2 Expansion End

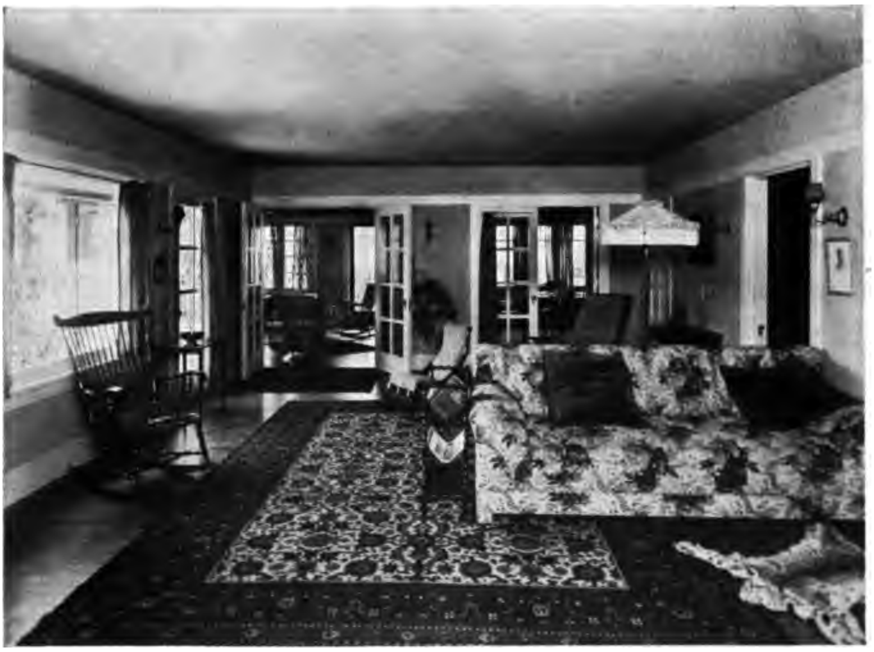
Plate VIII. Detail Drawing of Girders and Bracing for Plate-Girder Span


1 Cov. Pl. 16" × 1/2" × 33'-1" Top and Bottom
1 Cov. Pl. 16" × 1/2" × 48'-2" Top and Bottom
1 Cov. Pl. 16" × 1/2" × 73'-10" Top only
G1
P4
P5
P7
P8
L7
L8
G2
CF1
GIRDERS ARE SYMMETRICAL ABOUT C. LINE EXCEPT AS SHOWN
73'-10" back to back of angles
36'-11"
G1
C. to C. of Bearings
1 Cover Pl. Bottom only
P4
P5
P7
L7
L8
CF2
Material Soft O.H. Steel
Rivets
Open Holes except as noted
RR
West Seneca, Erie Co., N.Y.
One S.T. Deck Pl. Gr. Span 73'-10" O.to.O.
GIRDERS AND BRACING



FLOOR PLANS OF W. A. CHILDS HOUSE AT RIVERSIDE, CALIFORNIA
George A. Clark, Architect, Pasadena, California
For exterior see page 91; for interiors see opposite page





TWO VIEWS OF LIVING ROOM IN W. A. CHILDS RESIDENCE AT RIVERSIDE, CALIFORNIA

George A. Clark, Architect, Pasadena, California
For exterior see page 91; for plans see opposite page

to permit it being swung into place without interfering with the heads of the flange-rivets in the girders.

(5) Always use a plate, not a washer, at the intersection of the diagonals of cross-frame.

The various parts of Sheet 2, Plate VIII, will now be taken up in detail and described and commented upon.

The Webs. As a usual thing webs are never specified in fractions of an inch. If so, the next inch in width must be ordered and then after the flange angles are riveted on, the projecting portion is cut off—an expensive operation. Webs are ordered in even inch widths and the distance back to back of angles is made ¼ inch or ½ inch greater than the width of the web plate. This is sufficient to prevent any irregularities in rolling from projecting above the flange angles. Some engineers favor the web planed down from the greater width and claim that the bearing of the web on the sole or base plate thus obtained is a great advantage. The advantage is slight, however, and unless specially instructed to detail it that way, it should not be done. The web splices should, as before mentioned, be at a panel point of the lateral system. In some cases the web plates butt up against each other, being planed to an even bearing. In most cases, however, the ends of the webs are sheared off and the customary ¼-inch clearance is allowed. In this case the sum of the lengths of the webs as given is 2 (25′-¾″)+23′-5¾″=75′-9¼″, while the "overall" distance is 73′-10″ or ¾ inch less, which is taken up by the distance between webs at the splice and by the small amount, ¼ inch, which web is below the backs of the angles at the ends. This shows the webs to be ¼ inch apart at the splices. It is unnecessary to place any dimensions or notes on the drawing calling attention to this fact since the shop will make this allowance unless instructed otherwise. In case the webs are to be close together, a note must be placed on the drawing at the splice, reading "Webs planed to even bearing."

Web Splices. Web splices may be of two forms, viz, that as indicated on Plate VIII which takes shear only, and the moment web splice. The proper manner to detail a moment web splice is as shown in Fig. 71. In the simple shear splice both the splice stiffeners and the splice plates may and should have the same spacing as the intermediate stiffeners, and the rivet lines should be spaced so as to correspond to the spacing in the flange angles. In the moment splice this should be done if possible, but this is seldom the case. However, in case of more than one splice occurring in half of the girder, they should all be made alike, being figured for the one with the greater stress. Since a splice plate is a species of filler, it should be given a mark so that in case of other splices occurring the mark and not all the dimensions should be placed upon it.

Stiffeners. All stiffeners except the second from the end should have the outstanding leg on the side of the gauge line away from the center of the girder. As a rule, the end stiffeners should have enough rivets to take up the end shear, and the intermediate stiffeners should have sufficient to take up the shear at that point. This would, if carried out, require a different number of rivets in each stiffener. Common practice requires that the spacing in all stiffeners should be the same and that this spacing should be the same as in the end stiffeners. In some cases, such as in heavy girders, it is not possible to do this on account of the large number required in the end stiffeners, and the rivet spacing is then made the same in all the intermediate stiffeners.

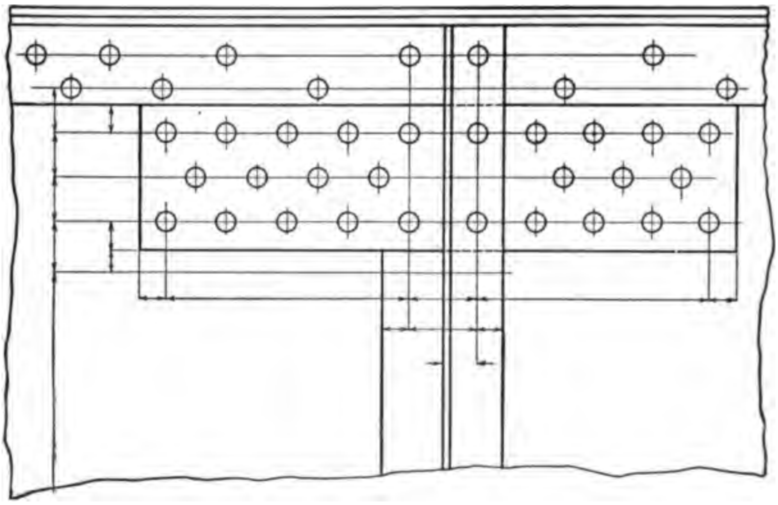


Fig. 71 Method of Detailing a Moment Web Splice

The rivet spacing should not exceed 4¾ or 5 inches at the most and should be so placed that it should be symmetrical about the center. When the web plates are of even inch width, the ¼ inch may be put into one odd space at the center in order to avoid ⅛ inch in the spacing. It may be necessary to put in a few more rivets than are computed as necessary, but the advantage gained by thus making the punching of the plate on the multiple punch possible, makes this advisable. Fig. 72 shows this method of detailing. In order to make the shear plate at the web splice efficient to some degree in withstanding the moment—for although it is not computed to take moment yet it does in reality—the rivets near the flange are placed close together for a few spaces. If the space changes after that, it should increase towards the middle of the web, except in such a case as Fig. 72 where the center space may or may not be as great as those on either side of it.

In double-gauge flange angles the rivet in the stiffener should be in the inner guage line of the flange angle as shown, and no rivet should come closer than 1½ inches to the end of a filler.

Each like stiffener should be given a mark and in case others of the same kind both in size and punching occur, the mark may be used instead of the material notation and dimension. Those crimped will be given different mark even if size and punching are the same. It should be noted that some of the stiffener angles differ only from the fact that they have holes in their outer leg to which the cross-frames are connected, hence a different mark.

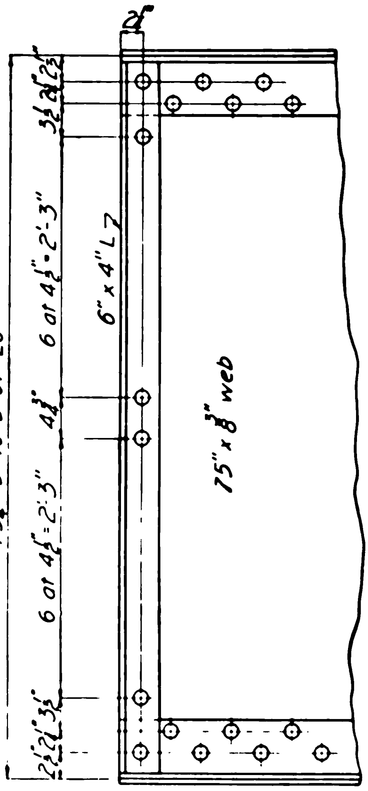


Fig. 72. Method of Detailing Web Plate Stiffeners

The length of stiffeners listed on the drawing is the distance inside of flange angle legs. Without further instructions they will be ground by the shopmen so as to have a snug fit.

Fillers. Fillers are placed under angles that are crimped since the angles are only crimped $\frac{3}{8}$ inch and not the entire $\frac{7}{8}$ inch which is the thickness of the flange angles. The fillers are given marks for the same reasons and in accordance with the same rules that apply to stiffeners.

Flange Angles. In case of double gauge on the 6-inch flange angle it is better to put the $2\frac{1}{2}$-inch gauge on the inside, no matter

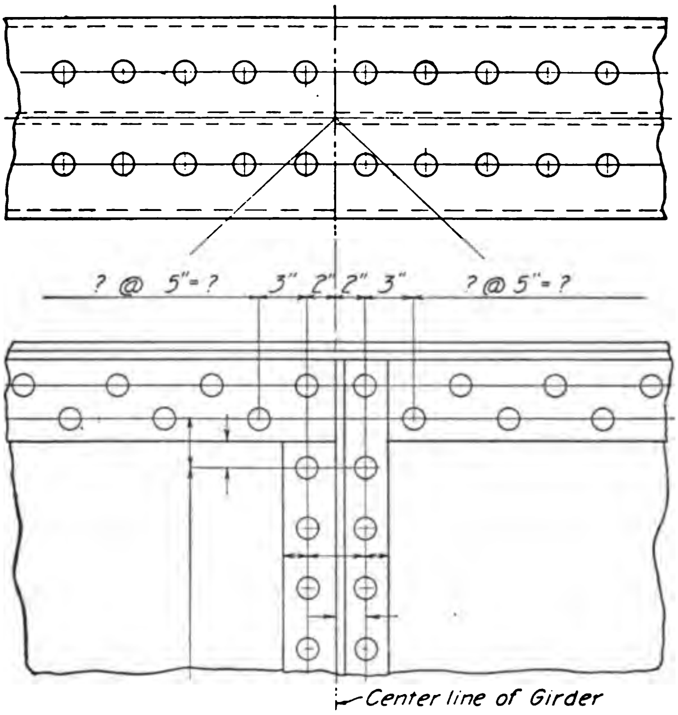


Fig. 73. Method of Detailing Rivet Spacing with Flange Angles

what the thickness may be, since by this operation the rivets in the horizontal flange, providing that is a double-gauge line, may be more advantageously spaced on account of the fact that the required stagger will be less.

The rivet spacing in the vertical leg of the flange angles should increase from the end towards the center and should remain the same, as far as possible, between any two stiffeners, any changes necessary being made near the stiffeners. Since a rivet must always be in the inner gauge line at a stiffener, an even number of spaces must be between any two stiffener gauge lines, since the rivets must stagger. This brings one rivet in the center of the girder, which can not occur in case there is a splice at the center of the girder. The stagger may then be broken as in Fig. 73, the stiffener angle being placed as shown and the rivet spacing being symmetrical on each side of the center of the girder.

Between the stiffeners at the end, the spacing should be the same as it is between the next two stiffeners.

The spacing at any point should never exceed the computed spacing unless constructive reasons require it. On account of rivet-driving clearances, a $\frac{7}{8}$-inch rivet can not be driven any closer than $1\frac{1}{4}$ inches to another member. Therefore, rivets can not be driven any closer to the stiffener than $1\frac{1}{4}$ inches, see Fig. 74. For this particular sized stiffener, the minimum spacings next to it will be $3\frac{1}{4}$ inches and $2\frac{3}{4}$ inches as seen in Fig. 74. The rivet-spacing multiplication table, Table X, will be found very helpful in spacing the rivets here.

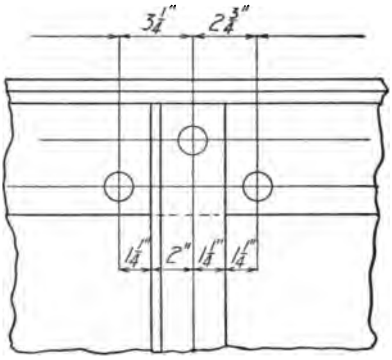


Fig. 74. Minimum Rivet Spacing for Stiffener Angles

Since the single gauge is used in the top flange and, according to the stress sheet, two rivet holes are taken out of each angle, it is possible to space the rivets in the outstanding leg and cover plates without reference to those in the other leg of the angle, due care being taken that they do not come closer than $1\frac{1}{4}$ inches to the outstanding stiffener leg. No special rule governs the spacing in the cover plates, the only requirement being those of the specifications, and that the number of rivets from the center of the span to the end of the cover plate or the number of rivets from the end of one cover plate to the end of another shall be

$$n = \frac{(\text{net area of cover plate})\ s}{v}$$

where n=the number required; s=the allowable unit flange stress;

and v = the value of a rivet in single shear or bearing in the cover plate, whichever is the smaller. For the first cover plate on top of the flange angles this equation gives

$$n = \frac{[(16 \times \frac{9}{16}) - 2\ (\frac{7}{8} + \frac{1}{8})\ \frac{9}{16}] \times 10000}{6013}$$

$$= 13$$

which shows the number 78 to be amply sufficient in this respect. A clause in most specifications requiring the maximum spacing to be not greater than 16 times the thinnest plate and not greater than 6 inches, further governs the number, which would be 50 by this requirement. Most engineers, notwithstanding the specifications, require the majority of the spacing to be within 5 inches.

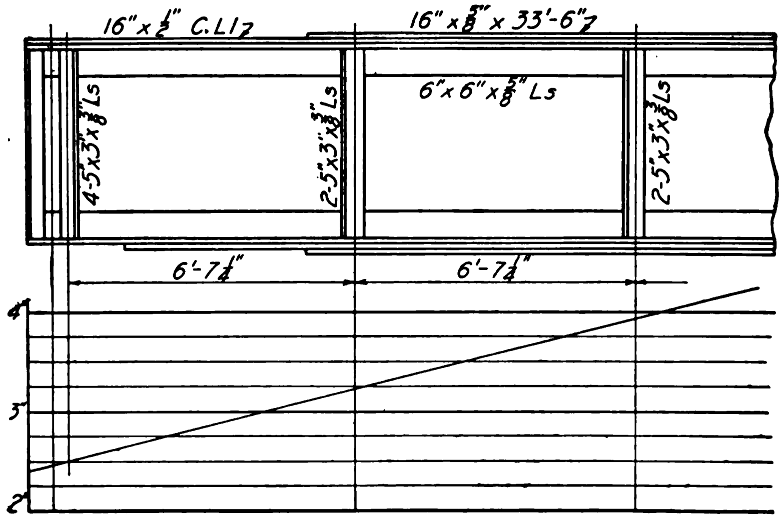


Fig. 75. Typical Stiffener and Rivet Spacing Diagram

In case the spacing in the top flange is on a double-gauge line, care must be taken to see that the minimum stagger, Table IX, is not violated. In such cases it is customary to place a rivet in the inner gauge line of one leg opposite a rivet in the outer gauge line of the other leg, and to do this until a stiffener interrupts, when spacings are made with the observance of Table IX, until the rivets can be placed opposite again.

In order to illustrate the above principles in regard to spacing when double-gauge lines are used on both legs and the maximum spacing for any particular distance is shown by the rivet curve, an example will be given. Let the stiffeners and the rivet-spacing diagram be as in Fig. 75. This shows the allowable rivet spacing to be 2½ inches at the second, 3¼ inches at the third, and 3⅞ inches at the fourth stiffener, the distance between stiffeners being 6′-7¼″. Let it be required to determine the rivet spacing between the second and fourth stiffeners

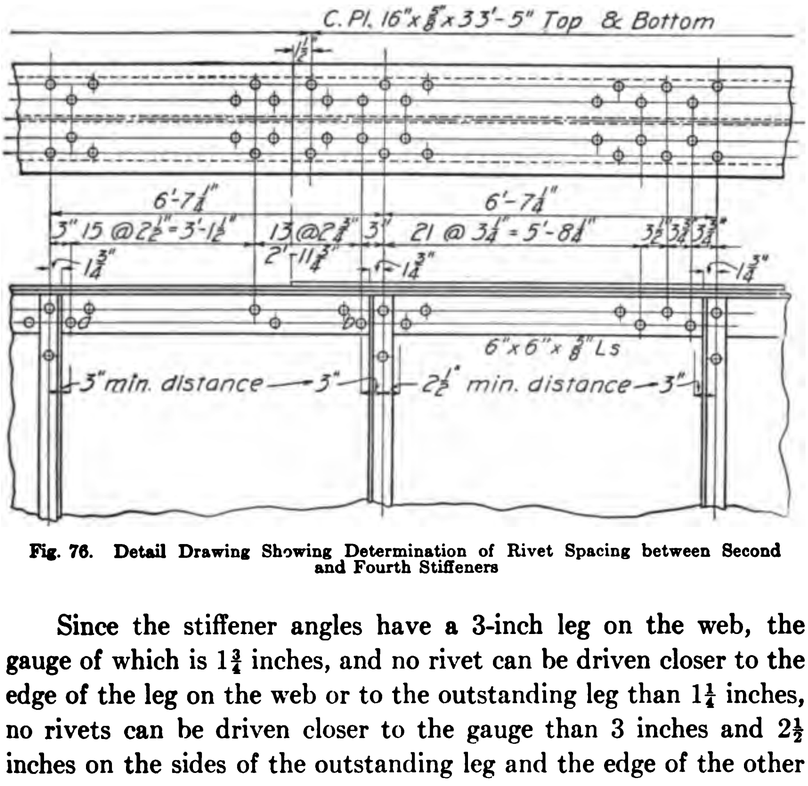


Fig. 76. Detail Drawing Showing Determination of Rivet Spacing between Second and Fourth Stiffeners

Since the stiffener angles have a 3-inch leg on the web, the gauge of which is 1¾ inches, and no rivet can be driven closer to the edge of the leg on the web or to the outstanding leg than 1¼ inches, no rivets can be driven closer to the gauge than 3 inches and 2½ inches on the sides of the outstanding leg and the edge of the other leg, respectively, see Fig. 76. Since 3 inches is the minimum distance it must be used at stiffener (2) notwithstanding the fact that the spacing diagram requires not less than 2½ inches. This leaves (6′-7¼″) $-3''=6'\text{-}4\frac{1}{4}''$ from that rivet to the one in the gauge at the top of stiffener (3), no attention being paid to 3 inches, the minimum distance here, since it is less than the 3¼ inches required by the diagram.

An odd number of spaces must be used since the last rivet is on the other gauge line; and from the rivet-spacing diagram it is seen that the spacing can not exceed 2¾ inches until half way between the two stiffeners, and that a space or two of 3¼ inches would be allowed at stiffener (3).

By consulting Table X it is seen that 29 spaces at 2½ inches are equal to 6′-0½″. Now (6′-4¼″)−(6′-0½″)=3¾″ or 15 fourths ($\frac{15}{4}$), from which it is seen that if ¼ inch was added to 15 of the 29 @ 2½″, the result would be all that is desired; but this would leave the last space 2¾ inches and by Fig. 76 it is seen that it must be at least 3 inches. By making the last space 3 inches, which is ½ inch, or $\frac{2}{4}$ greater than 2½ inches, there remain 28 spaces between rivet *a* and rivet *b*, Fig. 76, and only $\frac{15}{4}-\frac{2}{4}=\frac{13}{4}$ left. If, therefore, ¼ inch be added to 13 of the 2½-inch spaces, making 13 of 2½″+¼″=3¾″ each, the spacing will be correct. It is:

1 space at 3″ =0′-3″
15 " " 2½″=3′-1½″
13 " " 2¾″=2′-11¾″
1 " " 3″ =0′-3″
Total=6′-7¼″

In a similar manner the second space between stiffeners has its rivet spacing determined. Here it is seen that the rivet spacing may start at 3¼ inches, can not exceed 3½ inches until past the middle, and can have a few spaces at 3¾ inches at the stiffener. By Table X it is seen that 24 spaces at 3¼ inches equal 6′-6″. Now (6′7¼″)—(6′-6″)=1¼″ or $\frac{5}{4}$ and if one of the 24 spaces be increased ¼ inch and two of them are increased ½ inch the entire $\frac{5}{4}$ inch will be used up and the spacing will have been completed. It is·

21 spaces at 3¼″=5′-8¼″
1 " " 3½″=0′-3½″
2 " " 3¾″=0′-7½″
Total=6′-7¼″

In a similar manner almost any combination can be made to fill out any dimension.

The rivets in the horizontal flange of the angle and the cover plate are, when the spacing is greater than $2\frac{5}{8}$ inches, placed opposite those in the vertical flanges as is shown in Fig. 76, since according to Table IX, Y being $(2\frac{1}{2}''-\frac{5}{8}'')=1\frac{7}{8}''$, no stagger is required, and where the spacing is less than $2\frac{5}{8}$ inches it is changed so as to be 3 inches or more. In such cases as this it is not necessary to give spacing in the cover plates, a note, "Spacing same as in vertical legs" or "Spacing same as in web" being all that is required.

After all the spacing in the cover plates has been determined, it may be necessary to change it slightly in order to allow for better spacing in the connection plates, but it is common practice to make the connection plates conform to the spacing in the cover plates since by so doing the additional cost of templets for the horizontal legs of angles is saved; and although a few additional templets for connection plates may be required the saving is considerable.

Cover Plates. The actual lengths required are given on the stress sheet, but when the preliminary layout is made and the material ordered, the plates are ordered longer in order that they will at least be the required length when they are on the girder. The cover plates should be stopped so that the last rivet is $1\frac{1}{2}$ inches from the end, and in the case of double-gauge angles this rivet must be on the outer gauge, see Fig. 76. The single gauge is to be recommended, providing sufficient rivets can be gotten in. At the ends of the cover plates it is not necessary to give the distances to the edges. The dimensions should go on as in Plate VIII and Fig. 76. The material notation should be put on as shown, all cover plates on both top and bottom, which are of the same section and length, being listed at the top. Any plate which is special to the bottom, is listed there.

The beginner should be careful to note that the bottom cover plate next to the flange angles does not run the entire length of the girder, and accordingly he should not run his rivet spacing in the bottom flange through to the end but should stop at the end of the cover plate. This is a common error for beginners.

Cross Frames. The cross frames may be detailed as shown in Plate VIII or as shown in Fig. 77. A layout of the plates must be made; the working point being taken at the intersection of the angle gauges, as in Fig. 77, or at some point which is approximately in the line connecting these points, see Plate VIII. The latter method has the advantage in that it allows the point to be so chosen that the ends of the diagonals will be about ¼ inch from both the stiffener and the top angle, thus making a smaller plate. The bevels are not stated on the diagonals since the dimensions are given directly.

The end distances should be given or, if not, a note stating their value should be on the sheet. The distance, intersection to intersection and end hole to end hole, should always be given, likewise the distance to the center of any group of holes. The rivet spacing may then be measured from these points. It was formerly customary to give the distance *a*, Fig. 77, but it is unnecessary and it is not now put on the drawing. Attention is called to the detailing of the diagonals in C. F. 1, the center line being half way between the gauges and a rivet placed on it at the ends.

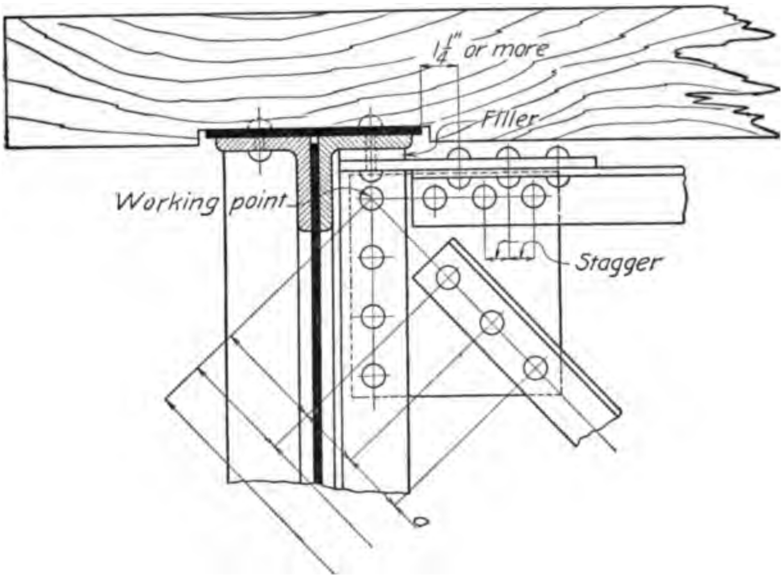


Fig. 77. Detailing of Cross Frames

Rivet clearances should receive close attention. The first rivet in the horizontal leg of the top and bottom struts should be at least 1¼ inches away from the edge of the cover plate, and it and all others should so stagger with those in the vertical flange that the field rivets may be driven. In case the frames are as in Plate VIII, the clearances of the rivets should be looked after and the spacing in the cover plates be so arranged as to have one rivet on the gauge of the angle.

In cases where there is not a cover plate or where the cover plate is thin, the tie may, on account of its being notched $\frac{1}{2}$ inch, press down on the rivet heads of the cross frames. This may be avoided either by cutting out the tie or by placing fillers as shown in Fig. 77. Since the tie is notched at $\frac{1}{2}$ inch and the head of a $\frac{7}{8}$-inch rivet is $\frac{5}{8}$ inch, then the cover plate thickness added to that of the angle must be at least $(\frac{1}{2}+\frac{5}{8})=1\frac{1}{8}$ inches before a filler is required; and the thickness of the filler required in any case is

$$t=1\tfrac{1}{8}''-s$$

where s is the sum of the thicknesses of the cover plate and flange angle, or flange angle alone in case there is no cover plate. Of course no filler is required at the bottom. All intermediate cross frames should be alike, and the end cross frames should be like each other. In Plate VIII, the angles are $\frac{7}{8}$ inch and the first cover plate $\frac{5}{16}$ inch, the sum being $(\frac{7}{8}+\frac{5}{16})=1\frac{3}{16}$ inches, which is greater than $1\frac{1}{8}$, no filler is required.

The top angles should have their horizontal leg detailed with the cross frame. This will save many dimensions on the lateral systems when they are detailed.

Lateral Systems. The lateral systems should be detailed in place whenever possible.

All the panels of the lateral systems should be of the same length. If this is not possible, the shortest panels should be at the ends. It is seldom possible to make all the panels equal when a rolled-steel masonry plate is used. In case of the cast-steel pedestals, the dimensions of the top may be so chosen as to have all the panels of the lateral system equal. This will make the lengths of all angles with the same sized legs on connection plates equal.

The angles may be detailed as shown in Plate VIII or as in Fig. 78. In either case the distance between intersections and between end holes must be given. The rivet spacing is measured back from these reference points, being determined from the layout of the plate. The distance from the working point out to the first hole should be given. The end distance should be given or else noted somewhere else. The plate should, when a double-gauge line is used, be made to take in both rows of rivets; however, as mentioned before, the double-gauge line should only be used when unavoidable.

The working point should be in the center of the web and on the gauge line of the stiffener angle when the method used in Fig. 78 is used, except in cases where a splice is used in the center of the girder, and then the intersection or working point should be at the center of the girder, see Fig. 73. All of the methods, Plate VIII and Figs. 78a and 78b, are in common use. The author prefers those shown in 78b or Plate VIII. Sufficient clearance should be between the cross frame and stiffener, see Fig. 78b.

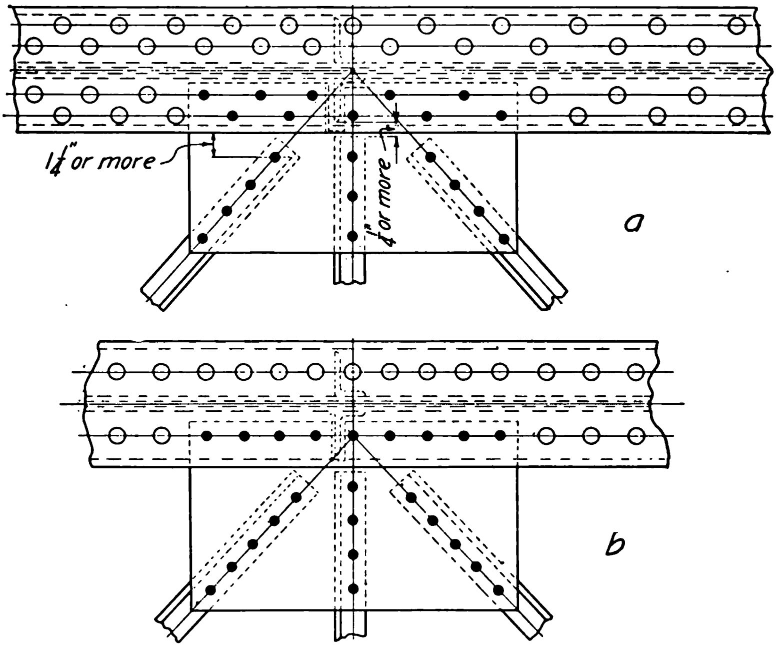


Fig. 78. Detailing of Angles in Lateral Systems

Each different angle, as in the case of stiffeners, should have a different mark. In such cases the mark is all that is necessary to designate another angle exactly like it, thus much repetition in detailing is avoided. The lateral systems, Plate VIII, are good examples of the efficient use of the marking system.

The size of the connection plates is determined from the layouts, the rivet spacing and clearances all being taken from the layout also.

The edge distances of the working ends and edges are shown only when greater than 1¼ inches, and in some cases even then the plates are kept rectangular throughout except in the case of the smaller ones. Few sizes for many plates give evidence of good detailing, and Plate VIII exemplifies this. It might be noted that with single gauge lines in the cover plates, the plates can be detailed more economically than when the double-gauge lines are used in the angles.

A notch must be cut in the plates to allow the stiffener angle to clear. This must be carefully located and detailed for each plate where it differs in the least, and all plates to which any one notch applies should be noted directly with the detail.

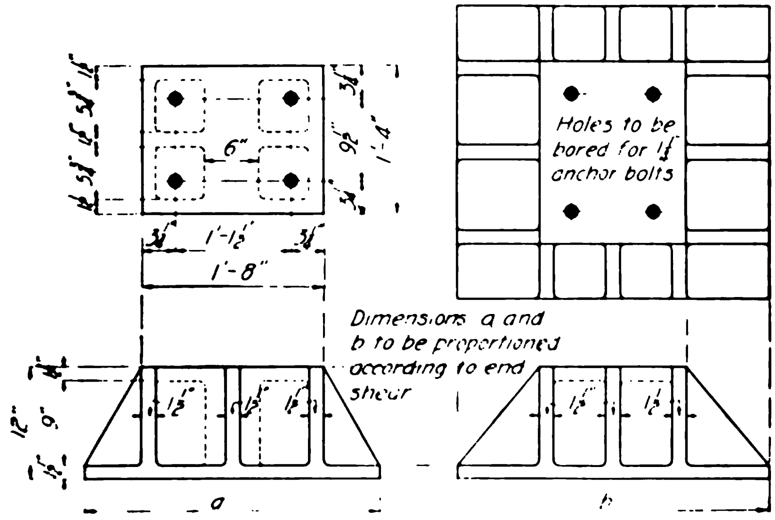

Holes to be bored for 1½" anchor bolts
6"
1'-1½"
1'-8"
1'-4"
Dimensions a and b to be proportioned according to end shear
a
b

Fig. 79 Details of Cast-Steel Bearing

Bearings. These may be as shown on Plate VIII and should be detailed in that manner, or they may consist of cast-steel pedestals with or without rollers, Figs. 79 and 80. The rollers may be either circular or segmental. In the latter case they should, in case the abutment or pier is liable to settle, have a tooth on each end of the lower plates, otherwise the movement of the girder and the movement due to the settlement of the abutment will cause the rollers to tilt over so far that they will not move back under movements due to temperature.

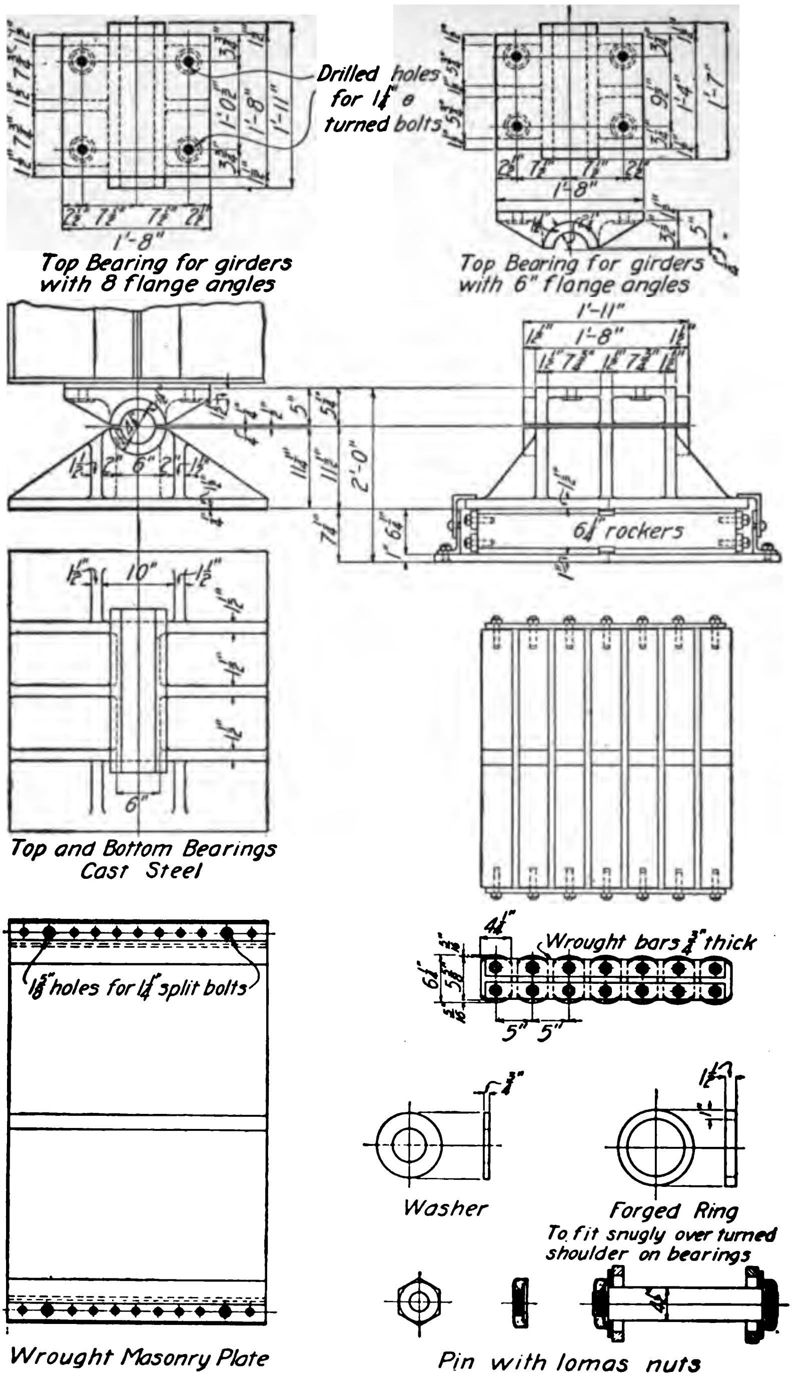

Drilled holes for 1¼" ⌀ turned bolts
1'-8"
Top Bearing for girders with 8 flange angles
Top Bearing for girders with 6" flange angles
1'-11"
1'-8"
2'-0"
1'-6¼"
6¼" rockers
10"
6"
Top and Bottom Bearings Cast Steel
1⅝" holes for 1¼" split bolts
Wrought Masonry Plate
Wrought bars ¾" thick
5" 5"
Washer
Forged Ring
To fit snugly over turned shoulder on bearings
Pin with lomas nuts

Fig. 80. Pin-Bearing Shoe and Rockers for Plate-Girder Bridges

Detailing of Compression Members. The first thing necessary is to determine the pin plates and the number of rivets required. This is done by a method already discussed.

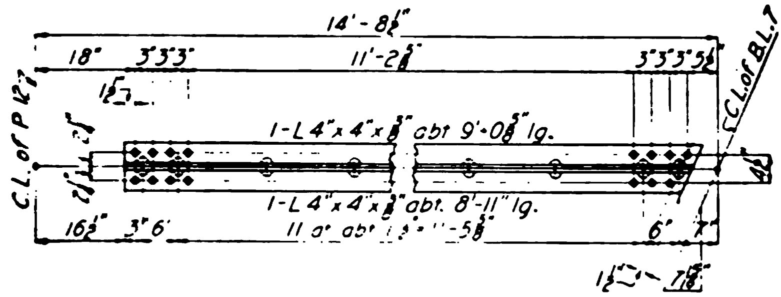


Fig. 81. Detail of a Two-Angle Compression Member

The rivet clearances and also the clearances required in order that each member may fit in with the adjacent ones in the structure, should receive the most careful consideration.

The compression members consisting of two angles riveted should be riveted together at distances throughout their length not greater than 12 inches. The clauses of the specifications relative to lattice bars, see Table XIV, and batten plates should be carefully read and followed. The dimensions necessary in compression members of two angles are the same as those required in diagonals of plate girders. The spacing of the rivets which rivet the angles together need not be given but noted as "Rivets spaced about 12

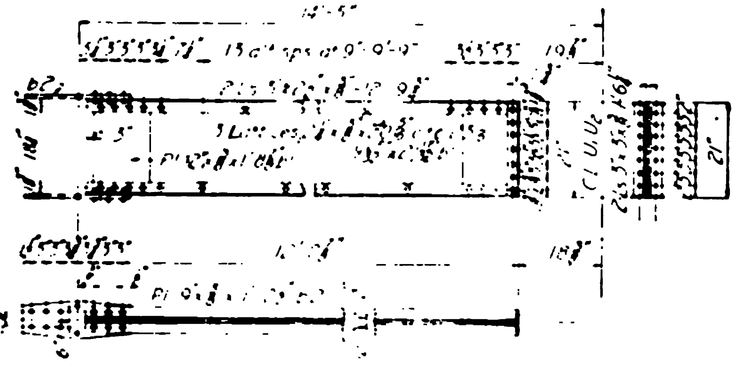


Fig. 82 Detail of Compression Member When Angles are Latticed

inches centers." Fig. 81 is a detail of a two-angle compression member.

When two or four angles are latticed, they should have tie plates at their ends unless otherwise specified. In such cases the method of detailing is shown in Fig. 82. The ends may or may not be alike. The left-hand end is the most usual method of connection.

Compression members consisting of channels and lacing bars and tie plates are very common. Their design is given in Bridge Engineering and the clauses of the specifications cover the details. The pin plates should be on the inside, not the back, of the channel. Fig. 83 represents a typical detail of this class of member.

Compression members of cover plates and channels are used in light bridges. The detailing of such a class is shown by Fig. 84.

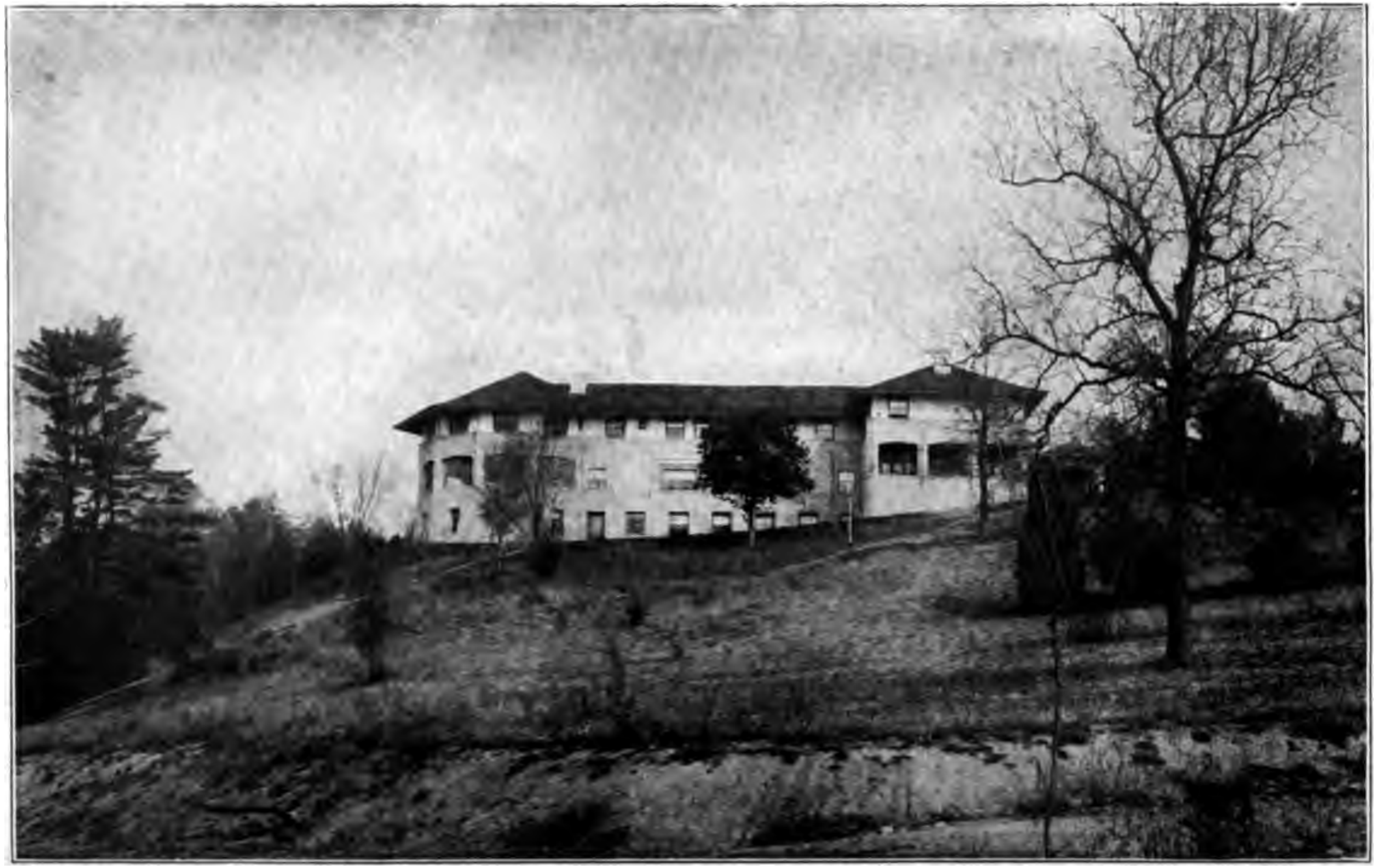
Heavy compression members are made up of angles and plates. The detailing of such members requires considerable care in order that the clearances may be sufficient. Fig. 85 shows a top chord section $U_0 U_2$ of a riveted railroad bridge and fairly well represents the detailing of that type of member.

Detailing of Built-Up Tension Members. The detail of these members is no different from the detailing of compression members of the same class, except that care must be taken not to reduce the section beyond the required amount, by taking out too many rivet holes. Those clauses of the specifications relating to batten or tie plates and lattice bars apply here as well as to compression members.

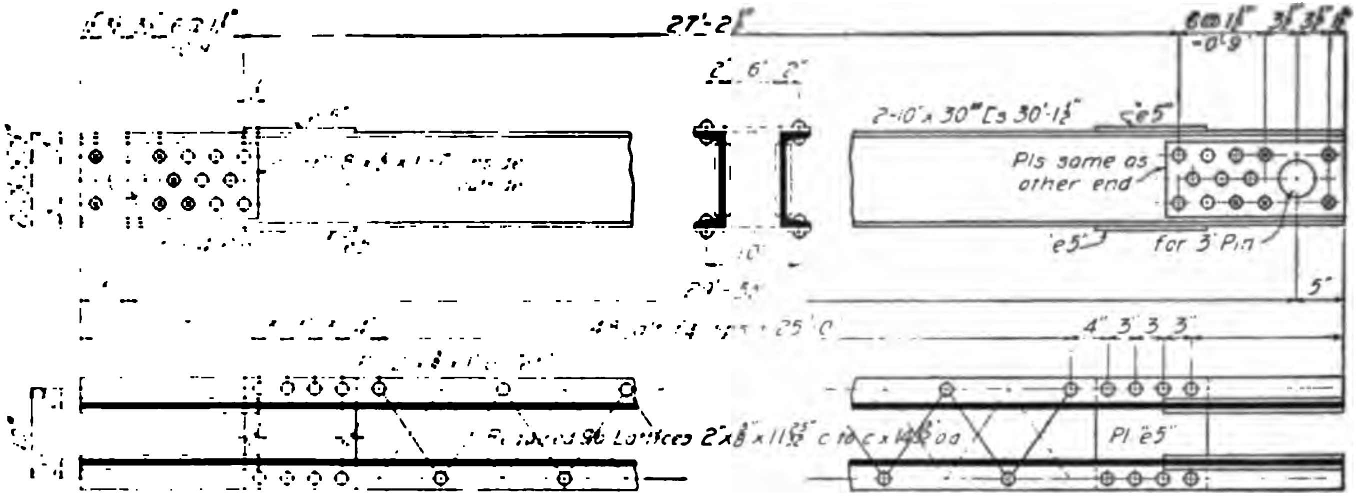
Built-up tension members must be symmetrical about the neutral axis.

Facilitation of Erection. In detailing, it should be kept in mind that while there are many ways to detail a piece so that the shop and field will get it right, yet some of them are such that the fabrication and the erection will be greatly facilitated if they are used. The rules to facilitate fabrication are the principles laid down in the previous pages. While experience is necessary in order that the erection will be facilitated by the correctly planned details of the draftsman, yet many points tending to this may be put in the form of rules or instructions. The following will, if attended to, tend to prevent delays and will facilitate erection.

(1) The first consideration for ease and safety in erection should be to so arrange all details, joints, and connections that a



ALPHA DELTA PHI CHAPTER-HOUSE AT CORNELL UNIVERSITY, ITHACA, NEW YORK
Dean and Dean, Architects, Chicago, Illinois
For other views see pages 58 and 130



Dotted lines indicate lattices at top

[illegible] Detail of Compression Member Made up of Channels, Lacing Bars, Tie Plates

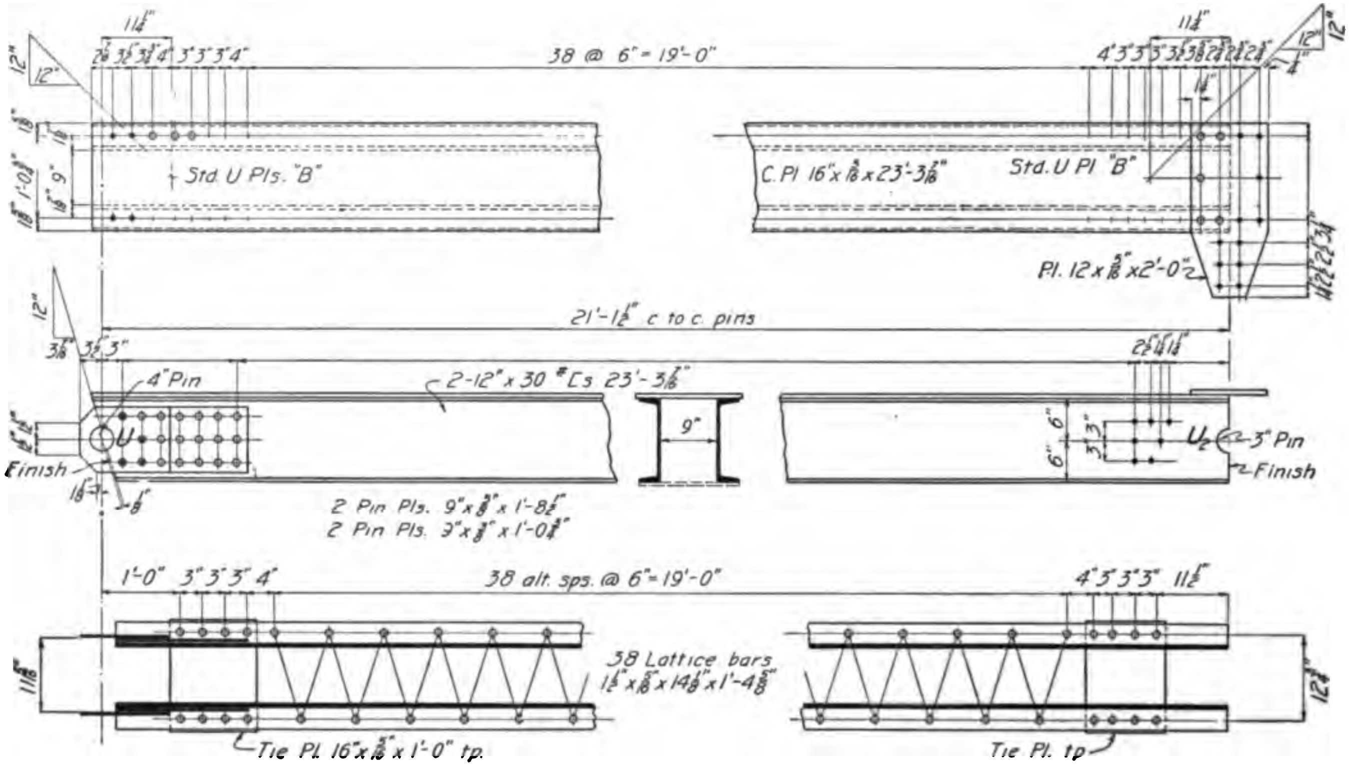


Fig. 84. Detail of Compression Member Using Cover Plates and Channels

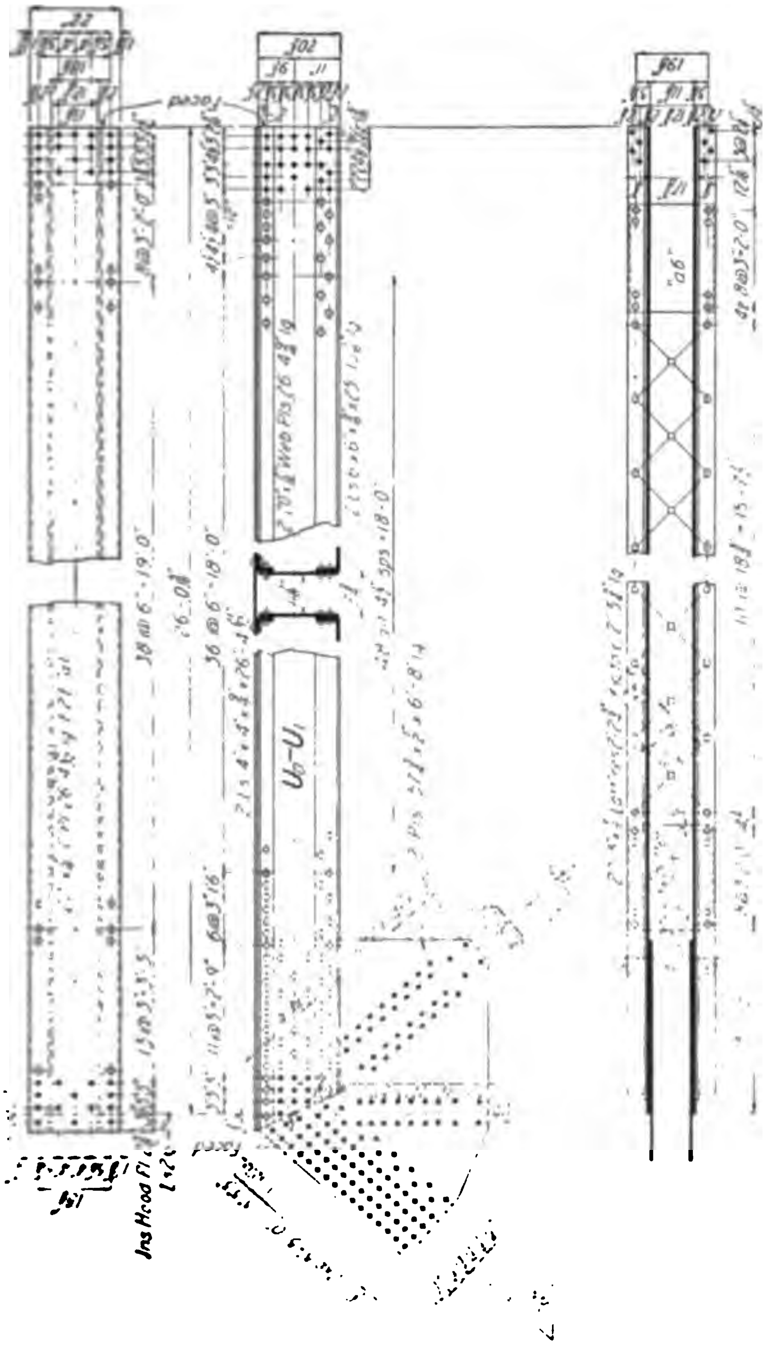


Fig. 85. Detail of a Top Chord of Riveted Railroad Bridge

structure may be connected, made self-sustaining and safe in the shortest time possible.

(2) Entering connections of any character should be avoided when possible, notably on top chords, floor beam and stringer connections, splices in girders, etc.

(3) When practicable, joints should be so arranged as to avoid having to put members together by entering them on end, as it is often impossible to get the necessary clearance in which to do this.

(4) In all through spans floor connections should be so arranged that the floor system can be put in place after the trusses or girders have been erected in their final position, and vice versa, so that the trusses or girders can be erected after the floor system has been set in place.

(5) All lateral bracing, hitch-plates, rivets in laterals, etc., should, as far as possible, be kept clear of the bottom of the ties, it being very expensive to cut out ties to clear such obstructions.

(6) Lateral plates should be shipped loose, or bolted on, so that they do not project outside of the member, whenever there is danger of them being broken off in unloading and handling.

(7) Loose fillers should be avoided. They should be tacked on with rivets, countersunk where necessary.

(8) In elevated railroad work, viaducts, and similar structures, where longitudinal girders frame into cross girders, shelf angles should be provided on the latter. In these structures the expansion joints should be so arranged that the rivets connecting the fixed span to the cross girder can be driven after the expansion span is in place.

(9) In viaducts, etc., two spans, abutting on a bent, should be so arranged that either span can be set in place entirely independent of the other. The same thing applies to girder spans of different depth resting on the same bent.

(10) Holes for anchor bolts should be so arranged that the holes in the masonry can be drilled and the bolts put in place after the structure has been erected complete. In concrete masonry they should be set very carefully according to data furnished by the Bridge Company.

(11) In structures consisting of more than one span a separate bed-plate should be provided for each shoe. This is particularly important where an old structure is to be replaced; if two shoes were put on one bed-plate or two spans connected on the same pin, it

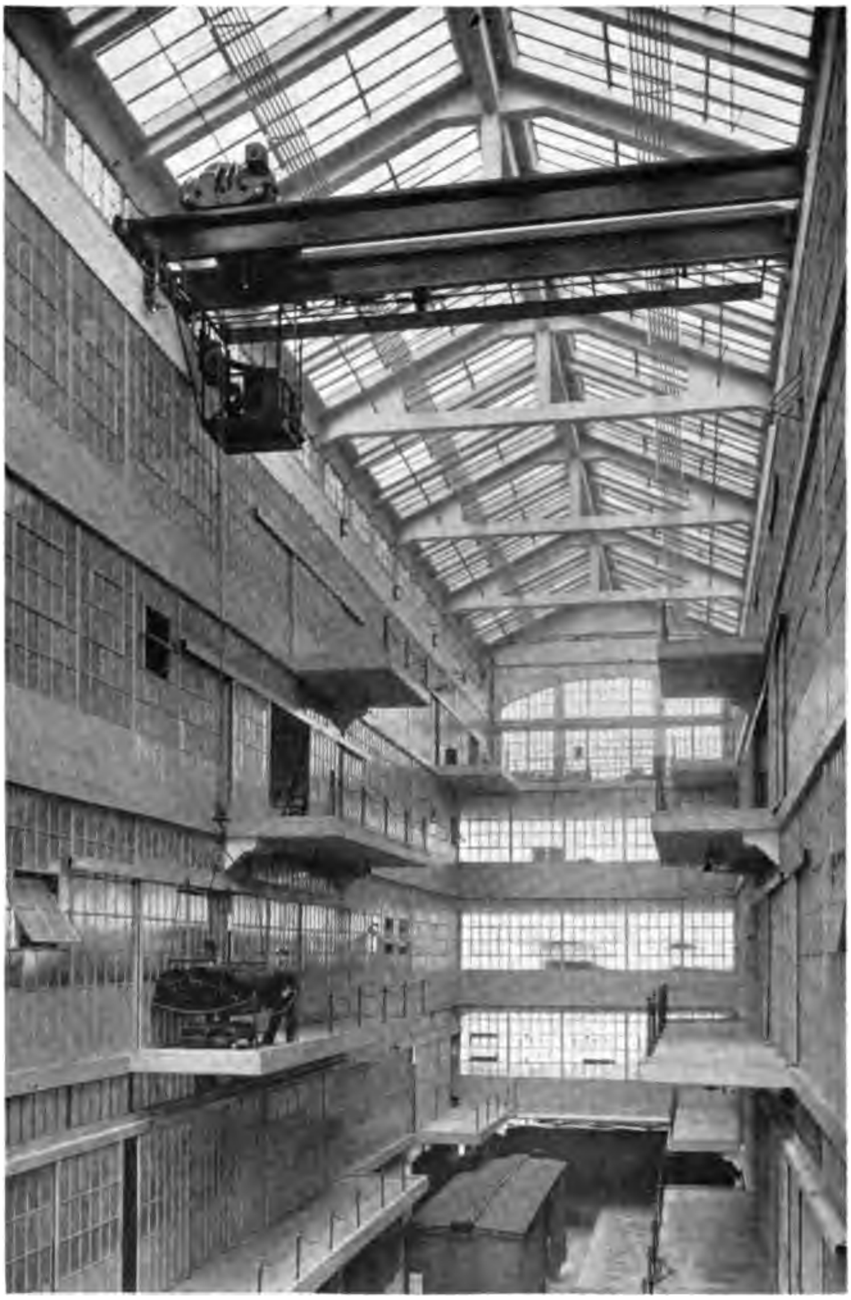
would necessitate removing two old spans in order to erect one new one.

(12) In pin-connected spans the sections of top chords nearest the center should be made with at least two pinholes. On skew spans the chord splices should be so located that two opposite panels can be erected without moving the traveler.

(13) Tie plates should be kept far enough away from the joints, and enough rivets should be countersunk inside the chord, to allow of eye bars and other members being easily set in place.

(14) Posts with channels or angles turned out and notched at the ends should, whenever possible, be avoided.

In conclusion, it may be said that the author has written this treatise with the idea of preventing the beginner from falling into the more common errors of judgment, as well as helping him to become proficient in detailing according to good common practice.



CHICAGO SERVICE BUILDING OF FORD MOTOR COMPANY, SHOWING MODERN CONCRETE CONSTRUCTION

The roof trusses are of concrete, and flat-slab floor construction is used throughout.

Courtesy of the Condron Company, Chicago

STATICS.

This subject, called Statics, is a branch of **Mechanics. It deals with** principles **relating** especially to forces **which act upon bodies at rest, and** with their useful applications.

There are two quite different methods of **carrying on the discussions and** computations. In one, the quantities **under consideration are** represented by lines and the discussion **is wholly by means of** geometrical figures, and computations are **carried out by means of** figures drawn to scale; this is called the *graphical method.* In the other, the quantities under consideration are **represented** by symbols as in ordinary Algebra and Arithmetic, **and the** discussions and computations are carried on by the methods **of those** branches and Trigonometry; this is called the *algebraic method.* In this paper, both methods are employed, and generally, **in a given case**, the more suitable of the two.

I. PRELIMINARY.

1. Force. The student, no doubt, has a **reasonably clear idea as to what is meant by** force, yet it may be well to repeat here a **few definitions** relative to it. By force is meant simply a *push* or *pull.* Every force has **magnitude**, and to express the magnitude **of a given** force we state how many times greater it is than some **standard** force. Convenient standards are those of weight and **these are almost** always used in this connection. Thus when we **speak of a** force of 100 pounds we mean a force equal to the **weight of** 100 pounds.

We say that a force has **direction**, and we mean by this the **direction in** which the force would move the body upon which it **acts if it** acted alone. Thus, Fig. 1 represents a body being **pulled** to the right by means of a cord; the direction of the force **exerted** upon the body is horizontal and to the right. The direction **may** be indicated by any line drawn in the figure parallel to **the cord** with an arrow on it pointing to the right.

We say also that a force has a **place of application**, and we **mean by that** the part or place on the body to which the force is applied. When the place of application is small so that it may be regarded as a point, it is called the "point of application." Thus the place of application of the pressure (push or force) which a locomotive wheel exerts on the rail is the part of the surface of the rail in contact with the wheel. For practically all purposes this pressure may be considered as applied at a point (the center of the surface of contact), and it is called the point of application of the force exerted by the wheel on the rail.

A force which has a point of application is said to have **a line of action,** and by this term is meant the line through the point of application of the force parallel to its direction. Thus, in the Fig. 1, the line of action of the force exerted on the body is the line representing the string. Notice clearly the distinction between the direction and line of action of the force; the direction of the force in the illustration could be represented by any horizontal line in the figure with an arrowhead upon it pointing toward the right, but the line of action can be represented only by the line representing the string, indefinite as to length, but definite in position.

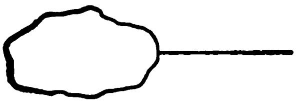


Fig. 1.

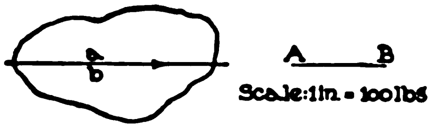
That part of the direction of a force which is indicated by means of the arrowhead on a line is called **the sense of the force.** Thus the sense of the force of the preceding illustration is toward the right and not toward the left.

2. Specification and Graphic Representation of a Force. For the purposes of statics, a force is completely specified or described if its

(1) magnitude, (2) line of action, and (3) sense are known or given.

These three elements of a force can be represented graphically, that is by a drawing. Thus, as already explained, the straight line (Fig. 1) represents the line of action of the force exerted upon the body; an arrowhead placed on the line pointing toward the right gives the sense of the force; and a definite length marked off on the line represents to some scale the magnitude of the force. For example, if the magnitude is 50 pounds, then to a scale of 100 pounds to the inch, one-half of an inch represents the magnitude of the force.

It is often convenient, especially when many forces are concerned in a single problem, to use two lines instead of one to represent a force—one to represent the magnitude and one the line of action, the arrowhead being placed on either. Thus Fig. 2 also represents the force of the preceding example, AB (one-half inch long) representing the magnitude of the force and *ab* its line of action. The line AB might have been drawn anywhere in the figure, but its length is definite, being fixed by the scale.



(a) (b)

Fig. 2.

The part of a drawing in which the body upon which forces act is represented, and in which the lines of action of the forces are drawn, is called the **space diagram** (Fig. 2*a*). If the body were drawn to scale, the scale would be a certain number of inches or feet to the inch. The part of a drawing in which the force magnitudes are laid off (Fig. 2*b*) is called by various names; let us call it the **force diagram.** The scale of a force diagram is always a certain number of pounds or tons to the inch.

3. **Notation.** When forces are represented in two separate diagrams, it is convenient to use a special notation, namely: a capital letter at each end of the line representing the magnitude of the force, and the same small letters on opposite sides of the line representing the action line of the force (see Fig. 2). When we wish to refer to a force, we shall state the capital letters used in the notation of that force; thus "force AB" means the force whose magnitude, action line, and sense are represented by the lines AB and *ab*.

In the algebraic work we shall usually denote a force by the letter F.

4. **Scales.** In this subject, scales will always be expressed in feet or pounds to an inch, or thus, 1 inch = 10 feet, 1 inch = 100 pounds, etc. The number of feet or pounds represented by one inch on the drawing is called the *scale number.*

To find the length of the line to represent a certain distance or force, divide the distance or force by the scale number; the quotient is the length to be laid off in the drawing. To find the

magnitude of a distance or a force represented by a certain line in a drawing, multiply the length of the line by the scale number; the product is the magnitude of the distance or force, as the case may be.

The scale to be used in making drawings depends, of course, upon how large the drawing is to be, and upon the size of the quantities which must be represented. In any case, it is convenient to select the scale number so that the quotients obtained by dividing the quantities to be represented may be easily laid off by means of the divided scale which is at hand.

Examples. 1. If one has a scale divided into 32nds, what is the convenient scale for representing 40 pounds, 32 pounds, 56 pounds, and 70 pounds?

According to the scale, 1 inch = 32 pounds, the lengths representing the forces are respectively:

$$\frac{40}{32} = 1\tfrac{1}{4};\ \frac{32}{32} = 1;\ \frac{56}{32} = 1\tfrac{3}{4};\ \frac{70}{32} = 2\tfrac{3}{16}\ \text{inches.}$$

Since all of these distances can be easily laid off by means of the "sixteenths scale," 1 inch = 32 pounds is convenient.

2. What are the forces represented by three lines, 1.20, 2.11, and 0.75 inches long, the scale being 1 inch = 200 pounds?

According to the rule given in the foregoing, we multiply each of the lengths by 200, thus:

$$1.20 \times 200 = 240 \text{ pounds.}$$
$$2.11 \times 200 = 422 \text{ pounds.}$$
$$0.75 \times 200 = 150 \text{ pounds.}$$

EXAMPLES FOR PRACTICE.

1. To a scale of 1 inch = 500 pounds, how long are the lines to represent forces of 1,250, 675, and 900 pounds?

Ans. 2.5, 1.35, and 1.8 inches

2. To a scale of 1 inch = 80 pounds, how large are the forces represented by 1¼ and 1.6 inches?

Ans. 100 and 128 pounds.

5. Concurrent and Non-concurrent Forces. If the lines of action of several forces intersect in a point they are called concurrent forces, or a concurrent system, and the point of intersection is called the *point of concurrence* of the forces. If the lines of action of several forces do not intersect in the same point, they are called non-concurrent, or a non-concurrent system.

We shall deal only with forces whose lines of action lie in the same plane. It is true that one meets with problems in which there are forces whose lines of action do not lie in a plane, but such problems can usually be solved by means of the principles herein explained.

6. **Equilibrium and Equilibrant.** When a number of forces act upon a body which is at rest, each tends to move it; but the effects of all the forces acting upon that body may counteract or neutralize one another, and the forces are said to be *balanced* or *in equilibrium*. Any one of the forces of a system in equilibrium balances all the others. A single force which balances a number of forces is called the *equilibrant* of those forces.

7. **Resultant and Composition.** Any force which would produce the same effect (so far as balancing other forces is concerned) as that of any system, is called the *resultant* of that system. Evidently the resultant and the equilibrant of a system of forces must be equal in magnitude, opposite in sense, and act along the same line.

The process of determining the resultant of a system of forces is called composition.

8. **Components and Resolution.** Any number of forces whose combined effect is the same as that of a single force are called *components* of that force. The process of determining the components of a force is called *resolution*. The most important case of this is the resolution of a force into two components.

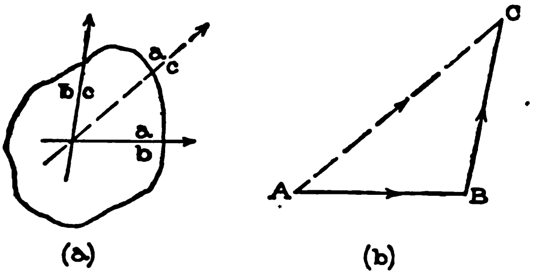
II. CONCURRENT FORCES; COMPOSITION AND RESOLUTION.

9. **Graphical Composition of Two Concurrent Forces.** *If two forces are represented in magnitude and direction by AB and BC (Fig. 3), the magnitude and direction of their resultant is represented by AC.* This is known as the "triangle law."

The line of action of the resultant is parallel to AC and passes through the point of concurrence of the two given forces; thus the line of action of the resultant is *ac*.

The law can be proved experimentally by means of two spring balances, a drawing board, and a few cords arranged as shown in

Fig. 4. The drawing board (not shown) is set up vertically, then from two nails in it the spring balances are hung, and these in turn support by means of two cords a small ring A from which a heavy body (not shown) is suspended. The ring A is in equilibrium under the action of three forces, a downward force equal to the weight of the suspended body, and two forces exerted by the upper cords whose values or magnitudes can be read from the spring balances. The first force is the equilibrant of the other two. Knowing the weight of the suspended body and the readings of the balances, lay off AB equal to the pull of the right-hand upper string according to some convenient scale, and BC parallel to the left-hand upper string and equal to the force exerted by it. It will then be found that the line joining A and C is vertical, and equals (by scale) the weight of the suspended body. Hence AC, with arrowhead pointing down, represents the equilibrant of the two upward pulls on the ring; and with arrowhead pointing up, it represents the resultant of those two forces.



(a) (b)

Fig. 3.

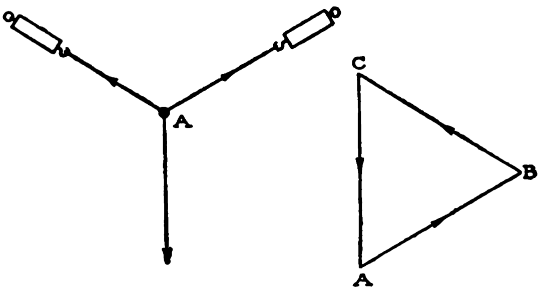


Fig. 4.

Notice especially how the arrowheads are related in the triangle (Fig. 3), and be certain that you understand this law before proceeding far, as it is the basis of most of this subject.

Examples. Fig. 5 represents a board 3 feet square to which forces are applied as shown. It is required to compound or find the resultant of the 100- and 80-pound forces.

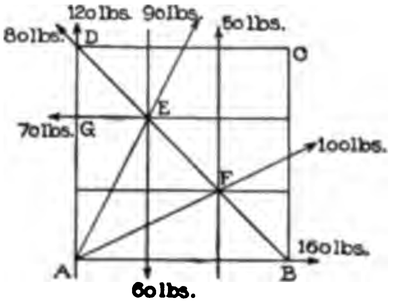


Fig. 5.

First we make a drawing of the board and mark upon it the lines of action of the two forces whose resultant is to be found, as in Fig. 6. Then by some convenient scale, as 100 pounds to the inch, lay off from any convenient point A, a line AB in the direction of the 100-pound force, and make AB one inch long, representing 100 pounds by the scale. Then from B lay off a line BC in the direction of the second force and make BC, 0.8 of an inch long, representing 80 pounds by the scale. Then the line AC, with the arrow pointing *from A to C*, represents the magnitude and direction of the resultant. Since AC equals 1.06 inch, the resultant equals

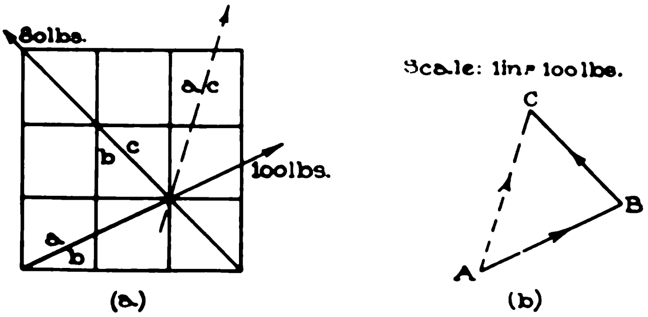


Fig. 6.

$$1.06 \times 100 = 106 \text{ pounds.}$$

The line of action of the resultant is *ac*, parallel to AC and passing through the intersection of the lines of action (the point of concurrence) of the given forces. To complete the notation, we mark these lines of action *ab* and *bc* as in the figure.

EXAMPLES FOR PRACTICE.*

1. Determine the resultant of the 100- and the 120-pound forces represented in Fig. 5.

Ans. The magnitude is 188 pounds; the force acts upward through A and a point 1.62 feet to the right of D.

2. Determine the resultant of the 120- and the 160-pound forces represented in Fig. 5.

Ans. The magnitude is 200 pounds; the force acts upward through A and a point 9 inches below C.

10. Algebraic Composition of Two Concurrent Forces. If the angle between the lines of action of the two forces is not 90 degrees, the algebraic method is not simple, and the graphical is usually preferable. If the angle is 90 degrees, the algebraic method is usually the shorter, and this is the only case herein explained.

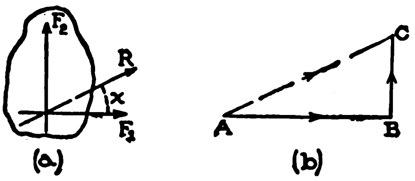


Fig. 7.

Let F_1 and F_2 be two forces acting through some point of a body as represented in Fig. 7*a*. AB and BC represent the magnitudes and direction of F_1 and F_2 respectively; then, according to the triangle law (Art. 9), AC represents the magnitude and direction of the resultant of F_1 and F_2, and the line marked R (parallel to AC) is the line of action of that resultant. Since ABC is a right triangle,

$$(AC)^2 = (AB)^2 + (BC)^2$$

and,

$$\tan CAB = \frac{BC}{AB}.$$

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[*] Use sheets of paper not smaller than large letter size, and devote a full sheet to each example. In reading the answers to these examples, remember that the board on which the forces act was stated to be 3 feet square.

Now let R denote the resultant. Since AC, AB, and BC represent R, F_1, and F_2 respectively, and angle CAB = x,

$$R^2 = F_1^2 + F_2^2; \text{ or } R = \sqrt{F_1^2 + F_2^2};$$

and,

$$\tan x = F_2 \div F_1.$$

By the help of these two equations we compute the magnitude of the resultant and inclination of its line of action to the force F_1.

Example. It is required to determine the resultant of the 120- and the 160-pound forces represented in Fig. 5.

Let us call the 160-pound force F_1; then,

$$R = \sqrt{160^2 + 120^2} = \sqrt{25{,}600 + 14{,}400}$$

$$= \sqrt{40{,}000} = 200 \text{ pounds};$$

and,

$$\tan x = \frac{120}{160} = \frac{3}{4}; \text{ hence } x = 36^\circ\ 52'.$$

The resultant therefore is 200 pounds in magnitude, acts through A (Fig. 5) upward and to the right, making an angle of 36° 52′ with the horizontal.

EXAMPLES FOR PRACTICE.

1. Determine the resultant of the 50- and 70-pound forces represented in Fig. 5.

Ans. { R = 86 pounds;
angle between R and 70-pound force = 35° 32′.

2. Determine the resultant of the 60- and 70-pound forces represented in Fig 5.

Ans. { R = 92.2 pounds;
angle between R and 70-pound force = 40° 36′.

11. **Force Polygon.** If lines representing the magnitudes and directions of any number of forces be drawn continuous and so that the arrowheads on the lines point the same way around on the series of lines, the figure so formed is called the *force polygon* for the forces. Thus ABCD (Fig. 8) is a force polygon for the 80-, 90-, and 100-pound forces of Fig. 5, for AB, BC, and CD represent the magnitudes and directions of those forces respectively, and the arrowheads point in the same way around, from A to D.

A number of force polygons can be drawn for any system of forces, no two alike. Thus $A_1 B_1 C_1 D_1$ and $A_2 B_2 C_2 D_2$ are other force polygons for the same three forces, 80, 90, and 100 pounds. Notice that $A_3 B_3 C_3 D_3$ is not a force polygon for the three forces although the lines represent the three forces in magnitude and direction. The reason why it is not a force polygon is that the arrowheads do not all point the same way around.

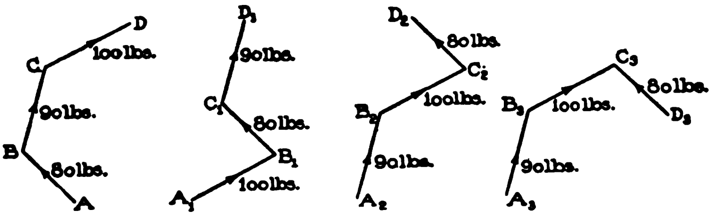


Fig. 8.

A force polygon is not necessarily a closed figure. If a force polygon closes for a system of concurrent forces, then evidently the resultant equals zero.

EXAMPLE FOR PRACTICE.

Draw to the same scale as many different force polygons as you can for the 100-, 120- and 160-pound forces of Fig. 5. Bear in mind that the arrowheads on a force polygon point the same way around.

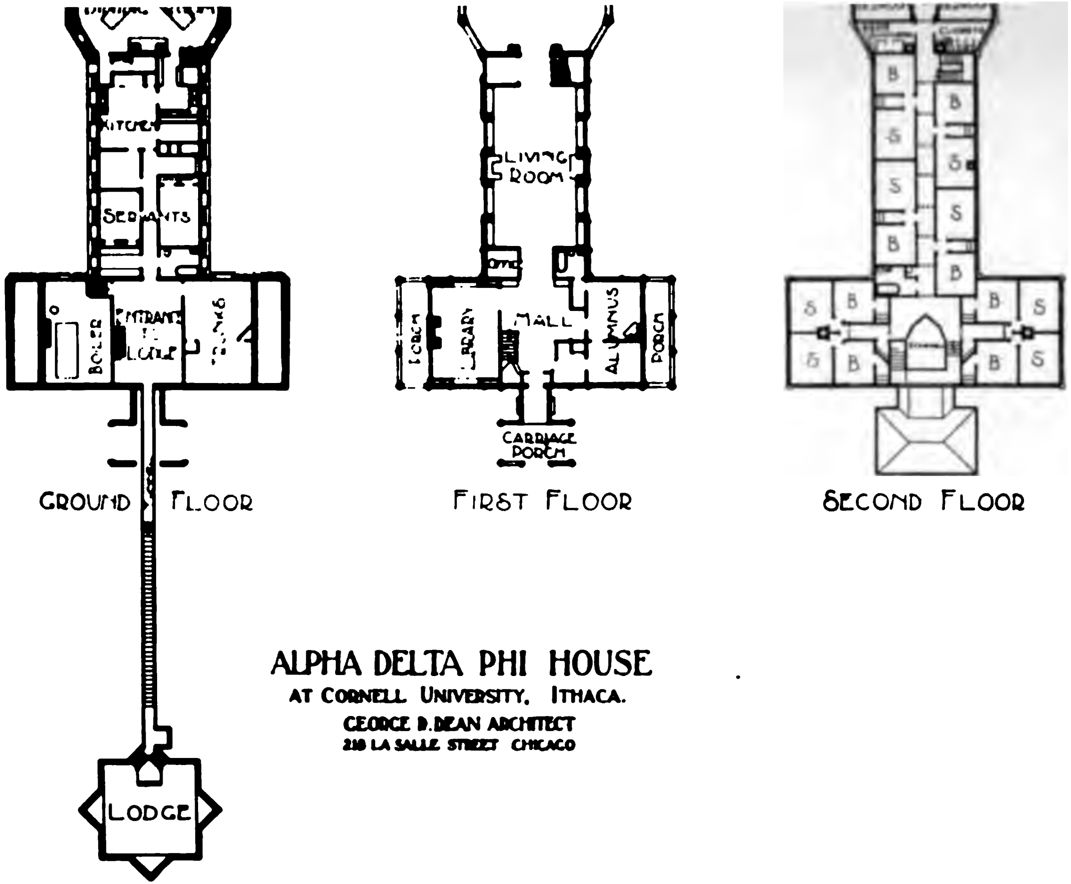
12. Composition of More Than Two Concurrent Forces. The graphical is much the simpler method; therefore the algebraic one will not be explained. The following is a rule for performing the composition graphically:

(1). Draw a force polygon for the given forces.

(2). Join the two ends of the polygon and place an arrowhead on the joining line pointing from the beginning to the end of the polygon. That line then represents the magnitude and direction of the resultant.

(3). Draw a line through the point of concurrence of the given forces parallel to the line drawn as directed in (2). This line represents the action line of the resultant.

Example. It is required to determine the resultant of the four forces acting through the point E (Fig. 5).



PLANS OF ALPHA DELTA PHI CHAPTER-HOUSE AT CORNELL UNIVERSITY, ITHACA, NEW YORK

Dean and Dean, Architects, Chicago, Illinois

For other views see pages 58 and 122

First, make a drawing of the board and indicate the lines of action of the forces as shown in Fig. 9, but without lettering. Then to construct a force polygon, draw from any convenient point A, a line in the direction of one of the forces (the 70-pound force), and make AB equal to 70 pounds according to the scale (70 ÷ 100 = 0.7 inch). Then from B draw a line in the direction of the next force (80-pound). and make BC equal to 0.8 inch, representing 80 pounds. Next draw a line from C in the direction of the third force (90-pound), and make CD equal to 0.9 inch, representing 90 pounds. Finally draw a line from D in the direction of the last force, and make DE equal to 0.6 inch, representing 60 pounds. The force polygon is ABCDE, beginning at A and ending at E.

The second step is to connect A and E and place an arrowhead on the line pointing from A to E. This represents the magnitude and direction of the resultant. Since AE = 1.16 inches, the resultant is a force of

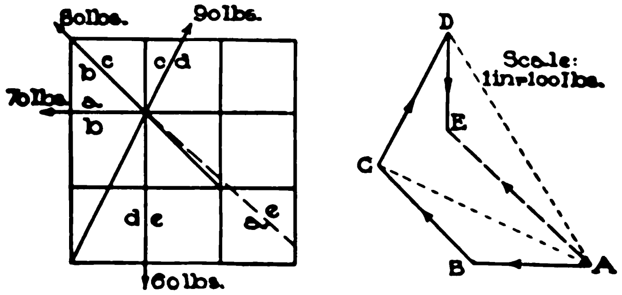


Fig. 9.

$$1.16 \times 100 = 116 \text{ pounds.}$$

The third step is to draw a line *ae* through the point of concurrence and parallel to AE. This is the line of action of the resultant. (To complete the notation the lines of action of the 70-, 80-, 90- and 60-pound forces should be marked *ab*, *bc*, *cd*, and *de* respectively.)

That the rule for composition is correct can easily be proved. According to the triangle law, AC (Fig. 9), with arrowhead pointing from A to C, represents the magnitude and direction of the resultant of the 70- and 80-pound forces. According to the law, AD, with arrowhead pointing from A to D, represents the magnitude and direction of the resultant of AC and the 90-pound force, hence also of the 70-, 80-, and 90-pound forces. According to the law, AE with arrowhead pointing from A to E, represents the magnitude and direction of the resultant of AD and the 60-pound force. Thus we see that the foregoing rule and the triangle law lead to the same result, but the application of the rule is shorter as in it we do not need the lines AC and AD.

EXAMPLES FOR PRACTICE.

1. Determine the resultant of the four forces acting through the point A (Fig. 5).

Ans. { 380 pounds acting upward through A and a point 0.45 feet below C.

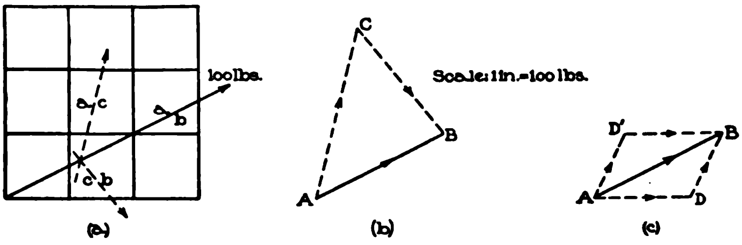


Fig. 10.

2. Determine the resultant of the three forces acting at the point F (Fig. 5).

Ans. { 155 pounds acting upward through F and a point 0.57 feet to left of C.

13. Graphical Resolution of Force into Two Concurrent Components. This is performed by applying the triangle law inversely. Thus, if it is required to resolve the 100-pound force of Fig. 5 into two components, we draw first Fig. 10 (a) to show the line of action of the force, and then AB, Fig. 10 (b), to represent the magnitude and direction. Then draw from A and B any two lines which intersect, mark their intersection C, and place arrowheads on AC and CB, pointing from A to C and from C to B. Also draw two lines in the space diagram parallel to AC and CB and so that they intersect on the line of action of the 100-pound force, *ab*.

The test of the correctness of a solution like this is to take the two components as found, and find their resultant; if the resultant thus found agrees in magnitude, direction, and sense with the given force (originally resolved), the solution is correct.

Notice that the solution above given is not definite, for the lines drawn from A and B were drawn at random. A force may therefore be resolved into two components in many ways. If, however, the components have to satisfy conditions, there may be but one solution. In the most important case of resolution, the lines of action of the components are given; this case is definite, there being but one solution, as is shown in the following example.

Example. It is required to resolve the 100-pound force (Fig. 5) into two components acting in the lines AE and AB.

Using the space diagram of Fig. 10, draw a line AB in Fig. 10 (c) to represent the magnitude and direction of the 100-pound force, and then a line from A parallel to the line of action of either of the components, and a line from B parallel to the other, thus locating D (or D′). Then AD and DB (or AD′ and D′B) represent the magnitudes and directions of the required components.

EXAMPLES FOR PRACTICE.

1. Resolve the 160-pound force of Fig. 5 into components which act in AF and AE.

Ans. The first component equals 238½ pounds, and its sense is from A to F; the second component equals 119½ pounds, and its sense is from E to A.

2. Resolve the 50-pound force of Fig. 5 into two components, acting in FA and FB.

Ans. The first component equals 37.3 pounds, and its sense is from A to F; the second component equals 47.0 pounds, and its sense is from B to F.

14. Algebraic Resolution of a Force Into Two Components. If the angle between the lines of action of the two components is not 90 degrees, the algebraic method is not simple and the graphical method is usually preferable. When the angle is 90 degrees, the algebraic method is usually the shorter, and this is the only case herein explained.

Let F (Fig. 11) be the force to be resolved into two components acting in the lines OX and OY. If AB is drawn to represent the magnitude and direction of F, and lines be drawn from A and B parallel to OX and OY, thus locating C, then AC and BC with arrowheads as shown represent the magnitudes and directions of the required components.

Now if F' and F'' represent the components acting in OX and OY, and x and y denote the angles between F and F', and F and F'' respectively, then AC and BC represent F' and F'', and the angles BAC and ABC equal x and y respectively. From the right triangle ABC it follows that

$$F' = F \cos x, \quad \text{and } F'' = F \cos y.$$

If a force is resolved into two components whose lines of action are at right angles to each other, each is called a *rectangular component* of that force. Thus F' and F'' are rectangular components of F.

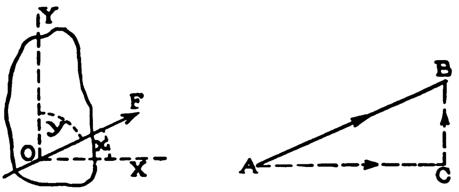


Fig. 11.

The foregoing equations show that *the rectangular component of a force along any line equals the product of the force and the cosine of the angle between the force and the line.* They show also that *the rectangular component of a force along its own line of action equals the force, and its rectangular component at right angles to the line of action equals zero.*

Examples. 1. A force of 120 pounds makes an angle of 22 degrees with the horizontal. What is the value of its component along the horizontal? *

Since cos 22° = 0.927, the value of the component equals 120 × 0.927 = 111.24 pounds.

2. What is the value of the component of the 90-pound force of Fig. 5 along the vertical?

First we must find the value of the angle which the 90-pound force of Fig. 5 makes with the vertical.

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* When nothing is stated herein as to whether a component is rectangular or not, then rectangular component is meant.

Since $\tan EAG = \frac{EG}{AG} = \frac{1}{2}$, angle $EAG = 26° 34'$.

Hence the value of the desired component equals

$$90 \times \cos 26° 34' = 90 \times 0.8944 = 80.50 \text{ pounds.}$$

EXAMPLES FOR PRACTICE.

1. Compute the horizontal and vertical components of a force of 80 pounds whose angle with the horizontal is 60 degrees

Ans. $\begin{cases} \text{40 pounds.} \\ \text{69.28 pounds.} \end{cases}$

2. Compute the horizontal and vertical components of the 100-pound force in Fig. 5. What are their senses?

Ans. $\begin{cases} \text{89.44 pounds to the right.} \\ \text{44.72 pounds upwards.} \end{cases}$

3. Compute the component of the 70-pound force in Fig. 5 along the line EA. What is the sense of the component?

Ans. 31.29 pounds; E to A.

III. CONCURRENT FORCES IN EQUILIBRIUM.

15. Condition of Equilibrium Defined. By condition of equilibrium of a system of forces is meant a relation which they must fulfill in order that they may be in equilibrium or a relation which they fulfill when they are in equilibrium.

In order that any system may be in equilibrium, or be balanced, their equilibrant, and hence their resultant, must be zero, and this is a condition of equilibrium. If a system is known to be in equilibrium, then, since the forces balance among themselves, their equilibrant and hence their resultant also equals zero. This (the necessity of a zero resultant) is known as the general condition of equilibrium for it pertains to all kinds of force systems. For special kinds of systems there are special conditions, some of which are explained in the following.

16. Graphical Condition of Equilibrium. *The "graphical condition of equilibrium" for a system of concurrent forces is that the polygon for the forces must close.* For if the polygon closes, then the resultant equals zero as was pointed out in Art 11.

By means of this condition we can solve problems relating to concurrent forces which are known to be in equilibrium. The most common and practically important of these is the following:

The forces of a concurrent system in equilibrium are all known except two, but the lines of action of these two are known; it is required to determine their magnitudes and directions. This problem arises again and again in the "analysis of trusses" (Arts. 23 to 26) but will be illustrated first in simpler cases.

Example. 1. Fig. 12 represents a body resting on an inclined plane being prevented from slipping down by a rope fastened to it as shown. It is required to determine the pull or tension on the rope and the pressure of the plane if the body weighs 120 pounds and the surface of the plane is perfectly smooth.*

There are three forces acting upon the body, namely, its weight directly downwards, the pull of the rope and the reaction or pressure of the plane which, as explained in the footnote, is perpendicular to the plane. We now draw the polygon for these forces making it close; thus draw AB (1.2 inches long) to represent the magnitude and direction of the weight, 120 pounds, then from A a line parallel to either one of the other forces, from B a line parallel to the third, and mark the intersection of these two lines C; then ABCA is the polygon. Since the arrowhead on AB must point down and since the arrowheads in any force polygon must point the same way around, those on BC and CA must point as shown.

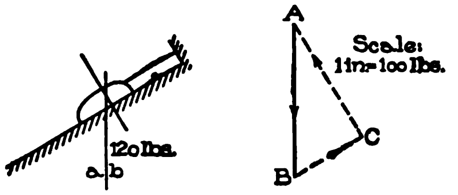

Fig. 12.

Hence BC (0.6 inch, or 60 pounds) represents the magnitude and direction of the pull of the rope and CA (1.04 inches, or 104 pounds) represents the magnitude and direction of the pressure of the plane on the body.

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* By "a perfectly smooth" surface is meant one which offers no resistance to the sliding of a body upon it. Strictly, there are no such surfaces, as all real surfaces exert more or less frictional resistance. But there are surfaces which are practically perfectly smooth. We use perfectly smooth surfaces in some of our illustrations and examples for the sake of simplicity, for we thus avoid the force of friction, and the reaction or force exerted by such a surface on a body resting upon it is perpendicular to the surface.

2. A body weighing 200 pounds is suspended from a small ring which is supported by means of two ropes as shown in Fig. 13. It is required to determine the pulls on the two ropes.

There are three forces acting on the ring, namely the downward force equal to the weight of the body and the pulls of the two ropes. Since the ring is at rest, the three forces balance or are in equilibrium, and hence their force polygon must close. We proceed to draw the polygon and in making it close, we shall determine the values of the unknown pulls. Thus, first draw AB (1 inch long) to represent the magnitude and direction of the known force, 200 pounds; the arrowhead on it must point down. Then from A a line parallel to one of the ropes and from B a line parallel to the other and mark their intersection C. ABCA is the polygon for the three forces, and since in any force polygon the arrows point the same way around, we place arrowheads on BC and CA as shown. Then BC and CA represent the magnitudes and directions of the pulls exerted on the ring by the right- and left-hand ropes respectively.

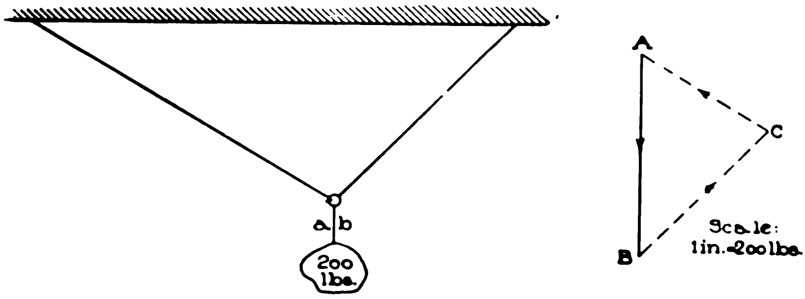

Fig. 13.

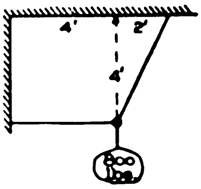

Fig. 14.

BC 0.895 inches and represents 179 pounds.
CA 0.725 inches and represents 145 pounds.

The directions of the pulls are evident in this case and the arrowheads are superfluous, but they are mentioned to show how to place them and what they mean so that they may be used when necessary. To complete the notation, the rope at the right should be marked *bc* and the other *ca*.

EXAMPLES FOR PRACTICE.

1. Fig. 14 represents a body weighing 800 pounds suspended from a ring which is supported by two ropes as shown. Compute the pulls on the ropes.

$$\text{Ans.}\begin{cases}\text{Pull in the horizontal rope} = 400 \text{ pounds.}\\ \text{Pull in the inclined rope} = 894 \text{ pounds.}\end{cases}$$

2. Suppose that in Fig. 12 the rope supporting the body on the plane is so fastened that it is horizontal. Determine the pull on the rope and the pressure on the plane if the inclination of the plane to the horizontal is 30 degrees and the body weighs 120 pounds.

$$\text{Ans.}\begin{cases}\text{Pull} = 68.7 \text{ pounds.}\\ \text{Pressure} = 138 \text{ pounds.}\end{cases}$$

3. A sphere weighing 400 pounds rests in a V-shaped trough, the sides of which are inclined at 60 degrees with the horizontal. Compute the pressures on the sphere.

Ans. 400 pounds.

17. Algebraic Conditions of Equilibrium. Imagine each one of the forces of a concurrent system in equilibrium replaced by its components along two lines at right angles to each other, horizontal and vertical for example, through the point of concurrence. Evidently the system of components would also be in equilibrium. Now since the components act along one of two lines (horizontal or vertical), all the components along each line must balance among themselves for if either set of components were not balanced, the body would be moved along that line. Hence we state that the conditions of equilibrium of a system of concurrent forces are that the resultants of the two sets of components of the forces along any two lines at right angles to each other must equal zero.

If the components acting in the same direction along either of the two lines be given the plus sign and those acting in the other direction, the negative sign, then it follows from the foregoing that the condition of equilibrium for a concurrent system is that *the algebraic sums of the components of the forces along each of two lines at right angles to each other must equal zero.*

Examples. 1. It is required to determine the pull on the rope and the pressure on the plane in Example 1, Art. 16 (Fig. 12), it being given that the inclination of the plane to the horizontal is 30 degrees.

Let us denote the pull of the rope by F_1 and the pressure of the plane by F_2. The angles which these forces make with the horizontal are 30° and 60°, respectively; hence

the horizontal component of $F_1 = F_1 \times \cos 30° = 0.8660\ F_1$,
and " " " " $F_2 = F_2 \times \cos 60° = 0.5000\ F_2$;
also " " " " the weight $= 0$.

The angles which F_1 and F_2 make with the vertical are 60° and 30° respectively, hence

the vertical component of $F_1 = F_1 \times \cos 60° = 0.5000\ F_1$,
and the vertical component of $F_2 = F_2 \times \cos 30° = 0.8660\ F_2$;
also the vertical component of the weight $= 120$.

Since the three forces are in equilibrium, the horizontal and the vertical components are balanced, and hence

$$0.866\ F_1 = 0.5\ F_2$$

and

$$0.5\ F_1 + 0.866\ F_2 = 120.$$

From these two equations F_1 and F_2 may be determined; thus from the first,

$$F_2 = \frac{0.866}{0.5}\ F_1 = 1.732\ F_1.$$

Substituting this value of F_2 in the second equation we have

$$0.5\ F_1 + 0.866 \times 1.732\ F_1 = 120,$$

or

$$2\ F_1 = 120;$$

hence,

$$F_1 = \frac{120}{2} = 60 \text{ pounds},$$

and

$$F_2 = 1.732 \times 60 = 103.92 \text{ pounds}.$$

2. It is required to determine the pulls in the ropes of Fig. 13 by the algebraic method, it being given that the angles which the left- and right-hand ropes make with the ceiling are 30 and 70 degrees respectively and the body weighs 100 pounds.

Let us denote the pulls in the right- and left-hand ropes by F_1 and F_2 respectively. Then

the horizontal component of $F_1 = F_1 \times \cos 70^\circ = 0.342\ F_1$,
the horizontal component of $F_2 = F_2 \times \cos 30^\circ = 0.866\ F_2$,
the horizontal component of the weight $= 0$,
the vertical component of $F_1 = F_1 \times \cos 20^\circ = 0.9397\ F_1$,
the vertical component of $F_2 = F_2 \times \cos 60^\circ = 0.500\ F_2$,
and the vertical component of the weight $= 100$.

Now since these three forces are in equilibrium, the horizontal and the vertical components balance; hence

$$0.342\ F_1 = 0.866\ F_2$$

and

$$0.9397\ F_1 + 0.5\ F_2 = 100.$$

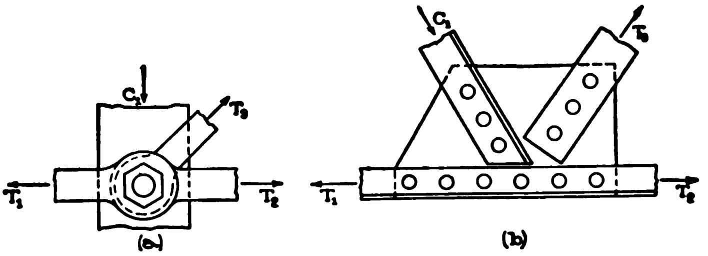

Fig. 15.

These equations may be solved for the unknown forces; thus from the first,

$$F_1 = \frac{0.866}{0.342} F_2 = 2.532\ F_2.$$

Substituting this value of F_1 in the second equation, we get

$$0.9397 \times 2.532\ F_2 + 0.5\ F_2 = 100,$$

or,

$$2.88\ F_2 = 100;$$

hence

$$F_2 = \frac{100}{2.88} = 34.72 \text{ pounds},$$

and

$$F_1 = 2.532 \times 34.72 = 87.91 \text{ pounds}.$$

EXAMPLES FOR PRACTICE.

1. Solve Ex. 1, Art. 16 algebraically. (First determine the angle which the inclined rope makes with the horizontal; you should find it to be 63° 26′.)

2. Solve Ex. 2, Art. 16 algebraically.
3. Solve Ex. 3, Art. 16 algebraically.

IV. ANALYSIS OF TRUSSES; "METHOD OF JOINTS."

18. **Trusses.** A truss is a frame work used principally to support loads as in roofs and bridges. Fig. 16, 25, 26 and 27 represent several forms of trusses. The separate bars or rods, $\overline{12}$, $\overline{23}$, etc. (Fig. 16) are called *members* of the truss and all the parts immediately concerned with the connection of a number of members at one place constitute a *joint*. A "pin joint" is shown in Fig. 15 (*a*) and a "riveted joint" in 15 (*b*).

19. **Truss Loads.** The loads which trusses sustain may be classified into fixed, or dead, and moving or live loads. A fixed, or dead load, is one whose place of application is fixed with reference to the truss, while a moving or live load is one whose place of application moves about on the truss.

Roof truss loads are usually fixed, and consist of the weight of the truss, roof covering, the snow, and the wind pressure, if any. Bridge truss loads are fixed and moving, the first consisting of the weights of the truss, the floor or track, the snow, and the wind pressure, and the second of the weight of the passing trains or wagons.

In this paper we shall deal only with trusses sustaining fixed loads, trusses sustaining moving loads being discussed later.

Weight of Roof Trusses. Before we can design a truss, it is necessary to make an estimate of its own weight; the actual weight can be determined only after the truss is designed. There are a number of formulas for computing the probable weight of a truss, all derived from the actual weights of existing trusses. If W denotes the weight of the truss, l the span or distance between supports in feet and a the distance between adjacent trusses in feet, then for steel trusses

$$W = al\left(\frac{l}{25} + 1\right);$$

and the weight of a wooden truss is somewhat less.

Roof Covering. The beams extending between adjacent trusses to support the roof are called *purlins*. On these there are sometimes placed lighter beams called *rafters* which in turn support *roof boards* or "*sheathing*" and the other covering. Sometimes the purlins are spaced closely, no rafters being used.

The following are weights of roof materials in *pounds per square foot* of roof surface:

Sheathing: Boards, 3 to 5.

Shingling: Tin, 1; wood shingles, 2 to 3; iron, 1 to 3; slate, 10; tiles, 12 to 25.

Rafters: 1.5 to 3.

Purlins: Wood, 1 to 3; iron, 2 to 4.

Snow Loads. The weight of the snow load that may have to be borne depends, of course, on location. It is usually taken from 10 to 30 pounds per square foot of area covered by the roof.

Wind Pressure. Wind pressure per square foot depends on the velocity of the wind and the inclination of the surface on which it blows to the direction of the wind. A horizontal wind blowing at 90 miles per hour *produces a pressure of about 40 pounds per square foot on a surface perpendicular to the wind, while on surfaces inclined, the pressures are as follows:

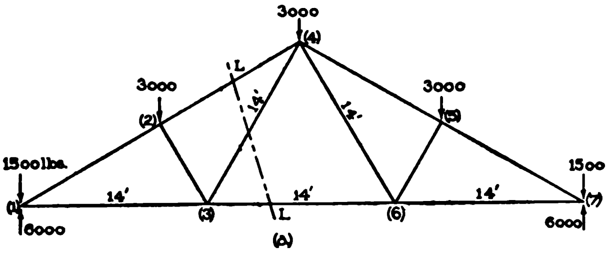

Fig. 16.

10° to horizontal, 15 lb. per sq. ft.
20° to horizontal, 24 lb. per sq. ft.
30° to horizontal, 32 lb. per sq. ft.
40° to horizontal, 36 lb. per sq. ft.
50°-90° to horizontal, 40 lb. per sq. ft.

The wind pressure on an inclined surface is practically perpendicular to the surface.

20. **Computation of "Apex Loads."** The weight of the roof covering including rafters and purlins comes upon the trusses at the points where they support the purlins; likewise the

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* For pressure per sq. ft. using other values than 90 miles, use formula $P = .004\ V^2$, where P = pressure per sq. ft. in lb. and V = velocity of wind in miles per hr.

pressure due to wind and snow. Sometimes all the purlins are supported at joints; in such cases the loads mentioned act upon the truss at its joints. However, the roof, snow, and wind loads are always assumed to be applied to the truss at the upper joints of the trusses. This assumption is equivalent to neglecting the bending effect due to the pressure of those purlins which are not supported at joints. This bending effect can be computed separately.

The weight of the truss itself is assumed to come upon the truss at its upper joints; this, of course, is not exactly correct. Most of the weight does come upon the upper joints for the upper members are much heavier than the lower and the assumption is in most cases sufficiently correct.

Examples. 1. It is required to compute the apex loads for the truss represented in Fig. 16, it being of steel, the roof such that it weighs 15 pounds per square foot, and the distance between adjacent trusses 14 feet.

The span being 42 feet, the formula for weight of truss (Art. 19) becomes

$$14 \times 42 \left(\frac{42}{25} + 1\right) = 1{,}575.84 \text{ pounds.}$$

The length 14 scales about 24¼ feet, hence the area of roofing sustained by one truss equals

$$48\tfrac{1}{2} \times 14 = 679 \text{ square feet,}$$

and the weight of the roofing equals

$$679 \times 15 = 10{,}185 \text{ pounds.}$$

The total load equals

$$1{,}575.84 + 10{,}185 = 11{,}760.84 \text{ pounds.}$$

Now this load is to be proportioned among the five upper joints, but joints numbered (1) and (7) sustain only one-half as much load as the others. Hence for joints (1) and (7) the loads equal

$$\frac{1}{8} \text{ of } 11{,}760 = 1{,}470,$$

and for (2), (4) and (5) they equal

$$\frac{1}{4} \text{ of } 11{,}760 = 2{,}940 \text{ pounds.}$$

As the weight of the truss is only estimated, the apex loads would be taken as 1,500 and 3,000 pounds for convenience.

2. It is required to compute the apex loads due to a snow load on the roof represented in Fig. 16, the distance between trusses being 14 feet.

The horizontal area covered by the roof which is sustained by one truss equals

$$42 \times 14 = 588 \text{ square feet.}$$

If we assume the snow load equal to 10 pounds per horizontal square foot, than the total snow load borne by one truss equals

$$588 \times 10 = 5{,}880 \text{ pounds.}$$

This load divided between the upper joints makes

$$\frac{1}{8} \times 5{,}880 = 735 \text{ pounds}$$

at joints (1) and (7); and

$$\frac{1}{4} \times 5{,}880 = 1{,}470 \text{ pounds}$$

at the joints (2), (4), and (5).

3. It is required to compute the apex loads due to wind pressure on the truss represented in Fig. 16, the distance between trusses being 14 ft.

The inclination of the roof to the horizontal can be found by measuring the angle from a scale drawing with a protractor or by computing as follows : The triangle $\overline{346}$ is equilateral, and hence its angles equal 60 degrees and the altitude of the triangle equals

$$14 \times \sin 60 = 12.12 \text{ feet.}$$

The tangent of the angle $\overline{413}$ equals

$$\frac{12.12}{21} = 0.577,$$

and hence the angle equals 30 degrees.

According to Art. 19, 32 pounds per square foot is the proper value of the wind pressure. Since the wind blows only on one side of the roof at a given time, the pressure sustained by one truss is the wind pressure on one half of the area of the roof sustained by one truss, that is

$$14 \times 24\tfrac{1}{4} \times 32 = 10{,}864 \text{ pounds.}$$

One half of this pressure comes upon the truss at joint (2) and one fourth at joints (1) and (4).

EXAMPLES FOR PRACTICE.

1. Compute the apex loads due to weight for the truss represented in Fig. 27 if the roofing weighs 12 pounds per square foot and the trusses (steel) are 12 feet apart.

Ans. As shown in Fig. 27.

2. Compute the apex loads due to a snow load of 20 pounds per square foot on the truss of Fig. 25, the distance between trusses being 15 feet.

Ans.
For joints (4) and (7), 1,200 pounds.
For joints (1) and (3), 3,600 pounds.
For joint (2), 4,800 pounds.

3. Compute the apex loads due to wind for the truss of Fig. 26, the distance between trusses being 15 feet.

Ans. Pressure equals practically 29 pounds per square foot. Load at joint (2) is 4,860 and at joints (1) and (3) 2,430 pounds.

21. **Stress in a Member.** If a truss is loaded only at its joints, its members are under either tension or compression, but the weight of a member tends to bend it also, unless it is vertical. If purlins rest upon members between the joints, then they also bend these members. We have therefore tension members, compression members, and members subjected to bending stress combined with tension or compression. Calling simple tension or compression *direct stress* as in "Strength of Materials," then the process of determining the direct stress in the members is called "analyzing the truss."

22. **Forces at a Joint.** By "forces at a joint" is meant all the loads, weights, and reactions which are applied there and the forces which the members exert upon it. These latter are pushes for compression members and pulls for tension members, in each case acting along the axis of the member. Thus, if the horizontal

and inclined members in Fig. 15 are in tension, they exert pulls on the joint, and if the vertical is a compression member, it exerts a push on the joint as indicated. *The forces acting at a joint are therefore concurrent and their lines of action are always known.*

23. **General Method of Procedure.** The forces acting at a joint constitute a system in equilibrium, and since the forces are concurrent and their lines of action are all known, we can determine the magnitude of two of the forces if the others are all known; for this is the important problem mentioned in Art. 16 which was illustrated there and in Art. 17.

Accordingly, after the loads and reactions on a truss, which is to be analyzed, have been ascertained*, we look for a joint at which only two members are connected (the end joints are usually such). Then we consider the forces at that joint and determine the two unknown forces which the two members exert upon it by methods explained in Arts. 16 or 17. The forces so ascertained are the direct stresses, or stresses, as we shall call them for short, and *they are the values of the pushes or pulls which those same members exert upon the joints at their other ends.*

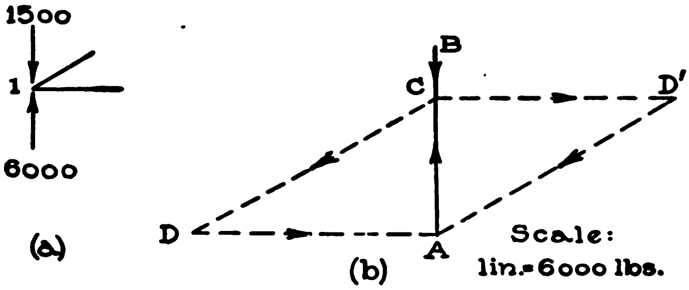

Fig. 17.

Next we look for another joint at which but two unknown forces act, then determine these forces, and continue this process until the stress in each member has been ascertained. We explain further by means of

Examples. 1. It is desired to determine the stresses in the

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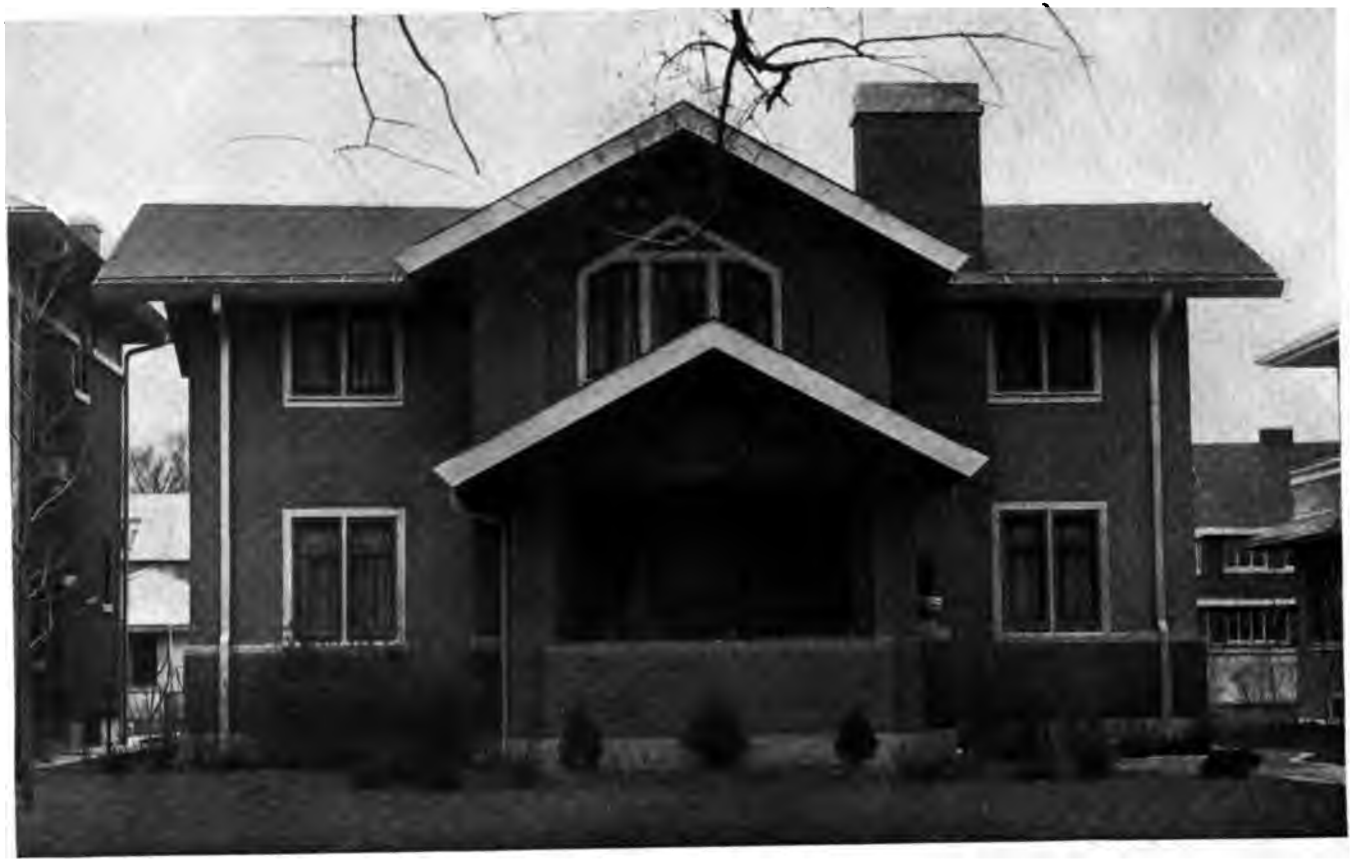
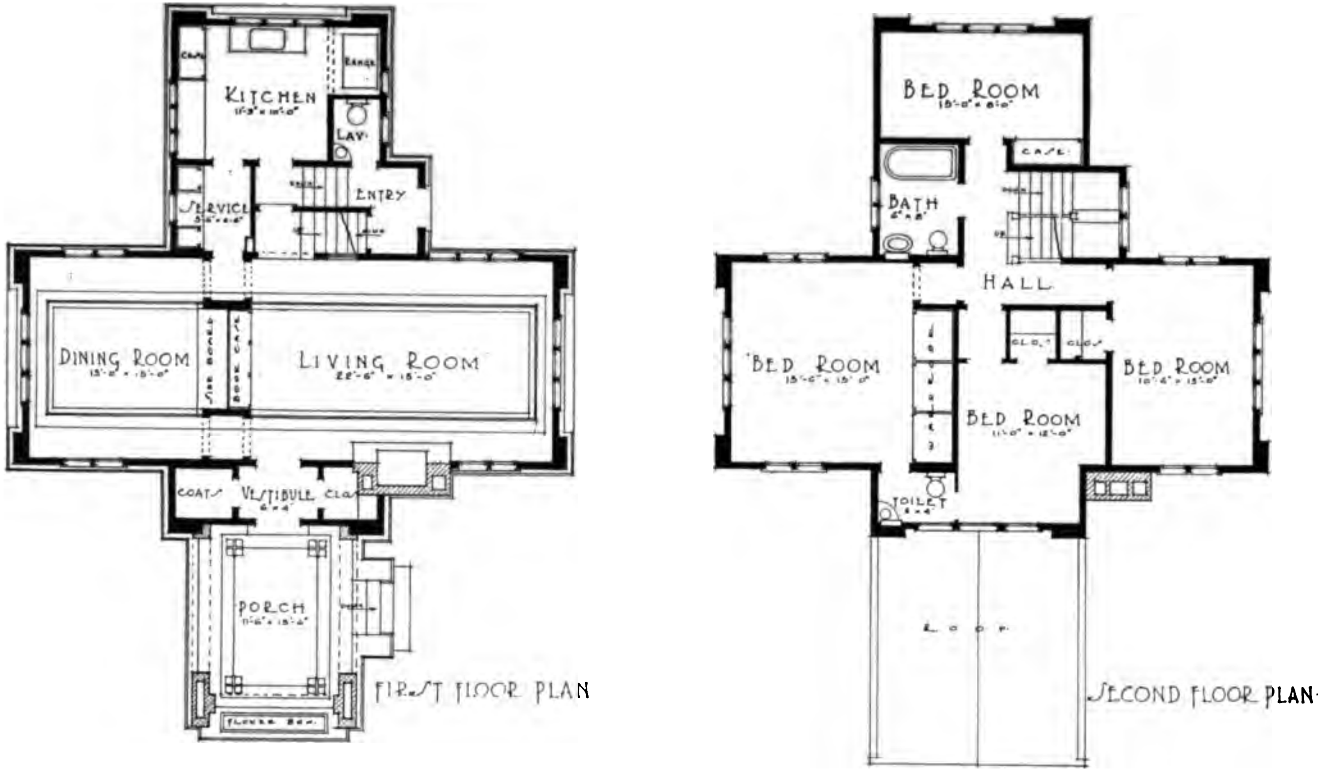
* How to ascertain the values of the reactions is explained in Art. 37. For the present their values in any given case are merely stated.



RESIDENCE OF JOSEPH S. GUY, AT OAK PARK, ILLINOIS
Tallmadge and Watson, Architects, Chicago
For floor plans see opposite page



FLOOR PLANS FOR RESIDENCE OF JOSEPH S. GUY, OAK PARK, ILLINOIS
Tallmadge and Watson, Architects, Chicago
For exterior view see opposite page

members of the steel truss, represented in Fig. 16, due to its own weight and that of the roofing assumed to weigh 12 pounds per square foot. The distance between trusses is 14 feet.

The apex loads for this case were computed in Example 1, Art. 20, and are marked in Fig. 16. Without computation it is plain that each reaction equals one-half the total load, that is, $\frac{1}{2}$ of 12,000, or 6,000 pounds.

The forces at joint (1) are four in number, namely, the left reaction (6,000 pounds), the load applied there (1,500 pounds), and the forces exerted by members $\overline{12}$ and $\overline{13}$. For clearness, we represent these forces so far as known in Fig. 17 (*a*); we can determine the two unknown forces by merely constructing a closed force polygon for all of them. To construct the polygon, we first represent the known forces; thus AB (1 inch long with arrowhead pointing up) represents the reaction and BC ($\frac{1}{4}$ inch long with arrowhead pointing down) represents the load. Then from A and C we draw lines parallel to the two unknown forces and mark their intersection D (or D′). Then the polygon is ABCDA, and CD (1.5 inches = 9,000 pounds) represents the force exerted by the member $\overline{12}$ on the joint and DA (1.3 inches = 7,800 pounds) represents the force exerted by the member $\overline{13}$ on the joint. The arrowheads on BC and CD must point as shown, in order that all may point the same way around, and hence the force exerted by member $\overline{12}$ acts toward the joint and is a push, and that exerted by $\overline{13}$ acts away from the joint and is a pull. It follows that $\overline{12}$ is in compression and $\overline{13}$ in tension.

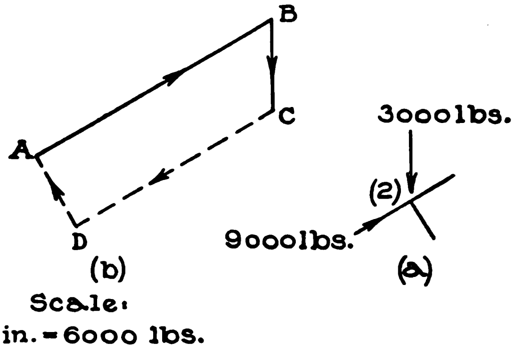


Fig. 18.

If D′ be used, the same results are reached, for the polygon is ABCD′A with arrowheads as shown, and it is plain that CD′ and DA also D′A and CD are equal and have the same sense. But one of these force polygons is preferable for reasons explained later.

Since $\overline{12}$ is in compression, it exerts a push (9,000 pounds) on joint (2) as represented in Fig. 18 (*a*), and since $\overline{13}$ is in tension it exerts a pull (7,800 pounds) on joint (3) as represented in Fig. 19 (*a*).

The forces at joint (2) are four in number, the load (3,000 pounds), the force 9,000 pounds, and the force exerted upon it by the members $\overline{24}$ and $\overline{23}$; they are represented as far as known in Fig. 18 (*a*). We determine the unknown forces by constructing a closed polygon for all of them. Representing the known forces first, draw AB (1.5 inches long with arrowhead pointing up) to represent the 9,000 pound force and BC (½ inch long with arrowhead pointing down) to represent the load of 3,000 pounds. Next from A and C draw lines parallel to the two unknown forces and mark their intersection D; then the force polygon is ABCDA and the arrowheads on CD and DA must point as shown. CD (1.25 inches = 7,500 pounds) represents the force exerted on joint (2) by $\overline{24}$; since it acts toward the joint the force is a push and member $\overline{24}$ is in compression. DA (0.43 inches = 2,580 pounds) represents the force exerted on the joint by member $\overline{23}$; since the force acts toward the joint it is a push and the member is in compression. Member 23 therefore exerts a push on joint (3) as shown in Fig. 19 (*a*).

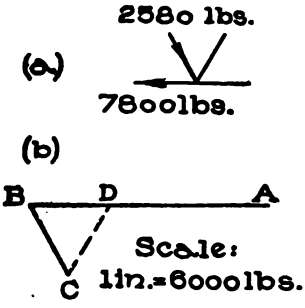


Fig. 19.

At joint (3) there are four forces, 7,800 pounds, 2,580 pounds, and the forces exerted on the joint by members $\overline{34}$ and $\overline{36}$. To determine these, construct the polygon for the four forces. Thus, AB (1.3 inches long with arrowhead pointing to the left) represents the 7,800-pound force and BC (0.43 inches long with arrowheads pointing down) represents the 2,580-pound force. Next draw from A and C two lines parallel to the unknown forces and mark their intersection D; then the force polygon is ABCDA and the arrowhead on CD and DA must point upward and to the right respectively. CD (0.43 inches = 2,580 pounds) represents the force exerted on the joint by member $\overline{34}$; since the force acts away from the joint it is a pull and the member is in tension. DA (0.87 inches = 5,220 pounds) represents the force exerted upon the joint by the member $\overline{36}$; since the force acts away from the joint, it is a pull and the member is in tension.

We have now determined the amount and kind of stress in members $\overline{12}$, $\overline{13}$, $\overline{23}$, $\overline{24}$, $\overline{34}$ and $\overline{36}$. It is evident that the stress in each of the members on the right-hand side is the same as the stress in the corresponding one on the left-hand side; hence further analysis is unnecessary.

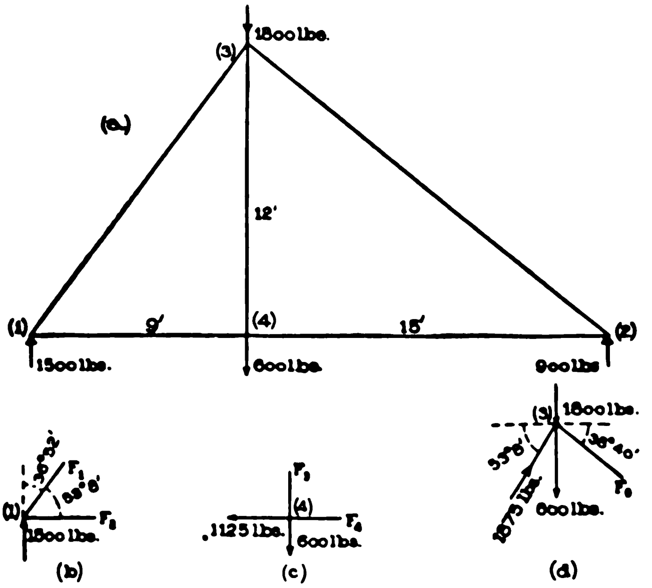


Fig. 20.

2. It is required to analyze the truss represented in Fig. 20 (*a*), the truss being supported at the ends and sustaining two loads, 1,800 and 600 pounds, as shown. (For simplicity we assumed values of the load; the lower one might be a load due to a suspended body. We shall solve algebraically.)

The right and left reactions equal 900 and 1,500 pounds as is shown in Example 1, Page 56. At joint (1) there are three forces, namely, the reaction 1,500 pounds and the forces exerted by members $\overline{13}$ and $\overline{14}$, which we will denote by F_1 and F_2 respect ively. The three forces are represented in Fig. 20 (*b*) as far as they are known. These three forces being in equilibrium, their horizontal and their vertical components balance. Since there are but two horizontal components and two vertical components it follows that (for balance of the components) F_1 must act downward and F_2 toward the right. Hence member $\overline{13}$ pushes on the joint and is under compression while member $\overline{14}$ pulls on the joint and is under tension. From the figure it is plain that

the horizontal component of $F_1 = F_1 \cos 53° \, 8' = 0.6 \, F_1$*,
the horizontal component of $F_2 = F_2$,
the vertical component of $F_1 = F_1 \cos 36° \, 52' = 0.8 \, F_1$,
and the vertical component of the reaction = 1,500.

Hence $$0.6 \, F_1 = F_2, \text{ and } 0.8 \, F_1 = 1{,}500;$$

or, $$F_1 = \frac{1{,}500}{0.8} = 1{,}875 \text{ pounds},$$

and $$F_2 = 0.6 \times 1{,}875 = 1{,}125 \text{ pounds}.$$

Since members $\overline{14}$ and $\overline{13}$ are in tension and compression respectively, $\overline{14}$ pulls on joint (4) as shown in Fig. 20 (*c*) and $\overline{13}$ pushes on joint (3) as shown in Fig. 20 (*d*).

The forces acting at joint (4) are the load 600 pounds, the pull 1,125 pounds, and the forces exerted by members $\overline{34}$ and $\overline{24}$; the last two we will call F_3 and F_4 respectively. The four forces being horizontal or vertical, it is plain without computation that for balance F_4 must be a pull of 1,125 pounds and F_3 one of 600 pounds. Since members $\overline{42}$ and $\overline{43}$ pull on the joint they are both in tension.

Member $\overline{43}$, being in tension, pulls down on joint (3) as shown in Fig. 20 (*d*). The other forces acting on that joint are the load 1,800 pounds, the push 1,875 pounds, the pull 600 pounds, and the force exerted by member $\overline{32}$ which we will call F_5. The only one of these forces having horizontal components are 1,875 and F_5; hence in order that these two components may balance, F_5 must act toward the left. F_5 is therefore a push and the member $\overline{32}$ is under compression.

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* The angles can be computed from the dimensions of the truss: often they can be ascertained easiest by scaling them with a protractor from a large size drawing of the truss.

The horizontal component of 1,875 = 1,875 × cos 53° 8′ = 1,125; and the horizontal component of $F_5 = F_5 \times \cos 38° 40' = 0.7808 F_5$.

Hence $$0.7808 F_5 = 1,125,$$

or, $$F_5 = \frac{1,125}{.7808} = 1,440 \text{ pounds.}$$

(This same truss is analyzed graphically later.)

24. **Notation for Graphical Analysis of Trusses.** The notation described in Art. 3 can be advantageously systematized in this connection as follows: Each triangular space in the diagram of the truss and the spaces between consecutive lines of action of the loads and reactions should be marked by a small letter (see

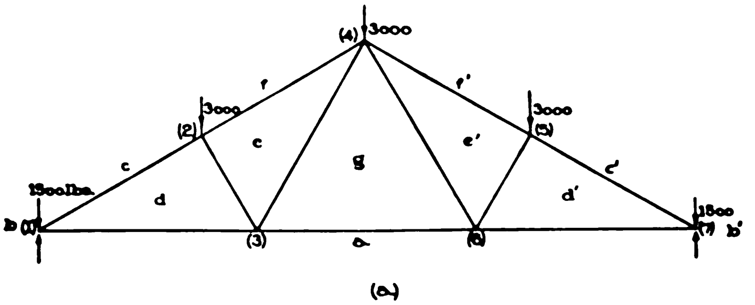


Fig. 21 *a*). Then the two letters on opposite sides of any line serve to denote that line and the same large letters are used to denote the force acting in that line. Thus *cd* (Fig. 21*a*) refers to the member 12 and CD should be used to stand for the force or stress in that member.

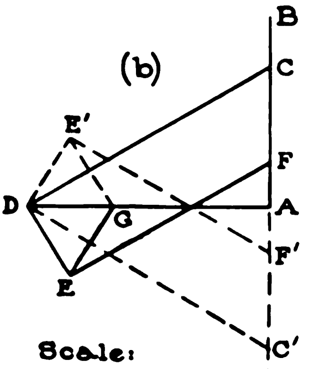
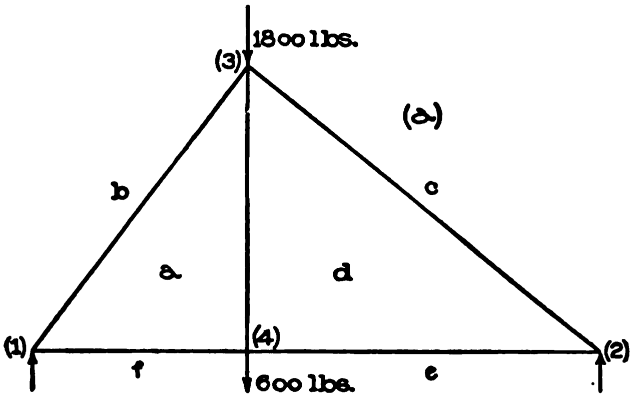


Fig. 21.

25. **Polygon for a Joint.** In drawing the polygon for all the forces at a joint, it is advantageous to represent the forces in the order in which they occur about the joint. Evidently there are always two possible orders thus (see Fig. 20 *d*) F_5, 600, 1,875, and 1,800 is one order around, and F_5, 1,800, 1,875, and 600 is another. The first is called a clockwise order and the second counter-clockwise.

A force polygon for the forces at a joint in which the forces are represented in either order in which they occur about the joint is called *a polygon for the joint*, and it will be called a clockwise or counter-clockwise polygon according as the order followed is clockwise or counter-clockwise. Thus in Fig. 17 (*b*), ABCDA is a clockwise polygon for joint (1). ABCD′A is a polygon for the *forces* at the joint; it is not a polygon for the *joint* because the order in which the forces are represented in that polygon is not the same as either order in which they occur about the joint.



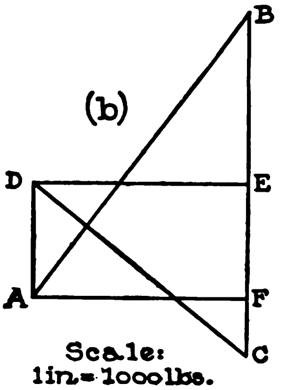


Fig. 22.

(Draw the counter-clockwise polygon for the joint and compare it with ABCDA and ABCD′A.)

26. **Stress Diagrams.** If the polygons for all the joints of a truss are drawn separately as in Example 1, Art. 23, the stress in each member will have been represented twice. It is possible to combine the polygons so that it will be unnecessary to represent the stress in any one member more than once, thus reducing the number of lines to be drawn. Such a combination of force polygons is called a stress diagram.

Fig. 21 (*b*) is a stress diagram for the truss of Fig. 21 (*a*) same as the truss of Fig. 16. It will be seen that the part of the stress diagram consisting of solid lines is a combination of separate polygons previously drawn for the joints on the left half of the truss (Figs. 17, 18 and 19.) It will also be seen that the polygons are all clockwise, but counter-clockwise polygons could be combined into a stress diagram.

To Construct a Stress Diagram for a Truss Under Given Loads.

1. Determine the reactions*.
2. Letter the truss diagram as explained in Art. 24.
3. Construct a force polygon for all the forces applied to the truss (loads and reactions) representing them in the order in which they occur around the truss, clockwise or counter-clockwise. (The part of this polygon representing the loads is called a load line.)
4. On the sides of that polygon, construct the polygons for all the joints. They must be clockwise or counter-clockwise according as the polygon for the loads and reactions is clockwise or counter-clockwise. (The first polygon for a joint must be drawn for one at which but two members are connected—the joints at the supports are usually such. Then one can draw in succession the polygons for joints at which there are not more than two unknown forces until the stress diagram is completed.)

Example. It is desired to construct a stress diagram for the truss represented in Fig. 22 (*a*), it being supported at its ends and sustaining two loads of 1,800 and 600 pounds as shown.

The right and left reactions are 900 and 1,500 pounds as is shown in Example 1, Art. 37. Following the foregoing directions we first letter the truss, as shown. Then, where convenient, draw the polygon for all the loads and reactions, beginning with any force, but representing them in order as previously directed. Thus, beginning with the 1,800-pound load and following the clockwise order for example, lay off a line 1.8 inch in length representing 1,800 pounds (scale 1,000 pounds to an inch); since the line of action of the force is *bc*, the line is to be marked BC and B should be placed at the upper end of the line for a reason which will presently appear. The next force to be represented is the right reaction, 900 pounds ; hence from C draw a line upward and 0.90 inch long. The line of action of this force being *ce*, the line just drawn should be marked CE and since C is already at the lower end, we mark the upper end E. (The reason for placing B at the upper end of the first line is now apparent.) The next force to be represented is the 600-pound load ; therefore we draw from E a line downward and 0.6 inch long, and since the line of action of that force is *ef*, mark the lower end of the line F. The next force to be represented is the left reaction, 1,500 pounds, hence we draw a line 1.5 inches long and upward from F. If the lines have been carefully laid off, the end of the last line should fall at B, that is, the polygon should close.

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* As already stated, methods for determining reactions are explained in Art. 37; for the present the values of the reactions in any example will be given.

We are now ready to draw polygons for the joints; we may begin at the right or left end as we please but we should bear in mind that the polygons must be clockwise because the polygon for the loads and reactions (BCEFB) is such an one. Beginning at the right end for example, notice that there are three forces there, the right reaction, *de* and *dc*. The right reaction is represented by CE, hence from E draw a line parallel to *de* and from C one parallel to *dc* and mark their intersection D. Then CEDC is the clockwise polygon for the right-hand joint, and since CE acts up, the arrows on ED and DC would point to the left and down respectively. It is better to place the arrows near the joint to which they refer than in the stress diagram; this is left to the student. The force exerted by member *ed* on joint (2) being a pull, *ed* is under tension, and since ED measures 1.12 inches, the value of that tension is 1,120 pounds. The force exerted by member *dc* on joint (2) being a push, *dc* is under compression, and since DC measures 1.44 inches, the value of that compression is 1,440 pounds.

The member *dc* being in compression, exerts a push on the joint (3) and the member *de* being in tension, exerts a pull on the joint (4). Next indicate this push and pull by arrows.

We might now draw the polygon for any one of the remaining joints, for there are at each but two unknown forces. We choose to draw the polygon for the joint (3). There are four forces acting there, namely, the 1,800-pound load, the push (1,440 pounds) exerted by *cd*, and the forces exerted by members *ad* and *ab*, unknown in amount and sense. Now the first two of these forces are already represented in the stress diagram by BC and CD, therefore we draw from D a line parallel to *da* and from B a line parallel to *ba* and mark their intersection A. Then BCDAB is the polygon for the joint, and since the arrowhead on BC and CD would point down and up respectively, DA acts down and AB up; hence place arrowheads in those directions on *da* and *ab* near the joint being considered. These arrows signify that member *da* pulls on the joint and *ba* pushes; hence *da* is in tension and *ba* in compression. Since DA and AB measure 0.6 and 1.88 inches respectively, the values of the tension and compression are 600 and 1,880 pounds.

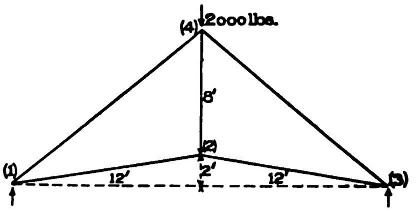


Fig. 23.

Next place arrowheads on *ab* and *ad* at joints (1) and (4) to represent a push and a pull respectively. There remains now but one stress undetermined, that in *af*. It can be ascertained by drawing the polygon for joint 1 or 4; let us draw the latter. There are four forces acting at that joint, namely, the 600-pound load, and the forces exerted by members *ed*, *da*, and *af*. The first three forces are already represented in the drawing by EF, DE and DA, and the polygon for those three forces (not closed) is ADEF. The fourth force must close the polygon, that is, a line from F parallel to *af* must pass through A, and if the drawing has been accurately done, it will pass through A. The polygon for the four forces then is ADEFA, and an arrowhead placed on FA ought to point to the left, but as before, place it in the truss diagram on *af* near joint (4). The force exerted by member *af* on joint (4) being a pull, *af* is under tension, and since AF measures 1.12 inches, the value of the tension is 1,120 pounds.

Since *af* is in tension it pulls on joint (1), hence we place an arrowhead on *af* near joint (1) to indicate that pull.

EXAMPLES FOR PRACTICE.

1. Construct a stress diagram for the truss of the preceding Example (Fig. 22*a*) making all the polygons counter-clockwise, and compare with the stress diagram in Fig. 22.

2. Determine the stresses in the members of the truss represented in Fig. 23 due to a single load of 2,000 pounds at the peak.

Ans. $\begin{cases} \text{Stresses in } \overline{12} \text{ and } \overline{23} = 1{,}510 \text{ pounds,} \\ \text{Stresses in } \overline{14} \text{ and } \overline{43} = 1{,}930 \text{ pounds,} \\ \text{Stress in } \overline{24} = 490 \text{ pounds.} \end{cases}$

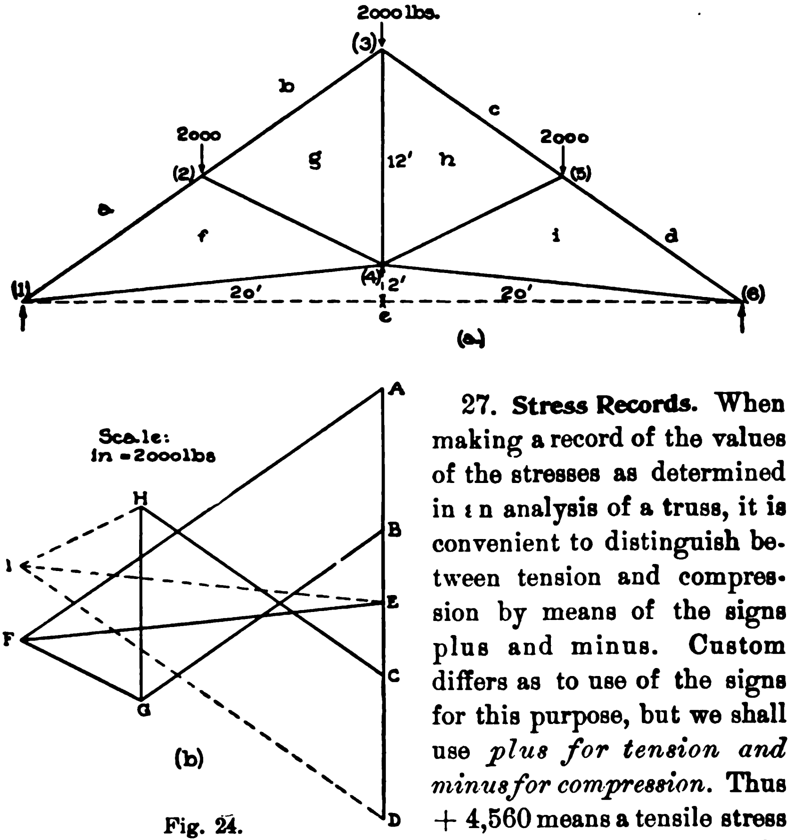


Fig. 24.

27. **Stress Records.** When making a record of the values of the stresses as determined in an analysis of a truss, it is convenient to distinguish between tension and compression by means of the signs plus and minus. Custom differs as to use of the signs for this purpose, but we shall use *plus for tension and minus for compression.* Thus + 4,560 means a tensile stress of 4,560 pounds, and – 7,500 means a compressive stress of 7,500 pounds.

The record of the stresses as obtained in an analysis can be conveniently made in the form of a table, as in Example 1 following, or in the truss diagram itself, as in Example 2 (Fig. 25).

As previously explained, the stress in a member is tensile or compressive according as the member pulls or pushes on the joints between which it extends. If the arrowheads are placed on the lines representing the members as was explained in Example 1 of Art. 26 (Fig. 22), the two arrowheads on any member

> **point toward each other on tension members,**
> **and from each other on compression members.**

If the system of lettering explained in Art. 24 is followed in the analysis of a truss, and if the first polygon (for the loads and reactions) is drawn according to directions (Art. 26), then the system of lettering will guide one in drawing the polygons for the joints as shown in the following illustrations. It must be remembered always that any two parallel lines, one in the truss and one in the stress diagram, must be designated by the same two letters, the first by small letters on opposite sides of it, and the second by the same capitals at its ends.

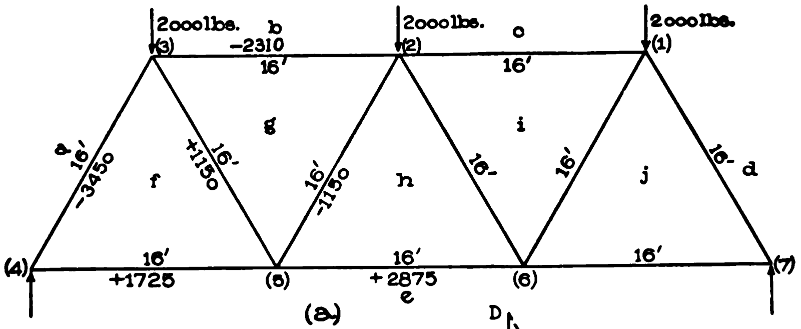
Examples. 1. It is required to construct a stress diagram for the truss represented in Fig. 24 supported at its ends and sustaining three loads of 2,000 pounds as shown. Evidently the reactions equal 3,000 pounds.

Following the directions of Art. 26, we letter the truss diagram, then draw the polygon for the loads and reactions. Thus, to the scale indicated in Fig. 24 (*b*), AB, BC, and CD represent the loads at joints (2), (3) and (5) respectively and DE and EA represent the right and the left reactions respectively. Notice that the polygon (ABCDEA) is a clockwise one.

At joint (1) there are three forces, the left reaction and the forces exerted by the members *af* and *fe*. Since the forces exerted by these two members must be marked AF and EF we draw from A a line parallel to *af* and from E one parallel to *ef* and mark their intersection F. Then EAFE is the polygon for joint (1), and since EA acts up (see the polygon), AF acts down and FE to the right. We, therefore, place the proper arrowheads on *af* and *fe* near (1), and record (see adjoining table) that the stresses in those members are compressive and tensile respectively. Measuring, we find that AF and FE equal 6,150 and 5,100 pounds respectively.

Member....	af	fe	bg	fg	gh
Stress......	- 6,150	+5,100	- 4,100	- 1,875	+2,720

We may next draw the polygon for joint (6) or (2) since there are but two unknown forces at each. At joint (2) for instance, the unknown forces are those exerted by *fg* and *bg*, and the known are the load *ab* (2,000 pounds) and the force exerted by *af*. Since the unknown forces must be marked FG and GB, draw from F a line parallel to *fg*, from B a line parallel to *bg*, and mark their intersection G. Then the polygon for the joint is FABGF, and since AB acts down (see the polygon) BG and GF act down and up respectively. Therefore, place the proper arrowheads on *bg* and *gf* near (2), and record that the stresses in those members are both compressive. Measuring, we find that BG and GF scale 4,100 and 1,875 pounds respectively.



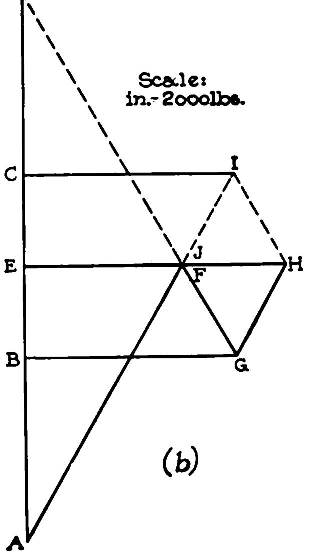


Fig. 25.

Now draw a polygon for joint (3) or (6) since there are but two unknown forces at each joint. At (3) for instance, the unknown forces are those exerted by *ch* and *gh*, the known forces being the load (2,000 pounds) and the force 4,100 pounds, exerted by *bg*. Since the unknown forces must be marked CH and GH, draw from C a line parallel to *ch*, from G one parallel to *gh*, and mark their intersection H. Then

GBCHG is the polygon for the joint, and since BC acts down (see the polygon) CH acts up and HG down. Therefore, place the proper arrowheads on *ch* and *hg* near (3), and record that the stresses in those members are compressive and tensile respectively. Measuring, we find that CH and HG scale 4,100 and 2,720 pounds respectively.

It is plain that the stress in any member on the right-hand side is the same as that in the corresponding member on the left, hence it is not necessary to construct the complete stress diagram.

2. It is required to analyze the truss of Fig. 25 which rests on end supports and sustains three loads each of 2,000 pounds as shown. Each member is 16 feet long.

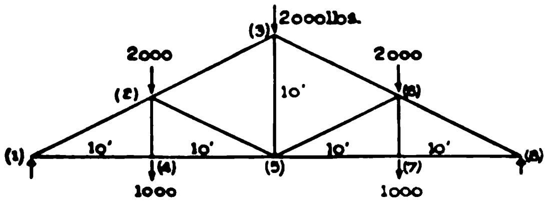


Fig. 26.

Evidently, reactions are each 3,000 pounds. Following directions of Art. 26, first letter the truss diagram and then draw a polygon for the loads and reactions representing them in either order in which they occur about the truss. DCBAED is a counter-clockwise polygon, DC, CB, and BA representing the loads at joints (1), (2) and (3), AE the left reaction, ED the right reaction.

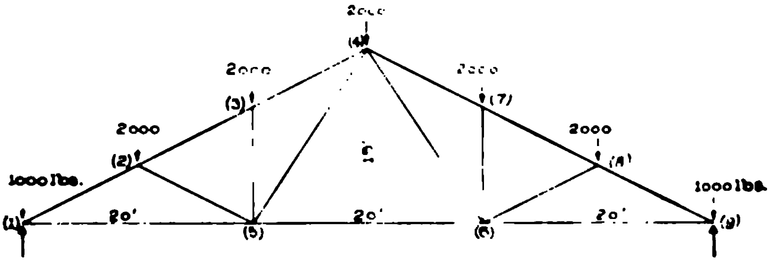


Fig. 27.

The construction of the polygons is carried out as in the preceding illustration, and little explanation is necessary. The polygon for joint (4) is AEFA, EF (1,725 pounds tension) representing the stress in *ef* and FA (3,450 pounds compression) that in *af*. The polygon for joint (3) is BAFGB, FG (1,150 pounds tension) representing the stress in *fg* and GB (2,300 pounds compression) that in *gb*. The polygon for joint (5) is GFEHG, EH (2,875 pounds tension) representing the stress in *eh* and HG (1,150 pounds compression) that in *hg*.

Evidently the stress in any member on the right side of the truss is like that in the corresponding member on the left, therefore it is not necessary to construct the remainder of the stress diagram.

EXAMPLES FOR PRACTICE.

1. Analyze the truss represented in Fig. 26, it being supported at its ends and sustaining three loads of 2,000 and two of 1,000 pounds as represented.

STRESS RECORD.

Member.......	$\overline{12}$	$\overline{23}$	$\overline{14}$	$\overline{45}$	$\overline{24}$	$\overline{25}$	$\overline{35}$
Stress.........	−8,950	−5,600	+8,000	+8,000	+1,000	−3,350	+3,000

2. Analyze the truss represented in Fig. 27, it being supported at its ends and sustaining five 2,000-pound loads and two of 1,000 as shown.

STRESS RECORD.

Member .	$\overline{12}$	$\overline{23}$	$\overline{34}$	$\overline{51}$	$\overline{52}$	$\overline{53}$	$\overline{54}$	$\overline{56}$
Stress . . .	− 11,200	− 8.900	− 8.900	+ 10.000	− 2.200	− 2,000	+3.600	+6,000

28. Analysis for Snow Loads. In some cases the apex snow loads are a definite fractional part of the apex loads due to the weights of roof and truss. For instance, in Examples 1 and 2, Pages 25 and 26, it is shown that the apex loads are 1,500 and 3,000 pounds due to weight of roof and truss, and 735 and 1,470 due to snow; hence the snow loads are practically equal to one-half of the permanent dead loads. It follows that the stress in any member due to snow load equals practically one-half of the stress in that member due to the permanent dead load. The snow load stresses in this case can therefore be obtained from the permanent load stresses and no stress diagram for snow load need be drawn.

In some cases, however, the apex loads due to snow at the various joints are not the *same* fractional part of the permanent load. This is the case if the roof is not all of the same slope, as for instance in Fig. 25 where a part of the roof is flat. In such a case the stresses due to the snow load cannot be determined from a stress diagram for the permanent dead load but a separate stress diagram for the snow load must be drawn. Such diagrams are drawn like those for permanent dead load.

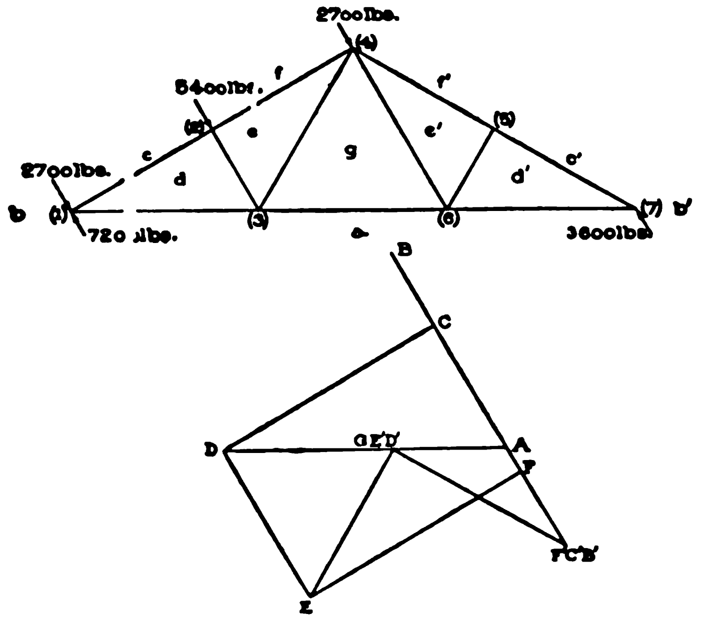


Fig. 28.

29. **Analysis for Wind Loads.** Stresses due to wind pressure cannot be computed from permanent load stresses; they can be most easily determined by means of a stress diagram. Since wind pressure exists only on one side of a truss at a time, the stresses in corresponding members on the right and left sides of a truss are unequal and the whole stress diagram must be drawn in analysis for "wind stresses." Moreover, where one end of the

truss rests on rollers, two stress diagrams must be drawn for a complete analysis, one for wind blowing on the right and one for wind blowing on the left (see Example 2 following).

Examples. 1. It is required to analyze the truss of Fig. 16 for wind pressure, the distance between trusses being 14 feet.

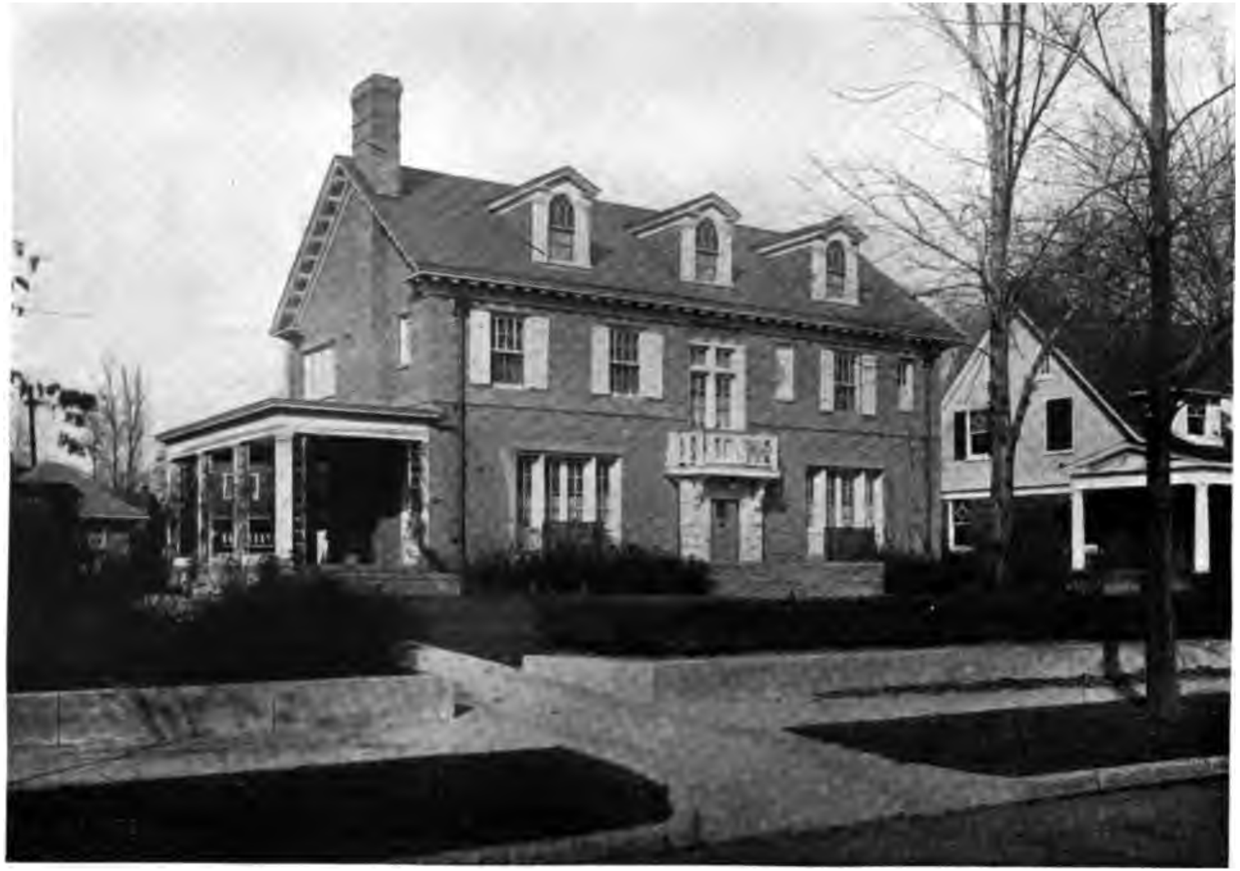
The apex loads for this case are computed in Example 3, Page 26, to be as represented in Fig. 28. Supposing both ends of the truss to be fastened to the supports, then the reactions (due to the wind alone) are parallel to the wind pressure and the right and left reactions equal 3,600 and 7,200 pounds as explained in Example 2, Page 57.

To draw a clockwise polygon for the loads and reactions, we lay off BC, CF, and FF′ to represent the loads at joints (1), (2), and (4) respectively; then since there are no loads at joints (5) and (7) we mark the point F′ by C′ and B′ also; then lay off B′A to represent the reaction at the right end. If the lengths are laid off carefully, AB will represent the reaction at the left end and the polygon is BCFF′C′B′AB.

At joint (1) there are four forces, the reaction, the load, and the two stresses. AB and BC represent the first two forces, hence from C draw a line parallel to *cd* and from A a line parallel to *ad* and mark their intersection D. Then ABCDA is the polygon for the joint and CD and DA represent the two stresses. The former is 7,750 pounds compression and the latter 9,000 pounds tension.

At joint (2) there are four forces, the stress in *cd* (7,750 pounds compression), the load, and the stresses in *fe* and *ed*. As DC and CF represent the stress 7,750 and the load, from F draw a line parallel to *fe* and from D a line parallel to *de*, and mark their intersection E. Then DCFED is the polygon for the joint and FE and ED represent the stresses in *fe* and *ed* respectively. The former is 7,750 pounds and the latter 5,400, both compressive.

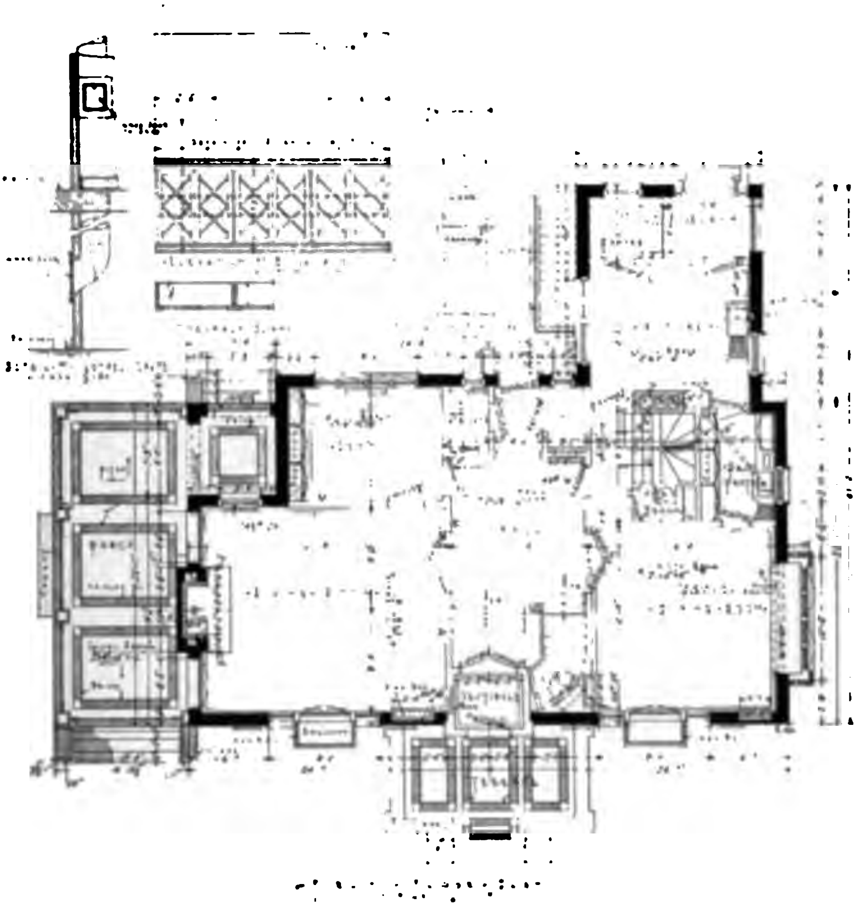
At joint (3) there are four forces, the stresses in *ad* (9,000 pounds), *de* (5,400 pounds), *eg* and *ga*. AD and DE represent the first two stresses; hence from E draw a line parallel to *eg* and from A a line parallel to *ag* and mark their intersection G. Then ADEGA is the polygon for the joint and EG and GA represent the stresses in *eg* and *ga* respectively The former is 5,400 and the latter 3,600 pounds, both tensile.



RESIDENCE OF CLIFFORD OFF AT PEORIA, ILLINOIS

Hewitt and Emerson, Architects, Peoria, Illinois

A rather interesting example of a symmetrical exterior fitted to an unsymmetrical plan. The house is of brick veneer construction with a slate roof. Cost about $14,000. For floor plans see opposite page.



FLOOR PLAN OF RESIDENCE OF CLIFFORD OFF AT PEORIA, ILLINOIS
Hewitt and Emerson, Architects, Peoria, Illinois
For exterior view see opposite page

At joint (4) there are five forces, the stresses in *eg* (5,400 pounds) and *ef* (7,750 pounds), the load, and the stresses in *f'e'* and *e'g*. GE, EF and FF' represent the first three forces; hence draw from F' a line parallel to *f'e'* and from G a line parallel to *e'g* and mark their intersection E'. (The first line passes through G, hence E' falls at G). Then the polygon for the joint is GEFF'E'G, and

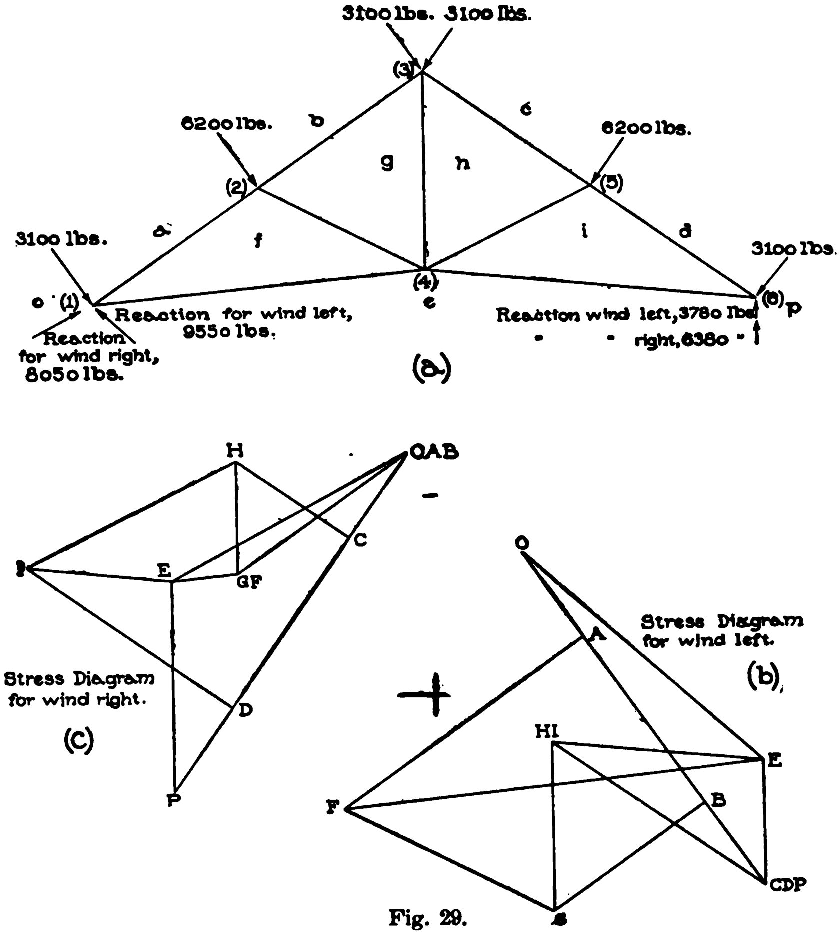

3100 lbs. 3100 lbs.
(3)
6200 lbs.
b
c
6200 lbs.
g
h
(2)
(5)
3100 lbs.
a
f
i
d
3100 lbs.
o
(1)
(4)
e
(6)
p
Reaction for wind left, 9550 lbs.
Reaction for wind right, 8050 lbs.
Reaction wind left, 3780 lbs.
" " right, 6380 "
(a)
H
OAB
C
E
GF
Stress Diagram for wind right.
(c)
D
P
O
Stress Diagram for wind left.
(b)
A
HI
E
F
B
CDP

Fig. 29.

F'E' (6,250 pounds compression) represents the stress in *f'e'*. Since E'G = 0, the wind produces no stress in member *ge'*.

At joint (5) three members are connected together and there is no load. The sides of the polygon for the joint must be parallel to the members joined there. Since two of those members are in the same straight line, two sides of the polygon will be parallel and it follows as a consequence that the third side must be zero. Hence the stress in the member $e'd'$ equals zero and the stresses in $f'e'$ and $d'c'$ are equal. This result may be explained slightly differently: Of the stresses in $e'f'$, $e'd'$, and $d'c'$ we know the first (6,250) and it is represented by E'F'. Hence we draw from F' a line parallel to $e'd'$ and one from E' parallel to $d'e$ and mark their intersection D'. Then the polygon for the joint is E'F'C'D'E'; C'D' (6,250 pounds compression) representing the stress in $c'd'$. Since E' and D' refer to the same point, E'D' scales zero and there is no stress in $e'd'$.

The stress in ad' can be determined in various ways. Since at joint (6) there are but two forces (the stresses in ge' and $e'd'$ being zero), the two forces must be equal and opposite to balance. Hence the stress in $d'a$ is a tension and its value is 3,600 pounds.

2. It is required to analyze the truss represented in Fig. 24 for wind pressure, the distance between trusses being 15 feet.

The length $\overline{13}$ equals $\sqrt{20^2 + 14^2}$ or

$$\sqrt{400 + 196} = 24.4 \text{ feet.}$$

Hence the area sustaining the wind pressure to be borne by one truss equals $24.4 \times 15 = 366$ square feet.

The tangent of the angle which the roof makes with the horizontal equals $14 \div 20 = 0.7$; hence the angle is practically 35 degrees. According to Art. 19, the wind pressures for slopes of 30 and 40 degrees are 32 and 36 pounds per square foot; hence for 35 degrees it is 34 pounds per square foot. The total wind pressure equals, therefore, $366 \times 34 = 12,444$, or practically 12,400 pounds

The apex load for

joint (2) is ½ of 12,400, or 6,200 pounds,
and for joints (1) and (3), ¼ of 12,400, or 3,100 pounds (see Fig. 29).

When the wind blows from the right the

load for joint (5) is 6,200 pounds, and
for joints (3) and (6) 3,100 pounds.

If the left end of the truss is fastened to its support and the right rests on rollers*, when the wind blows on the left side the right and left reactions equal 3,780 and 9,550 pounds respectively and act as shown. When the wind blows on the right side, the right and left reactions equal 6,380 and 8,050 pounds and act as shown. The computation of these reactions is shown in Example 1, Page 58.

For the wind on the left side, OA, AB, and BC (Fig. 29*b*) represent the apex loads at joints (1), (2) and (3) respectively and CE and EO represent the right and left reactions; then the polygon (clockwise) for the loads and reactions is OABCDPEO. The point C is also marked D and P because there are no loads at joints (5) and (6).

The polygon for joint (1) is EOAFE, AF and FE representing the stresses in *af* and *fe* respectively. The values are recorded in the adjoining table. The polygon for joint (2) is FABGF, BG and GF representing the stresses in *bg* and *fg*. The polygon for joint (3) is GBCHG, CH and HG representing the stresses in *ch* and *hg* respectively. At joint (5) there is no load and two of the members connected there are in the same line; hence there is no wind stress in the third member and the stresses in the other two members are equal. The point H is therefore also marked I to make HI equal to zero. The polygon for joint (5) is HCDIH.

STRESS RECORD.

Member.	Stress, Wind Left.	Stress, Wind Right.
af	- 8,850	6,300
fe	+ 12,700	2,000
bg	- 5,600	- 6,300
fg	- 7,000	0
hg	+ 5,100	+ 3,400
hi	0	7,000
ch	7,700	- 4,100
ie	+ 6,400	+ 4,400
di	- 7,700	7,500

At joint (4) there are four forces, all known except the one in *ie*. EF, FG, and GH represent the first three; hence the line joining I and E must represent the stress in *ie*. This line, if the drawing has been correctly and accurately made, is parallel to *ie*.

---

* Rollers to allow for free expansion and contraction of the truss would not be required for one as short as this. They are not used generally unless the truss is 55 feet or more in length.

For wind on right side, BC, CD, and DP Fig. 29(*c*) represent the loads at joints (3), (5) and (6) respectively and PE and EB the right and left reactions; then BCDPEB is the polygon for the loads and reactions. The point B is also marked A and O because there are no loads at joints (2) and (1).

The polygon for joint (6) is DPEID, EI and ID representing the stresses in *ei* and *id* respectively. The polygon for joint (5) is CDIHC, IH, and HC representing the stresses in *ih* and *hc* respectively. The polygon for joint (3) is BCHGB, HG, and GB representing the stresses in *hg* and *gb* respectively. The polygon for joint (2) is BGFAB, FA representing the stress in *fa*, and since GF equals zero there is no stress in *gf*.

At joint (1) there are three forces, the left reaction, AF and the stress in *fe*. This third force must close the polygon, so we join F and E and this line represents the stress in *fe*. If the work has been accurately done, FE will be parallel to *fe*.

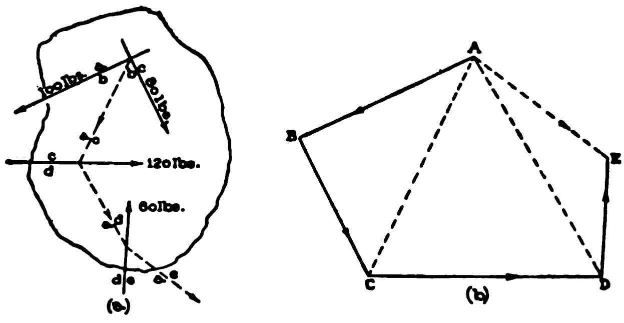


Fig 10.

EXAMPLE FOR PRACTICE.

Analyze the truss represented in Fig. 26 for wind pressure, the distance between trusses being 15 feet. (See Ex. 3, Page 27, for apex loads.) Assuming both ends of the truss fastened to the supports, the reactions are both parallel to the wind pressure and the reaction on the windward side equals 6,707 pounds and the other equals 3,053 pounds.

Ans. Stress Record for Wind Left.

Member.	Stress.
12	−8,500
14	+8,600
45	+8,600
24	0
23	− 5,000
25	− 6,080
35	+2,800
36	− 6,080
56	0
57	+4,200
68	− 6,200
67	0
78	+4,200

V. COMPOSITION OF NON-CONCURRENT FORCES.

30. Graphical Composition. As in composition of concurrent systems, we first compound any two of the forces by means of the Triangle Law (Art. 9), then compound the resultant of these two forces with the third, then compound the resultant of the first three with the fourth and so on until the resultant of all has been found. It will be seen in the illustration that the actual constructions are not quite so simple as for concurrent forces.

Example. It is required to determine the resultant of the four forces (100, 80, 120, and 60 pounds) represented in Fig. 30 (*a*).

If we take the 100- and 80-pound forces first, and from any convenient point A lay off AB and BC to represent the magnitudes and directions of those forces, then according to the triangle law AC represents the magnitude and direction of their resultant and its line of action is parallel to AC and passes through the point of concurrence of the two forces. This line of action should be marked *ac* and those of the 100- and 80-pound forces, *ab* and *bc* respectively.

If we take the 120-pound force as third, lay off CD to represent the magnitude and direction of that force; then AD represents the magnitude and direction of the resultant of AC and the third force, while the line of action of that resultant is parallel to AD and passes through the point of concurrence of the forces AC and CD. That line of action should be marked *ad* and that of the third force *cd*.

It remains to compound AD and the remaining one of the given forces, hence we lay off DE to represent the magnitude and direction of the fourth force; then AE represents the magnitude and direction of the resultant of AD and the fourth force (also of the four given forces). The line of action of the resultant is parallel to AE and passes through the point of concurrence of the forces AD and DE. That line should be marked *ae* and the line of action of the fourth face *de.*

It is now plain that the magnitude and direction of the resultant is found exactly as in the case of concurrent forces, but finding the line of action requires an extra construction.

31. **When the Forces Are Parallel or Nearly So,** the method of composition explained must be modified slightly because there is no intersection from which to draw the line of action of the resultant of the first two forces.

To make such an intersection available, resolve any one of the given forces into two components and imagine that force replaced by them; then find the resultant of those components and the other given forces by the methods explained in the preceding article. Evidently this resultant is the resultant of the given forces.

Example. It is required to find the resultant of the four parallel forces (50, 30, 40, and 60 pounds) represented in Fig. 31 (*a*).

Choosing the 30-pound force as the one to resolve, lay off AB to represent the magnitude and direction of that force and mark its line of action *ab*. Next draw lines from A and B intersecting at any convenient point O; then as explained in Art. 13, AO and OB (direction from A to O and O to B) represent the magnitudes and directions of two components of the 30-pound force, and the lines of action of those components are parallel to AO and OB and must intersect on the line of action of that force, as at 1. Draw next two such lines and mark them *ao* and *ob* respectively. Now imagine the 30-pound force replaced by its two components and then compound them with the 50-, 40- and 60-pound forces.

In the composition, the second component should be taken as the first force and the first component as the last. Choosing the 50-pound force as the second, lay off BC to represent the magnitude and direction of that force and mark the line of action *bc*. Then OC (direction O to C) represents the magnitude and direction of the resultant of OB and BC, and *oc* (parallel to OC and passing through the point of concurrence of the forces OB and BC) is the line of action.

Choosing the 40-pound force next, lay off CD to represent the magnitude and direction of that force and mark its line of action *cd*. Then OD (direction O to D) represents the magnitude and direction of the resultant of OC and CD, and *od* (parallel to OD and passing through the point of concurrence of the forces OC and CD) is the line of action of it.

Next lay off a line DE representing the magnitude and direction of the 60-pound force and mark the line of action *de*. Then OE (direction O to E) represents the magnitude and direction of the resultant of OD and DE, and *oe* (parallel to OE and passing through the point of concurrence of the forces OD and DE) is the line of action of it.

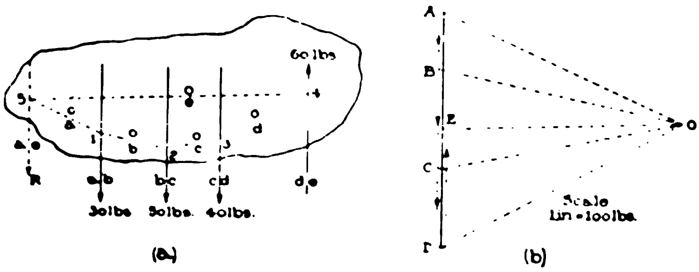


Fig. 31.

It remains now to compound the last resultant (OE) and the first component (AO). AE represents the magnitude and direction of their resultant, and *ae* (parallel to AE and passing through the point of concurrence of the forces OE and AO) is the line of action.

32. **Definitions and Rule for Composition.** The point O (Fig. 31) is called a *pole*, and the lines drawn to it are called *rays*. The lines *oa*, *ob*, *oc*, etc., are called *strings* and collectively they are called a *string polygon*. The string parallel to the ray drawn to the beginning of the force polygon (AO) is called the first string, and the one parallel to the ray drawn to the end of the force polygon is called the last string.

The method of construction may now be described as follows:

1. Draw a force polygon for the given forces. The line drawn from the beginning to the end of the polygon represents the magnitude and direction of the resultant.

2. Select a pole, draw the rays and then the string polygon. The line through the intersection of the first and last strings parallel to the direction of the resultant is the line of action of the resultant. (In constructing the string polygon, observe carefully that the two strings intersecting on the line of action of any one of the given forces are parallel to the two rays which are drawn to the ends of the line representing that force in the force polygon.)

EXAMPLES FOR PRACTICE.

1. Determine the resultant of the 50-, 70-, 80- and 120-pound forces of Fig. 5.

Ans. { 260 pounds acting upwards 1.8 and 0.1 feet to the right of A and D respectively.

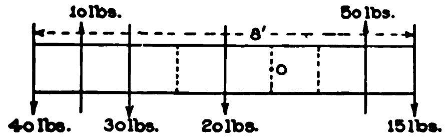


Fig. 32.

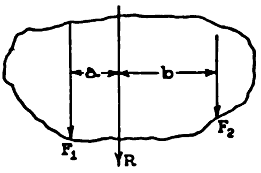


Fig. 33.

2. Determine the resultant of the 40-, 10-, 30- and 20-pound forces of Fig. 32.

Ans. { 80 pounds acting down 1$\frac{5}{8}$ feet from left end.

33. **Algebraic Composition.** The algebraic method of composition is best adapted to parallel forces and is herein explained only for that case.

If the plus sign is given to the forces acting in one direction, and the minus sign to those acting in the opposite direction, the magnitude and sense of the resultant is given by the algebraic sum of the forces; the magnitude of the resultant equals the value of the algebraic sum; the direction of the resultant is given by the sign of the sum, thus the resultant acts in the direction which has been called plus or minus according as the sign of the sum is plus or minus.

If, for example, we call up plus and down minus, the algebraic sum of the forces represented in Fig. 32 is

$$-40+10-30-20+50-15=-45;$$

hence the resultant equals 45 pounds and acts downward.

The line of action of the resultant is found by means of the principle of moments which is (as explained in "Strength of Materials") that *the moment of the resultant of any number of forces about any origin equals the algebraic sum of the moments of the forces.* It follows from the principle that the arm of the resultant with respect to any origin equals the quotient of the algebraic sum of the moments of the forces divided by the resultant; also the line of action of the resultant is on such a side of the origin that the sign of the moment of the resultant is the same as that of the algebraic sum of the moments of the given forces.

For example, choosing O as origin of moments in Fig. 32, the moments of the forces taking them in their order from left to right are

$$-40\times5=-200,\ +10\times4=+40,\ -30\times3=-90,$$
$$-20\times1=-20,\ -50\times2=-100,\ +15\times3=+45.^*$$

Hence the algebraic sum equals

$$-200+40-90-20-100+45=-325 \text{ foot-pounds.}$$

The sign of the sum being negative, the moment of the resultant about O must also be negative, and since the resultant acts down, its line of action must be on the left side of O. Its actual distance from O equals

$$\frac{325}{45}=7.22 \text{ feet.}$$

EXAMPLES FOR PRACTICE.

1. Make a sketch representing five parallel forces, 200, 150 100, 225, and 75 pounds, all acting in the same direction and 2 feet apart. Determine their resultant.

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* The student is reminded that when a force tends to turn the body on which it acts in the clockwise direction, about the selected origin, its moment is a given a plus sign, and when counter-clockwise, a minus sign.

Ans. $\left\{\begin{array}{l}\text{Resultant} = 750 \text{ pounds, and acts in the same} \\ \text{direction as the given forces and 4.47 feet to the} \\ \text{left of the 75-pound force.}\end{array}\right.$

2. Solve the preceding example, supposing that the first three forces act in one direction and the last two in the opposite direction.

Ans. $\left\{\begin{array}{l}\text{Resultant} = 150 \text{ pounds, and acts in the same} \\ \text{direction with the first three forces and 16.3 feet} \\ \text{to the left of the 75-pound force.}\end{array}\right.$

Two parallel forces acting in the same direction can be compounded by the methods explained in the foregoing, but it is sometimes convenient to remember that the resultant equals the sum of the forces, acts in the same direction as that of the two forces and between them so that the line of action of the resultant divides the distance between the forces inversely as their magnitudes. For example, let F_1 and F_2 (Fig. 33) be two parallel forces. Then if R denotes the resultant and a and b its distances to F_1 and F_2 as shown in the figure,

$$R = F_1 + F_2,$$

and

$$a : b :: F_2 : F_1.$$

34. **Couples.** Two parallel forces which are equal and act in opposite directions are called a couple. The forces of a couple cannot be compounded, that is, no single force can produce the same effect as a couple. The perpendicular distance between the lines of action of the two forces is called the *arm*, and the product of one of the forces and the arm is called the *moment of the couple.*

A plus or minus sign is given to the moment of a couple according as the couple turns or tends to turn the body on which it acts in the clockwise or counter-clockwise direction.

VI. EQUILIBRIUM OF NON-CONCURRENT FORCES.

35. **Conditions of Equilibrium of Non-Concurrent Forces Not Parallel** may be stated in various ways; let us consider four. First:

1. **The algebraic sums of the components of the forces along each of two lines at right angles to each other equal zero.**

2. The algebraic sum of the moments of the forces about any origin equals zero.

Second:

1. The sum of the components of the forces along any line equals zero.
2. The sums of the moments of the forces with respect to each of two origins equal zero.

Third:

The sums of the moments of the forces with respect to each of three origins equals zero.

Fourth:

1. The algebraic sum of the moments of the forces with respect to some origin equals zero.
2. The force polygon for the forces closes.

It can be shown that if any one of the foregoing sets of conditions are fulfilled by a system, its resultant equals zero. Hence each is called a set of conditions of equilibrium for a non-concurrent system of forces which are not parallel.

The first three sets are "algebraic" and the last is "mixed," (1) of the fourth, being algebraic and (2) graphical. There is a set of graphical conditions also, but some one of those here given is usually preferable to a set of wholly graphical conditions.

Like the conditions of equilibrium for concurrent forces, they are used to answer questions arising in connection with concurrent systems known to be in equilibrium. Examples may be found in Art. 37.

36. Conditions of Equilibrium for Parallel Non-Concurrent Forces. Usually the most convenient set of conditions to use is one of the following:

First:

1. The algebraic sum of the forces equals zero, and
2. The algebraic sum of the moments of the forces about some origin equals zero.

Second:

The algebraic sums of the moments of the forces with respect to each of two origins equal zero.

37. Determination of Reactions. The weight of a truss, its loads and the supporting forces or reactions are balanced and constitute a system in equilibrium. After the loads and weight are ascertained, the reactions can be determined by means of conditions of equilibrium stated in Arts. 35 and 36.

The only cases which can be taken up here are the following common ones: (1) The truss is fastened to two supports and (2) The truss is fastened to one support and simply rests on rollers at the other.

Case (1) Truss Fastened to Both Its Supports. If the loads are all vertical, the reactions also are vertical. If the loads are not vertical, we assume that the reactions are parallel to the resultant of the loads.

The algebraic is usually the simplest method for determining the reactions in this case, and two moment equations should be used. Just as when finding reactions on beams we first take moments about the right support to find the left reaction and then about the left support to find the right reaction. As a check we add the reactions to see if their sum equals the resultant load as it should.

Examples. 1. It is required to determine the reactions on the truss represented in Fig. 20, it being supported at its ends and sustaining two vertical loads of 1,800 and 600 pounds as shown.

The two reactions are vertical; hence the system in equilibrium consists of parallel forces. Since the algebraic sum of the moments of all the forces about any point equals zero, to find the left reaction we take moments about the right end, and to find the right reaction we take moments about the left end. Thus, if R_1 and R_2 denote the left- and right-reactions respectively, then taking moments about the right end,

$$(R_1 \times 24) - (1800 \times 15) - (600 \times 15) = 0,$$

or

$$24R_1 = 27{,}000 + 9{,}000 = 36{,}000;$$

hence

$$R_1 = \frac{36{,}000}{24} = 1{,}500 \text{ pounds.}$$

Taking moment about the left end,

$$-R_2 \times 24 + 1{,}800 \times 9 + 600 \times 9 = 0$$

or

$$24R_2 = 16{,}200 + 5{,}400 = 21{,}600;$$

hence

$$R_2 = 900 \text{ pounds.}$$

As a check, add the reactions to see if the sum equals that of the loads as should be the case. (It will be noticed that reactions on trusses and beams under vertical loads are determined in the same manner.)

2. It is required to determine the reactions on the truss represented in Fig. 28 due to the wind pressures shown (2,700, 5,400 and 2,700 pounds), the truss being fastened to both its supports.

The resultant of the three loads is evidently a single force of 10,800 pounds, acting as shown in Fig. 34. The reactions are parallel to this resultant; let R_1 and R_2 denote the left and right reactions respectively.

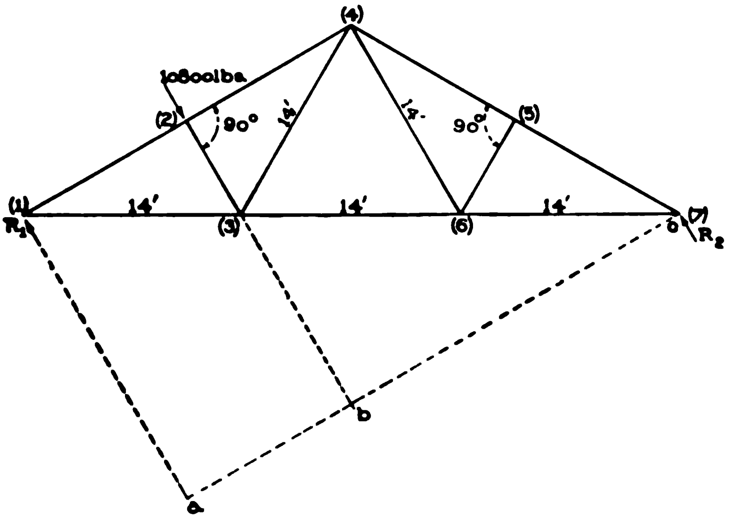


Fig. 34.

The line *abc* is drawn through the point 7 and perpendicular to the direction of the wind pressure; hence with respect to the right support the arms of R_1 and resultant wind pressure are *ac* and *bc*, and with respect to the left support, the arms of R_2 and the resultant wind pressure are *ac* and *ab*. These different arms can be measured from a scale drawing of the truss or be computed as follows: The angle 17*a* equals the angle 417, and 417 was shown to be 30 degrees in Example 3, Page 26. Hence

$$ab = 14 \cos 30°, \; bc = 28 \cos 30°, \; ac = 42 \cos 30°.$$

Since the algebraic sums of the moments of all the forces acting on the truss about the right and left supports equal zero,

$$R_1 \times 42 \cos 30^\circ = 10{,}800 \times 28 \cos 30^\circ,$$

and $$R_2 \times 42 \cos 30^\circ = 10{,}800 \times 14 \cos 30^\circ.$$

From the first equation,

$$R_1 = \frac{10{,}800 \times 28}{42} = 7{,}200 \text{ pounds},$$

and from the second,

$$R_2 = \frac{10{,}800 \times 14}{42} = 3{,}600 \text{ pounds}.$$

Adding the two reactions we find that their sum equals the load as it should.

Case (2) One end of the truss rests on rollers and the other is fixed to its support. The reaction at the roller end is always vertical, but the direction of the other is not known at the outset unless the loads are all vertical, in which case both reactions are vertical.

When the loads are not all vertical, the loads and the reactions constitute a non-concurrent non-parallel system and any one of the sets of conditions of equilibrium stated in Art. 35 may be used for determining the reactions. In general the fourth set is probably the simplest. In the first illustration we apply the four different sets for comparison.

Examples. 1. It is required to compute the reactions on the truss represented in Fig. 29 due to the wind pressures shown on the left side (3,100, 6,200 and 3,100 pounds), the truss resting on rollers at the right end and being fastened to its support at the left.

(*a*) Let R_1 and R_2 denote the left and right reactions. The direction of R_2 (at the roller end) is vertical, but the direction of R_1 is unknown. Imagine R_1 resolved into and replaced by its horizontal and vertical components and call them R_1' and R_1'' respectively (see Fig. 35.) The six forces, R_1', R_1'', R_2 and the three wind pressures are in equilibrium, and we may apply any one of the sets of statements of equilibrium for this kind of a system (see Art. 35) to determine the reactions. If we choose to use the first set we find, resolving forces along a horizontal line,

$$-R_1' + 3{,}100 \cos 55^\circ + 6{,}200 \cos 55^\circ + 3{,}100 \cos 55^\circ = 0;$$

resolving forces along a vertical line,

$$+R_1'' + R_2 - 3{,}100 \cos 35^\circ - 6{,}200 \cos 35^\circ - 3{,}100 \cos 35^\circ = 0;$$

taking moments about the left end,

$$+ 6{,}200 \times 12.2 + 3{,}100 \times 24.4 - R_2 \times 40 = 0.$$

From the first equation,

$$R_1' = 3{,}100 \cos 55^\circ + 6{,}200 \cos 55^\circ + 3{,}100 \cos 55^\circ = 7{,}113 \text{ pounds},$$

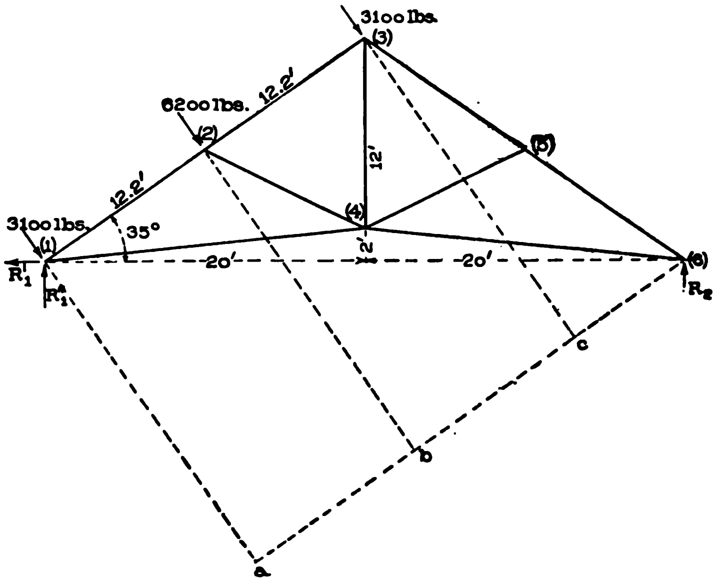


Fig. 35.

and from the third,

$$R_2 = \frac{6{,}200 \times 12.2 + 3{,}100 \times 24.4}{40} = 3{,}782 \text{ pounds.}$$

Substituting this value of R_2 in the second equation we find that

$$R_1'' = 3{,}100 \cos 35^\circ + 6{,}200 \cos 35^\circ + 3{,}100 \cos 35^\circ - 3{,}782$$
$$= 10{,}156 - 3{,}782 = 6{,}374 \text{ pounds.}$$

If desired, the reaction R_1 can now be found by compounding its two components R_1' and R_1''.

(*b*) Using the second set of conditions of equilibrium stated in Art. 35 we obtain the following three "equilibrium equations":
As in (1), resolving forces along the horizontal gives

$$-R_1' + 3{,}100 \cos 55^\circ + 6{,}200 \cos 55^\circ + 3{,}100 \cos 55^\circ = 0,$$

and taking moments about the left end,

$$6{,}200 \times 12.2 + 3{,}100 \times 24.4 - R_2 \times 40 = 0.$$

Taking moments about the right end gives

$$R_1'' \times 40 - 3{,}100 \times \overline{a6} - 6{,}200 \times \overline{b6} - 3{,}100 \times \overline{c6} = 0$$

Just as in (*a*), we find from the first and second equations the values of R_1' and R_2. To find R_1'' we need values of the arms $\overline{a6}$, $\overline{b6}$, and $\overline{c6}$. By measurement from a drawing we find that

$$\overline{a6} = 32.7, \overline{b6} = 20.5, \text{ and } \overline{c6} = 8.3 \text{ feet.}$$

Substituting these values in the third equation and solving for R_1'' we find that

$$R_1'' = \frac{3{,}100 \times 32.7 + 6{,}200 \times 20.5 + 3{,}100 \times 8.3}{40} = 6{,}355 \text{ pounds.}$$

(*c*) Using the third set of conditions of equilibrium stated in Art. 35 we obtain the following three equilibrium equations:
As in (*b*), taking moments about the right and left ends we get

$$R_1'' \times 40 - 3{,}100 \times 32.7 - 6{,}200 \times 20.5 - 3{,}100 \times 8.3 = 0,$$

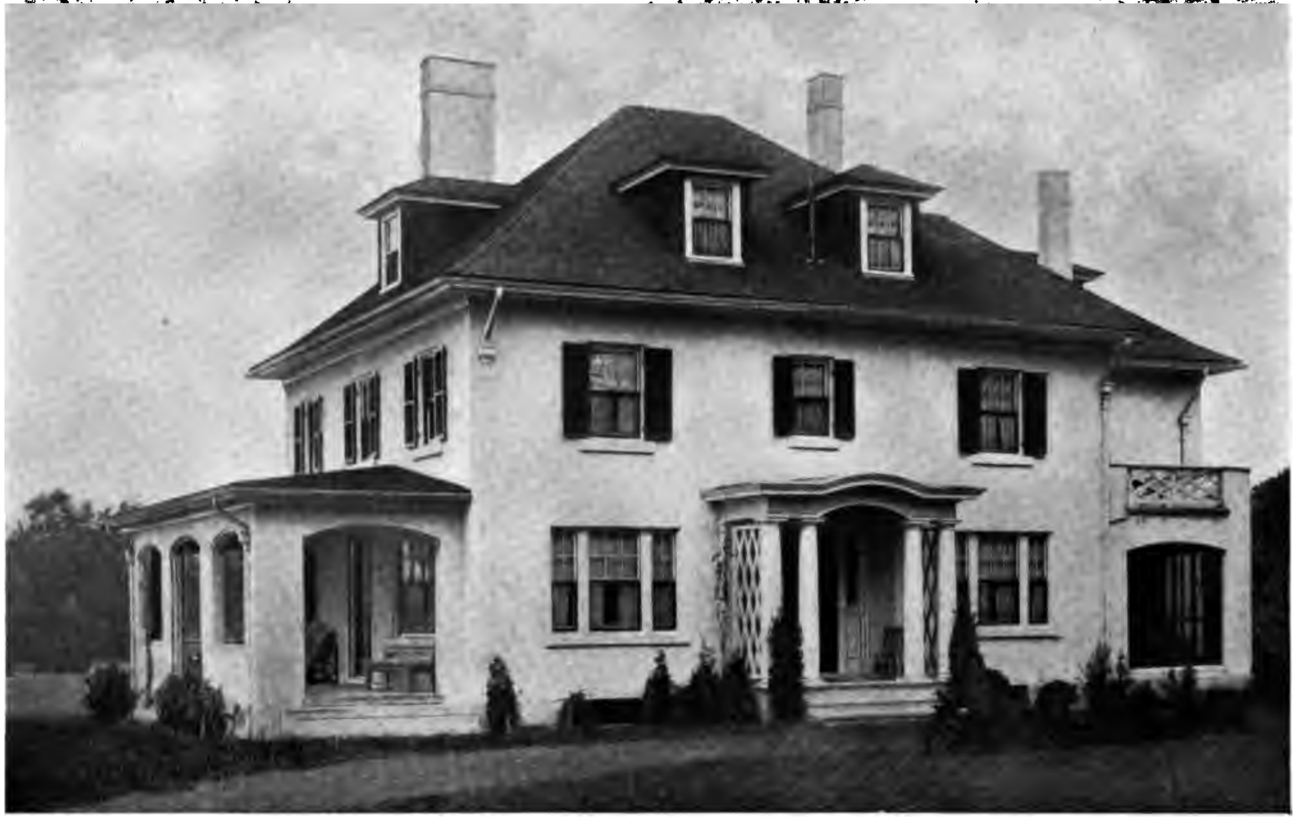
and $$-R_2 \times 40 + 6{,}200 \times 12.2 + 3{,}100 \times 24.4 = 0.$$

Choosing the peak of the truss as the origin of moments for the third equation we find that

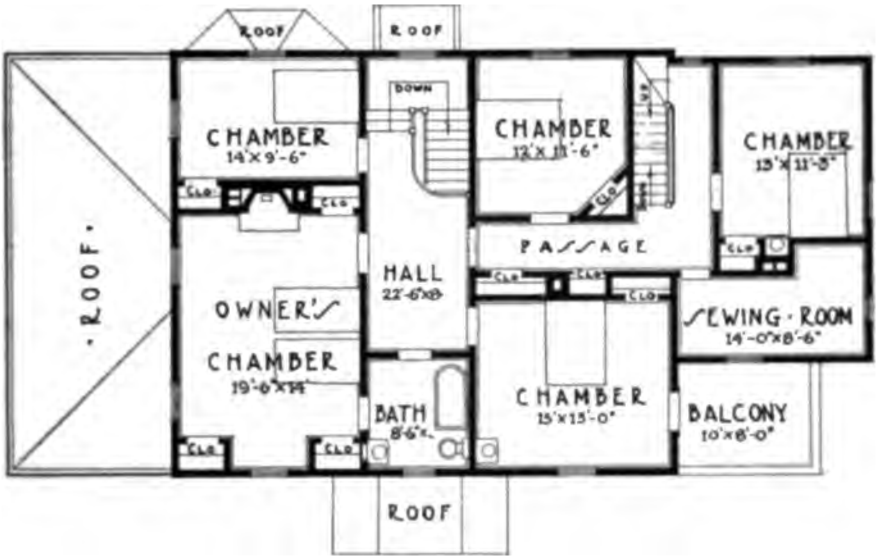
$$R_1' \times 14 + R_1'' \times 20 - 3{,}100 \times 24.4 - 6{,}200 \times 12.2 - R_2 \times 20 = 0.$$

As in (*b*) we find from the first two equations the values of R_1'' and R_2. These values substituted in the third equation change it to

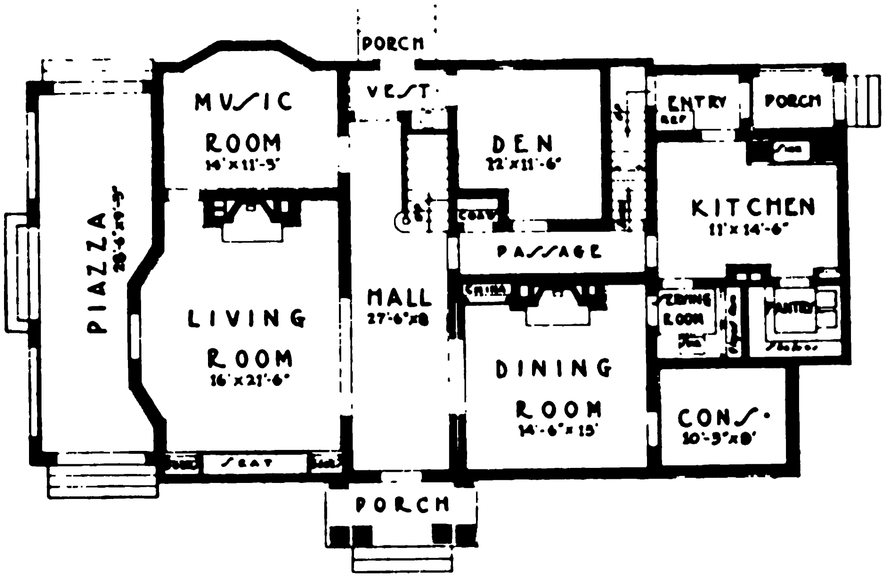
$$R_1' \times 14 + 6{,}373 \times 20 - 3{,}100 \times 24.4 - 6{,}200 \times 12.2 - 3{,}782 \times 20 = 0$$



RESIDENCE OF HERMAN ROSER, GLASTONBURY, CONNECTICUT
A. Raymond Ellis, Architect, Hartford, Connecticut
A family house for six. For plans see opposite page



SECOND · FLOOR · PLAN



FIRST · FLOOR · PLAN

FLOOR PLANS OF RESIDENCE OF HERMAN ROSER, GLASTONBURY, CONNECTICUT
A. Raymond Ellis, Architect, Hartford, Connecticut
For exterior view see opposite page

or

$$R_1' = \frac{-6{,}373 \times 20 + 3{,}100 \times 24.4 + 6{,}200 \times 12.2 + 3{,}782 \times 20}{14}$$

$$= 7{,}104.^*$$

(*d*) When using the fourth set of conditions we always determine the reaction at the roller end from the moment equation. Then, knowing the value of this reaction, draw the force polygon for all the loads and reactions and thus determine the magnitude and direction of the other reaction.

Taking moments about the left end, we find as in (*a*), (*b*), and

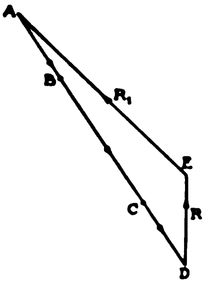


Fig. 36.

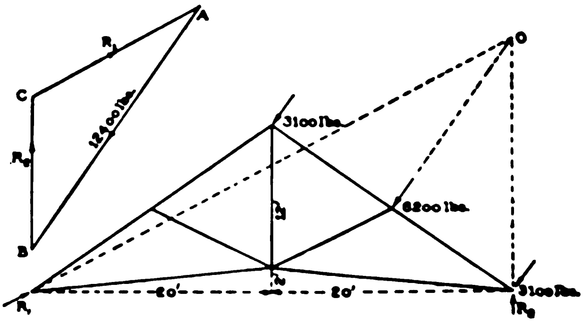


Fig. 37.

(*c*) that $R_2 = 3{,}782$. Then draw AB, BC and CD (Fig. 36) to represent the wind loads, and DE to represent R_2. Since the force polygon for all the forces must close, EA represents the magnitude and direction of the left reaction; it scales 9,550 pounds.

2. It is required to determine the reactions on the truss of the preceding illustration when the wind blows from the right.

The methods employed in the preceding illustration might be used here, but we explain another which is very simple. The truss and its loads are represented in Fig. 37. Evidently the resultant of the three wind loads equals 12,400 pounds and acts in the same line with the 6,200-pound load. If we imagine this resultant to replace the three loads we may regard the truss acted upon by three forces, the 12,400-pound force and the reactions, and these three forces as in equilibrium. Now when three forces are in equilibrium they must be concurrent or parallel, and since the resultant load (12,400 pounds) and R_2 intersect at O, the line of action of R_1 must also pass through O. Hence the left reaction acts through the left support and O as shown. We are now ready to determine the values of R_1 and R_2. Lay off AB to represent the resultant load, then from A and B draw lines parallel to R_1 and R_2, and mark their intersection C. Then BC and CA represent the magnitude and directions of R_2 and R_1 respectively; they scale 6,380 and 8,050 pounds.

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* The slight differences in the answers obtained from the different sets of equilibrium equations are due to inaccuracies in the measured arms of some of the forces.

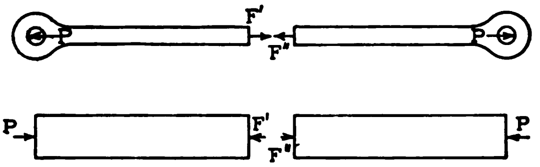

Fig. 38.

EXAMPLE FOR PRACTICE.

1. Determine the reactions on the truss represented in Fig. 26 due to wind pressure, the distance between trusses being 15 feet, supposing that both ends of the truss are fastened to the supports.

Ans. { Reaction at windward end is 6,682½ pounds.
Reaction at leeward end is 3,037½ pounds.

VII. ANALYSIS OF TRUSSES (CONTINUED); METHOD OF SECTIONS.

38. Forces in Tension and Compression Members. As explained in "Strength of Materials" if a member is subjected to forces, any two adjacent parts of it exert forces upon each other which hold the parts together. Figs. 38 (*a*) and 38 (*b*) show how these forces act in a tension and in a compression member. F' is the force exerted on the left part by the right, and F'' that exerted on the right part by the left. The two forces F' and F'' are equal, and in a tension member are pulls while in a compression member they are pushes.

39. Method of Sections. To determine the stress in a member of a truss by the method explained in the foregoing (the "method of joints"), we begin at one end of the truss and draw polygons for joints from that end until we reach one of the joints to which that member is connected. If the member is near the middle of a long truss, such a method of determining the stress in it requires the construction of several polygons. It is sometimes desirable to determine the stress in a member as directly as possible without having first determined stresses in other members. A method for doing this will now be explained; it is called the method of sections.

Fig. 39 (*a*) is a partial copy of Fig. 16. The line LL is intended to indicate a "section" of the truss "cutting" members $\overline{24}$, $\overline{34}$ and $\overline{36}$. Fig. 39 (*b*) and (*c*) represents the parts of the truss to the left and right of the section. By "part of a truss" we mean either of the two portions into which a section separates it when it cuts it completely.

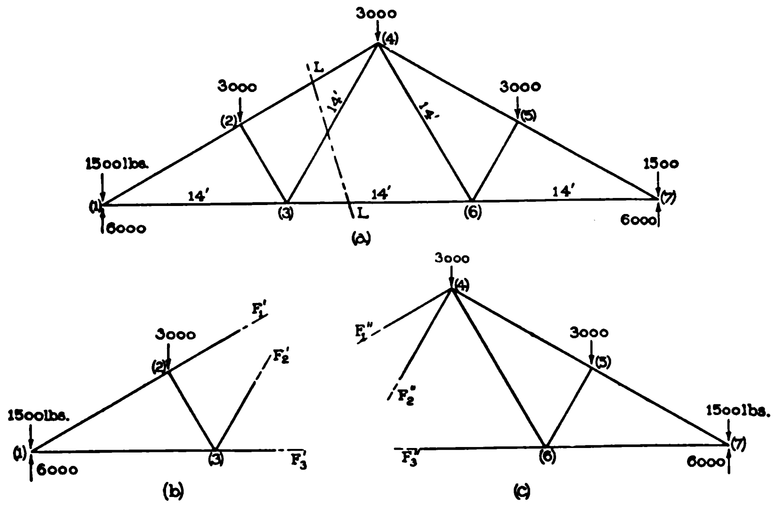

Fig. 39.

Since each part of the truss is at rest, all the forces acting on each part are balanced, or in equilibrium. The forces acting on each part consist of the loads and reactions applied to that part together with the forces exerted upon it by the other part. Thus the forces which hold the part in Fig. 39 (*b*) at rest are the 1,500- and 3,000-pound loads, the reaction 6,000 and the forces which the right part of the cut members exert upon the left parts. These latter forces are marked F_1', F_2' and F_3'; their senses are unknown.

but each acts along the axis of the corresponding member. The forces which hold the part in Fig. 39 (*c*) at rest are the two 3,000-pound loads, the 1,500-pound load, the right reaction 6,000 pounds and the forces which the left parts of the cut members exert upon the right parts. These are marked F_1'', F_2'' and F_3''; their senses are also unknown but each acts along the axis of the corresponding member. The forces F_1' and F_1'', F_2' and F_2'', and F_3' and F_3'' are equal and opposite; they are designated differently only for convenience.

If, in the system of forces acting on either part of the truss, there are not more than three unknown forces, then those three can be computed by "applying" one of the sets of the conditions of equilibrium stated in Art. 35.* In writing the equations of equilibrium for the system it is practially necessary to assume a sense for one or more of the unknown forces. *We shall always assume that such forces are pulls that is, act away from the part of the truss upon which they are exerted. Then if the computed value of a force is plus, the force is really a pull and the member is in tension and if the value is minus, then the force is really a push and the member is in compression.*

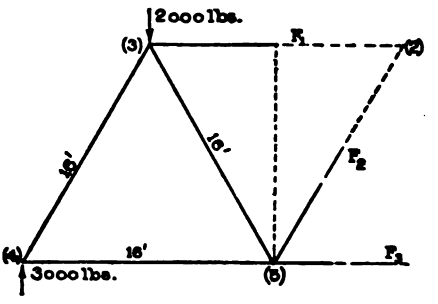

Fig. 40.

To determine the stress in any particular member of a truss in accordance with the foregoing, "pass a section" through the truss cutting the member under consideration, and then apply as many conditions of equilibrium to all the forces acting on either part of the truss as may be necessary to determine the desired force. In passing the section, care should be taken to cut as few members as possible, and never should a section be passed so as to cut more than three, the stresses in which are unknown; neither should the three be such that they intersect in a point.

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*If, however, the lines of action of the three forces intersect in a point then the statement is not true.

Examples. 1. It is required to determine the amount and kind of stress in the member $\overline{24}$ of the truss represented in Fig. 39 (*a*) when loaded as shown.

Having determined the reactions (6,000 pounds each) we pass a section through the entire truss and cutting $\overline{24}$; LL is such a section. Considering the part of the truss to the left of the section, the forces acting upon that part are the two loads, the left reaction and the forces on the cut ends of members $\overline{24}$, $\overline{34}$ and $\overline{36}$ (F_1', F_2', and F_3'). F_1' can be determined most simply by writing a moment equation using (3) as the origin, for with respect to that origin the moments of F_2' and F_3' are zero, and hence those forces will not appear in the equation. Measuring from a large scale drawing, we find that the arm of F_1' is 7 feet and that of the 3,000-pound load is 3.5 feet. Hence

$$(F_1' \times 7) + (6{,}000 \times 14) - (1{,}500 \times 14) - (3{,}000 \times 3.5) = 0$$

$$\text{or } F_1' = \frac{-(6{,}000 \times 14) + (1{,}500 \times 14) + (3{,}000 \times 3.5)}{7} = -7{,}500$$

The minus sign means that F_1' is a push and not a pull, hence the member $\overline{24}$ is under 7,500 pounds compression.

The stress in the member 24 may be computed from the part of the truss to the right of the section. Fig. 39 (*c*) represents that part and all the forces applied to it. To determine F_1'' we take moments about the intersection of F_2'' and F_3''. Measuring from a drawing we find that the arm of F_1'' is 7 feet,

that of the load at joint (4) is 7 feet,
that of the load at joint (5) is 17.5 feet,
that of the load at joint (7) is 28 feet,
and that of the reaction is 28 feet.

Hence, assuming F_1'' to be a pull,

$$-(F_1'' \times 7) + (3{,}000 \times 7) + (3{,}000 \times 17.5) + (1{,}500 \times 28) - (6{,}000 \times 28) = 0,$$

$$\text{or } F_1'' = \frac{(3{,}000 \times 7) + (3{,}000 \times 17.5) + (1{,}500 \times 28) - (6{,}000 \times 28)}{7} = -7{,}500$$

The minus sign means that F_1'' is a push, hence the member 24 is under compression of 7,500 pounds, a result agreeing with that previously found.

2. It is required to find the stress in the member *gh* of the truss represented in Fig. 25, due to the loads shown.

If we pass a section cutting *bg*, *gh* and *he*, and consider the left part, we get Fig. 40, the forces on that part being the 2,000-pound load, the left reaction, and the forces F_1, F_2 and F_3 exerted by the right part on the left. To compute F_2 it is simplest to use the condition that the algebraic sum of all the vertical components equals zero. Thus, assuming that F_2 is a pull, and since its angle with the vertical is 30°,

$$F_2 \cos 30° - 2{,}000 + 3{,}000 = 0; \text{ or,}$$

$$F_2 = \frac{2{,}000 - 3{,}000}{\cos 30°} = \frac{-1{,}000}{0.866} = -1{,}154.$$

The minus sign means that F_2 is a push, hence the member is under a compression of 1,154 pounds.

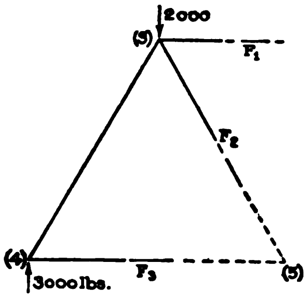


Fig. 41.

3. It is required to determine the stress in the member *bg* of the truss represented in Fig. 25, due to the loads shown.

If we pass a section cutting *bg* as in the preceding illustration, and consider the left part, we get Fig. 40. To compute F_1 it is simplest to write the moment equation for all the forces using joint 5 as origin. From a large scale drawing, we measure the arm of F_1 to be 13.86 feet hence, assuming F_1 to be a pull,

$$F_1 \times 13.86 - 2{,}000 \times 8 + 3{,}000 \times 16 = 0;$$

or,

$$F_1 = \frac{2{,}000 \times 8 - 3{,}000 \times 16}{13.86} = \frac{-32{,}000}{13.86}$$

$$= -2{,}310.$$

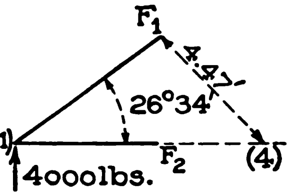


Fig. 42.

The minus sign means that F_1 is a push; hence the member is under a compression of 2,308 pounds.

The section might have been passed so as to cut members *bg*, *fg*, and *fe*, giving Fig. 41 as the left part, and the desired force might be obtained from the system of forces acting on that part (3,000, 2,000, F_1, F_2, and F_3.) To compute F_1 we take moments about the intersection of F_2 and F_3, thus

$$F_1 \times 13.86 - 2,000 \times 8 + 3,000 \times 16 = 0.$$

This is the same equation as was obtained in the first solution, and hence leads to the same result.

4. It is required to determine the stress in the member $\overline{12}$ of the truss represented in Fig. 26, due to the loads shown.

Passing a section cutting members $\overline{12}$ and 14, and considering the left part, we get Fig. 42. To determine F_1 we may write a moment equation preferably with origin at joint 4, thus:

$$F_1 \times 4.47 + 4,000 \times 10 = 0^*;$$

or,
$$F_1 = \frac{-4,000 \times 10}{4.47} = -8,948 \text{ pounds},$$

the minus sign meaning that the stress is compressive.

F_1 might be determined also by writing the algebraic sum of the vertical components of all the forces on the left part equal to zero, thus:

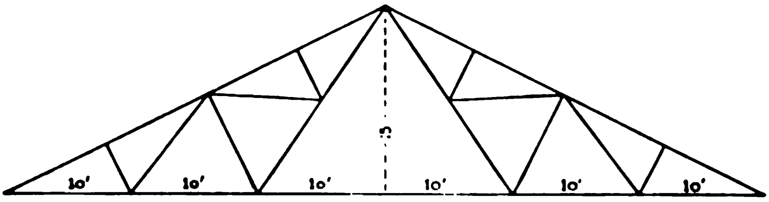


Fig. 43.

$$F_1 \sin 26° \, 34' + 4,000 = 0;$$

or,
$$F_1 = \frac{-4,000}{\sin 26° \, 34'} = \frac{-4,000}{0.447} = -8,948,$$

agreeing with the first result.

EXAMPLES FOR PRACTICE.

1. Determine by the method of sections the stresses in members $\overline{23}$, $\overline{25}$, and $\overline{45}$ of the truss represented in Fig. 26, due to the loads shown.

Ans. { Stress in $\overline{23}$ = − 5,600 pounds,
Stress in $\overline{25}$ = − 3,350 pounds,
Stress in $\overline{45}$ = + 8,000 pounds.

---

* The arm of F_1 with respect to (4) is 4.47 feet.

2. Determine the stresses in the members $\overline{12}$, $\overline{15}$, $\overline{34}$, and $\overline{56}$ of the truss represented in Fig. 27, due to the loads shown.

Ans.
Stress in $\overline{12}$ = − 11,170 pounds,
Stress in $\overline{15}$ = + 10,000 pounds,
Stress in $\overline{34}$ = − 8,940 pounds,
Stress in $\overline{56}$ = + 6,000 pounds.

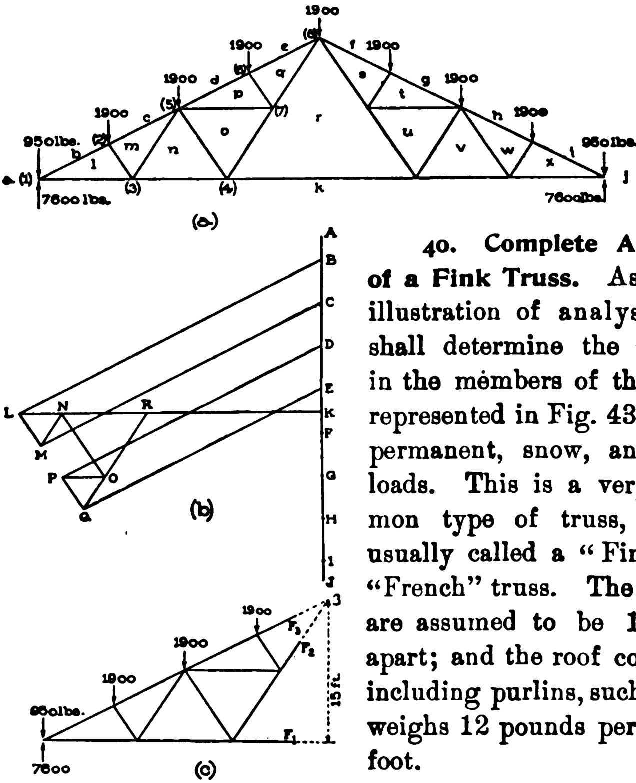


Fig. 44.

40. Complete Analysis of a Fink Truss. As a final illustration of analysis, we shall determine the stresses in the members of the truss represented in Fig. 43, due to permanent, snow, and wind loads. This is a very common type of truss, and is usually called a "Fink" or "French" truss. The trusses are assumed to be 15 feet apart; and the roof covering, including purlins, such that it weighs 12 pounds per square foot.

The length from one end to the peak of the truss equals

$$\sqrt{15^2 + 30^2} = \sqrt{1,125} = 33.54 \text{ feet},$$

hence the area of the roofing sustained by one truss equals

$$(33.54 \times 15)\ 2 = 1,006.2 \text{ square feet},$$

and the weight of that portion of the roof equals

$$1,006.2 \times 12 = 12,074 \text{ pounds}.$$

The probable weight of the truss (steel), according to the formula of Art. 19, is

$$15 \times 60\left(\frac{60}{25}+1\right)=3{,}060 \text{ pounds.}$$

The total permanent load, therefore, equals

$$12{,}074+3{,}060=15{,}134 \text{ pounds;}$$

the end loads equal $\frac{1}{16}$ of the total, or 950 pounds, and the other apex loads equal $\frac{1}{8}$ of the total, or 1,900 pounds.

Dead Load Stress. To determine the dead load stresses, construct a stress diagram. Evidently each reaction equals one-half the total load, that is 7,600 pounds; therefore ABCDEFGHIJKA (Fig. 44*b*) is a polygon for all the loads and reactions. First, we draw the polygon for joint 1; it is KABLK, BL and LK representing the stress in *bl* and *lk* (see record Page 72 for values). Next draw the polygon for joint 2; it is LBCML, CM and ML representing the stresses in *cm* and *ml*. Next draw the polygon for joint 3; it is KLMNK, MN and NK representing the stresses in *mn* and *nk*.

At each of the next joints (4 and 5), there are three unknown forces, and the polygon for neither joint can be drawn. We might draw the polygons for the joints on the right side corresponding to 1, 2, and 3, but no more until the stress in one of certain members is first determined otherwise. If, for instance, we determine by other methods the stress in *rk*, then we may construct the polygon for joint 4; then for 5, etc., without further difficulty.

To determine the stress in *rk*, we pass a section cutting *rk*, *qr*, and *eq*, and consider the left part (see Fig. 44*c*). The arms of the loads with respect to joint 8 are 7.5, 15, 22.5, and 30 feet; and hence, assuming F_1 to be a pull,

$$-F_1 \times 15 - 1{,}900 \times 7.5 - 1{,}900 \times 15 - 1{,}900 \times 22.5 - 950 \times 80 + 7{,}600 \times 30 = 0; \text{ or,}$$

$$F_1 = \frac{-1{,}900 \times 7.5 - 1{,}900 \times 15 - 1{,}900 \times 22.5 - 950 \times 30 + 7{,}600 \times 30}{15}$$
$$= 7{,}600 \text{ pounds}$$

Since the sign of F_1 is plus, the stress in *rk* is tensile.

Now lay off KR to represent the value of the stress in *kr* just found, and then construct the polygon for joint 4. The polygon is KNORK, NO and OR representing the stresses in *no* and *or*. Next draw the polygon for joint 5; it is ONMCDPO, DP and PO representing the stresses in *dp* and *po*. Next draw the polygon for joint 6 or joint 7; for 6 it is PDEQP, EQ and QP representing the stresses in *eq* and *qp*. At joint 7 there is now but one unknown force, namely, that in *qr*. The polygon for the three others at that joint is ROPQ; and since the unknown force must close the polygon, QR must represent that force, and must be parallel to *qr*.

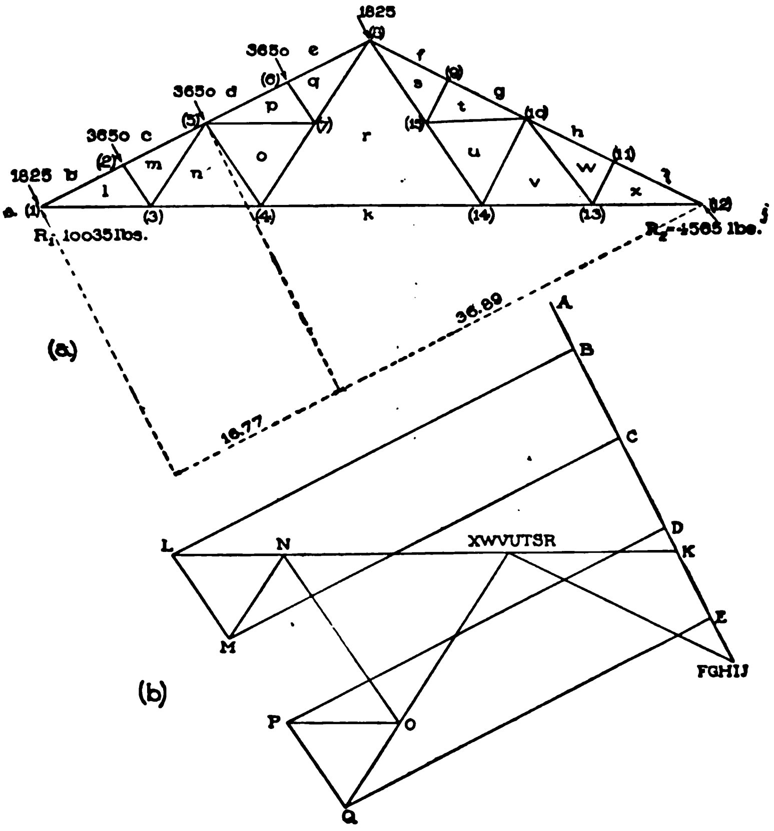


Fig. 45.

On account of the symmetry of loading, the stress in any member on the right side is just like that in the corresponding member on the left; hence, it is not necessary to draw the diagram for the right half of the truss.

Snow-Load Stress. The area of horizontal projection of the roof which is supported by one truss is 60 × 15 = 900 square feet; hence the snow load borne by one truss is 900 × 20 = 18,000 pounds, assuming a snow load of 20 pounds per horizontal square foot. This load is nearly 1.2 times the dead load, and is applied similarly to the latter; hence the snow load stress in any member equals 1.2 times the dead load stress in it. We record, therefore, in the third column of the stress record, numbers equal to 1.2 times those in the second as the snow-load stresses.

Wind Load Stress. The tangent of the angle which the roof makes with the horizontal equals $\frac{15}{30}$ or $\frac{1}{2}$; hence the angle is 26° 34′, and the value of wind pressure for the roof equals practically 29 pounds per square foot, according to Art. 19. As previously explained, the area of the roof sustained by one truss equals 1,006.2 square feet; and since but one-half of this receives wind pressure at one time, the wind pressure borne by one truss equals

$$503.1 \times 29 = 14{,}589.9, \text{ or practically } 14{,}600 \text{ pounds.}$$

When the wind blows from the left, the apex loads are as represented in Fig. 45a, and the resultant wind pressure acts through joint 5. To compute the reactions, we may imagine the separate wind pressures replaced by their resultant. We shall suppose that both ends of the truss are fixed; then the reactions will be parallel to the wind pressure. Let R_1 and R_2 denote the left and right reactions respectively; then, with respect to the right end, the arms of R_1 and the resultant wind pressure (as may be scaled from a drawing) are 16.77 + 36.89 and 36.89 feet respectively; and with respect to the left end, the arms of R_2 and the resultant wind pressure are 16.77 + 36.89 and 16.77 feet respectively.

Taking moments about the right end we find that

$$-14{,}600 \times 36.89 + R_1 \times (16.77 + 36.89) = 0;$$

or,
$$R_1 = \frac{14{,}600 \times 36.89}{16.77 + 36.89} = 10{,}035 \text{ pounds.}$$

Taking moments about the left end, we find that

$$14{,}600 \times 16.77 - R_2 \times (16.77 + 36.89) = 0;$$

or, $$R_2 = \frac{14{,}600 \times 16.77}{16.77 + 36.89} = 4{,}565 \text{ pounds.}$$

To determine the stresses in the members, we construct a stress diagram. In Fig. 45*b*, AB, BC, CD, DE, and EF represent the wind loads at the successive joints, beginning with joint 1. The point F is also marked G, H, I, and J, to indicate the fact that there are no loads at joints 9, 10, 11, and 12. JK represents the right reaction, and KA the left reaction.

We may draw the polygon for joint 1 or 12; for 1 it is KABLK, BL and LK representing the stresses in *bl* and *lk*. We may next draw the polygon for joint 2; it is LBCML, CM and ML representing the stresses in *cm* and *ml*.

Stress Record.

MEMBER.	STRESSES.					
	Dead Load.	Snow Load.	Wind Left.	Wind Right.	Resultant.	Resultant.
bl	-14,700	-17,600	-16,400	15,000	-48,700	-32,300
cm	-13,700	-16,400	-15,900	15,000	-46,000	-30,100
dp	-12,600	-15,100	-15,400	15,000	-43,100	-28,000
eq	-11,600	-13,900	-14,900	15,000	-40,400	-26,500
lm	- 1,650	- 2,000	- 3,700	0	- 7,350	- 5,350
mn	+ 1,650	+ 2,000	+ 3,700	0	+ 7,350	+ 5,350
no	- 3,300	- 4,000	- 7,400	0	-14,700	-10,700
op	+ 1,850	+ 2,200	+ 4,100	0	+ 8,150	+ 5,950
pq	- 1,650	- 2,000	- 3,700	0	- 7,350	- 5,350
rq	+ 5.000	+ 6,000	+11,000	0	+22,000	+16,000
ro	+ 3,400	+ 4,100	+ 7,400	0	+14,900	+10,800
kl	+13,300	+16,000	+18,300	+ 6,100	+47,600	+31,600
kn	+11,300	+13,600	+14,200	+ 6,100	+39,100	+25,500
kr	+ 8,000	+ 9,600	+ 6,100	+ 6,100	+23,700	+17,600
kv	+11,300	+13,600	+ 6,100	+14,200	+39,100	+25,500
kx	+13,300	+16,000	+ 6,100	+18,300	+47,600	+31,600
ru	+ 3,400	+ 4,100	0	+ 7,400	+14,900	+10,800
rs	+ 5,000	+ 6,000	0	+11.000	+22,000	+16,000
st	- 1,650	- 2,000	0	-3,700	- 7,350	- 5,350
tu	+ 1,850	+ 2,200	0	+ 4,100	+ 8,150	+ 5,950
uv	- 3,300	- 4,000	0	- 7,400	-14,700	-10,700
vw	+ 1,650	+ 2,000	0	- 3,700	+ 7,350	+ 5,350
wx	- 1,650	- 2,000	0	- 3,700	- 7,350	- 5,350
fs	-11,600	-13,900	15,000	-14,900	-40,400	-26,500
gt	-12,600	-15,100	15,000	-15,400	-43,100	-28,000
hw	-13,700	-16,400	15,000	-15,900	-46,000	-30,100
ix	-14,700	-17,600	15,000	-16,400	-48,700	-32,300

We may draw next the polygon for joint 8; it is KLMNK, MN and NK representing the stresses in *mn* and *nk*. No polygon for a joint on the left side can now be drawn, but we may begin at the right end. For joint 12 the polygon is JKXIJ, KX and XI representing the stresses in *kx* and *xi*.

At joint 11 there are three forces; and since they are balanced, and two act along the same line, those two must be equal and opposite, and the third must equal zero. Hence the point X is also marked W to indicate the fact that XW, or the stress in *xw*, is zero. Then, too, the diagram shows that WH equals XI. Having just shown that there is no stress in *xw*, there are but three forces at joint 13. Since two of these act along the same line, they must be equal and opposite, and the third zero. Therefore the point W is also marked V to indicate the fact that WV, or the stress in *wv*, equals zero. The diagram shows also that VK equals XK. This same argument applied to joints 9, 15, 10, and 14 successively, shows that the stresses in *st*, *tu*, *uv*, *ur*, and *sr* respectively equal zero. For this reason the point X is also marked UTS and R. It is plain, also, that the stresses in *sf* and *tg* equal those in *wh* and *xi*, and that the stress in *kr* equals that in *kv* or *kx*. Remembering that we are discussing stress due to wind pressure only, it is plain, so far as wind pressure goes, that the intermediate members on the right side are superfluous.

We may now resume the construction of the polygons for the joints on the left side. At joint 4, we know the forces in the members *kn* and *kr*; hence there are only two unknown forces there. The polygon for the joint is KNORK, NO and OR representing the stresses in *no* and *or*. The polygon for joint 5 may be drawn next; it is ONMCDPO, DP and PO representing the stresses in *dp* and *po*. The polygon for joint 6 or joint 7 may be drawn next; for 6 it is PDEQP, EQ and QP representing the stresses in *eq* and *qp*. At joint 7 there is but one unknown force, and it must close the polygon for the known forces there. That polygon is ROPQ; hence QR represents the unknown force. (If the work has been correctly and accurately done, QR will be parallel to *qr*).

When the wind blows upon the right side, the values of the reactions, and the stresses in any two corresponding members, are reversed. Thus, when the wind blows upon the left side, the stresses in *kl* and *kx* equal 18,300 and 6,100 pounds respectively; and when it blows upon the right they are respectively 6,100 and 18,300 pounds. It is not necessary, therefore, to construct a stress diagram for the wind pressure on the right. The numbers in the fifth column (see table, Page 72) relate to wind right, and were obtained from those in the fourth.

41. Combination of Dead, Snow, and Wind-Load Stresses. After having found the stress in any member due to the separate loads (dead, snow, and wind), we can then find the stress in that member due to any combination of loads, by adding algebraically the stresses due to loads separately. Thus, in a given member, suppose:

Dead-load stress = + 10,000 pounds,
Snow-load " = + 15,000 "
Wind-load " (right) = − 12,000 "
" " (left) = + 4,000 "

Since the dead load is permanent (and hence the dead-load stress also) the resultant stress in the member when there is a snow load and no wind pressure, is

$$+ 10{,}000 + 15{,}000 = + 25{,}000 \text{ pounds (tension)};$$

when there is wind pressure on the right, the resultant stress equals

$$+ 10{,}000 - 12{,}000 = - 2{,}000 \text{ pounds (compression)};$$

when there is wind pressure on the left, the resultant stress is

$$+ 10{,}000 + 4{,}000 = + 14{,}000 \text{ pounds (tension)};$$

and when there is a snow load and wind pressure on the left, the resultant stress is

$$+ 10{,}000 + 15{,}000 + 4{,}000 = + 29{,}000 \text{ pounds (tension)}.$$

If all possible combinations of stress for the preceding case be made, it will be seen that the greatest tension which can come upon the member is 29,000 pounds, and the greatest compression is 2,000 pounds.

In roof trusses it is not often that the wind load produces a "reversal of stress" (that is, changes a tension to compression, or *vice versa*); but in bridge trusses the rolling loads often produce reversals in some of the members. In a record of stresses the reversals of stress should always be noted, and also the value of the greatest tension and compression for each one.

The numbers in the sixth column of the record (Page 72) are the values of the greatest resultant stress for each member. It is sometimes assumed that the greatest snow and wind loads will not come upon the truss at the same time. On this assumption the resultant stresses are those given in the seventh column.

EXAMPLE FOR PRACTICE.

1. Compile a complete record for the stresses in the truss of Fig. 24, for dead, snow, and wind loads. See Example 1, Article 27, for values of dead-load stresses, and Example 2, Article 29, for values of the wind-load stresses. Assume the snow load to equal 1 2 times the dead load.

After the record is made, compute the greatest possible stress in each member, assuming that the wind load and snow load will not both come upon the truss at the same time.

The greatest resultant stresses are as follows:

Member	*af*	*fe*	*bg*	*fg*	*gh*	*hi*	*hc*	*ic*	*id*
Resultant...	-14,950	+17,800	10,400	8,875	+7,820	8,875	11,800	+11,500	-13,850

42. Truss Sustaining a Roof of Changing Slope. Fig 46 represents such a truss. The weight of the truss itself can be estimated by means of the formula of Art. 19. Thus if the distance between trusses equals 12 feet, the weight of the truss equals

$$W = 12 \times 32 \left(\frac{32}{25} + 1 \right) = 875 \text{ pounds.}$$

The weight of the roofing equals the product of the area of the roofing and the weight per unit area. The area equals 12 times the sum of the lengths of the members $\overline{12}$, $\overline{23}$, $\overline{34}$, and $\overline{45}$, that is, $12 \times 36\frac{1}{2} = 438$ square feet. If the roofing weighs 10 pounds per square foot, then the weight of roofing sustained by one truss equals $438 \times 10 = 4,380$ pounds. The total dead load then equals

$$875 + 4,380 = 5,255 \text{ pounds;}$$

and the apex dead loads for joints 2, 3, and 4 equal:

$$\frac{1}{4} \times 5{,}255 = 1{,}314 \text{ (or approximately 1,300) pounds;}$$

while the loads for joints 1 and 5 equal

$$\frac{1}{8} \times 5{,}255 = 657 \text{ (or approximately 650) pounds.}$$

The **snow loads** for the joints are found by computing the snow load on each separate slope of the roof. Thus, if the snow weighs 20 pounds per square foot (horizontal), the load on the portion $\overline{12}$ equals 20 times the area of the horizontal projection of the portion of the roof represented by $\overline{12}$. This horizontal projection equals 8 × 12 (= 96) square feet; snow load equals 96 × 20 (= 1,920) pounds. This load is to be equally divided between joints 1 and 2.

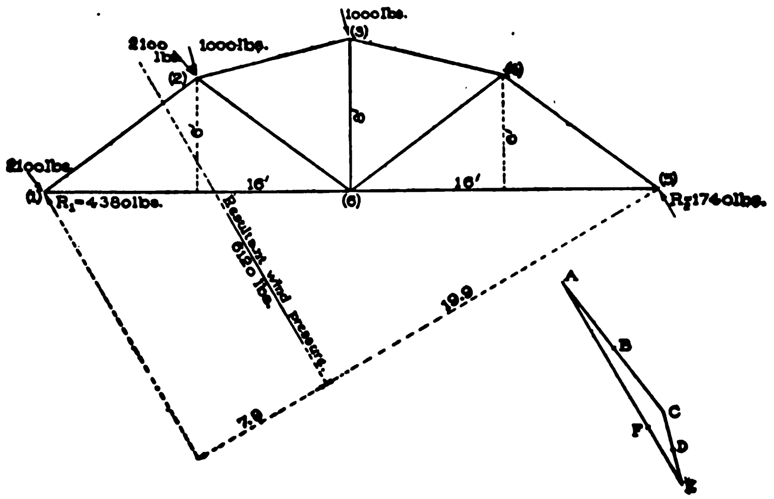
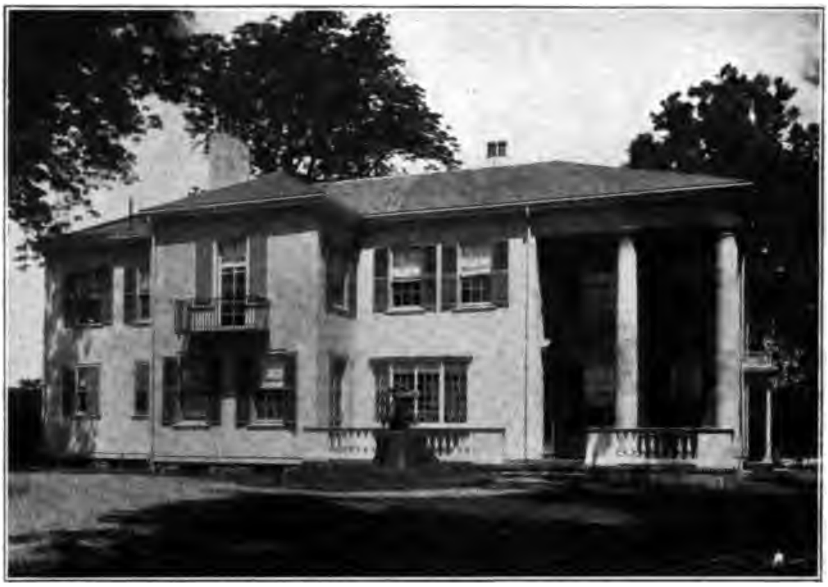
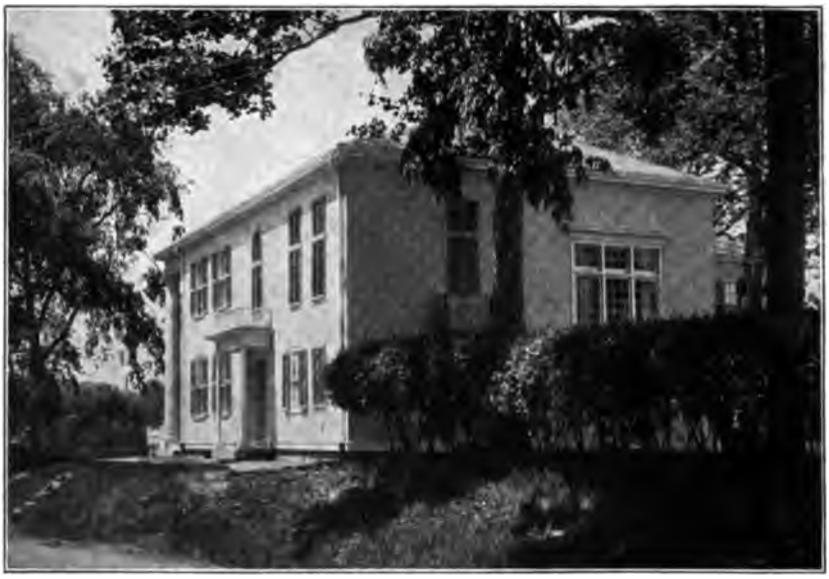


Fig. 46.

In a similar way the snow load borne by $\overline{23}$ equals 20 times the area of the horizontal projection of the roof represented by $\overline{23}$; this horizontal projection equals 8 × 12 (= 96) square feet as before, and the snow load hence equals 1,920 pounds also. This load is to be equally divided between joints 2 and 3.

Evidently the loads on parts $\overline{34}$ and $\overline{45}$ also equal 1,920 pounds each; hence the apex loads at joints 1 and 5 equal 960 pounds and at joints 2, 3 and 4, 1,920 pounds.





HOUSE AT FRAMINGHAM, MASSACHUSETTS, AS REMODELED FOR C. LA VERNE BUTLER

Frank Chouteau Brown, Architect, Boston, Massachusetts

Alterations on this plaster house cost $8000. Taking into consideration the changes made in the house and the work of repairing and raising the roof, etc., an entirely new house of this size could be built for this sum—certainly within $9000. For plans see page 219.

The wind load must be computed for each slope of the roof separately. The angles which $\overline{12}$ and $\overline{23}$ make with the horizontal, scale practically 37 and 15 degrees. According to the table of wind pressures (Art. 19), the pressures for these slopes equal about 35 and 20 pounds per square foot respectively. Since member $\overline{12}$ is 10 feet long, the wind pressure on the 37-degree slope equals $10 \times 12 \times 35 = 4{,}200$ pounds.

This force acts perpendicularly to the member $\overline{12}$, and is to be equally divided between joints 1 and 2 as represented in the figure. Since the member $\overline{23}$ is $8\frac{1}{4}$ feet long, the wind pressure on the 15-degree slope equals

$$8\tfrac{1}{4} \times 12 \times 20 = 1{,}980 \text{ or approximately } 2{,}000 \text{ pounds.}$$

This pressure acts perpendicularly to member $\overline{23}$, and is to be equally divided between joints 2 and 3 as represented.

The stress diagram for dead, snow, or wind load for a truss like that represented in Fig. 46, is constructed like those previously explained; but there are a few points of difference in the analysis for wind stress, and these will be explained in what follows.

Example. Let it be required to determine the stresses in the truss of Fig. 46, due to wind loads on the left as represented.

It is necessary to ascertain the reactions due to the wind loads; therefore, find the resultant of the wind pressures, see Art. 37; it equals 6,120 pounds and acts as shown. Now, if both ends of the truss are fastened to the supports, then the reactions are parallel to the resultant wind pressure, and their values can be readily found from moment equations. Let R_1 and R_2 denote the left and right reactions respectively; then, since the arms of R_1 and the resultant wind pressure with respect to the right end equal 27.8 and 19.9 feet respectively,

$$R_1 \times 27.8 = 6{,}120 \times 19.9 = 121{,}788\,;$$

hence, $$R_1 = \frac{121{,}788}{27.8} = 4{,}380 \text{ pounds approximately.}$$

Since the arms of R_2 and the resultant wind pressure with respect to the left support are 27.8 and 7.9 feet respectively,

$$R_2 \times 27.8 = 6{,}120 \times 7.9 = 48{,}348\,;$$

hence, $R_2 = \frac{48,348}{27.8} = 1,740$ pounds approximately.

The next step is to draw the polygon for the loads and reactions; so we draw lines AB, BC, CD, and DE to represent the loads at joint 1, the two at joint 2, and that at joint 3, respectively; and then EF to represent the right reaction. (If the reactions have been correctly determined and the drawing accurately done, then FA will represent the left reaction.)

The truss diagram should now be lettered (agreeing with the letters on the polygon just drawn), and then the construction of the stress diagram may be begun. Since this construction presents no points not already explained, it will not be here carried out.

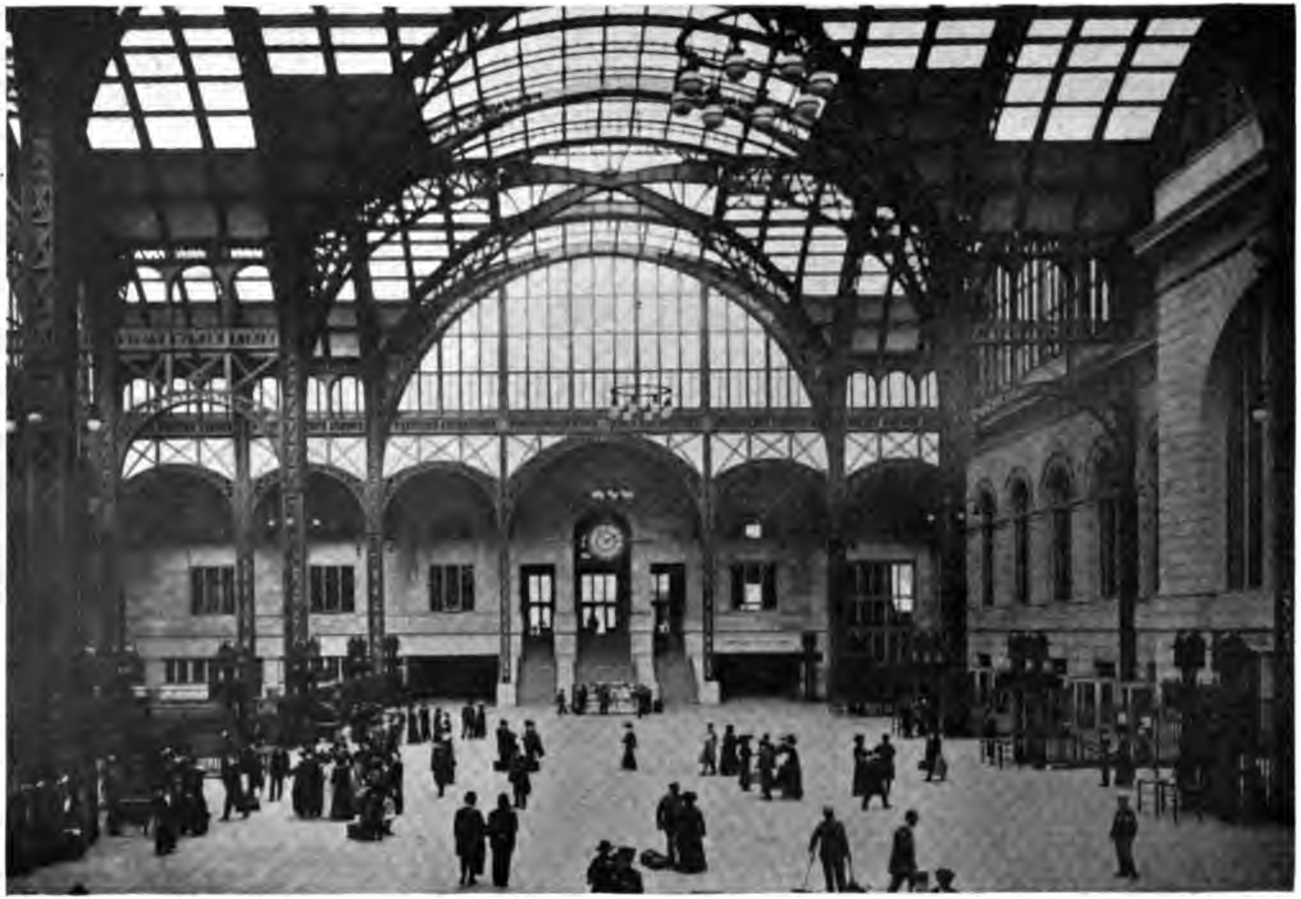
EXAMPLE FOR PRACTICE.

Analyze the truss of Fig. 46 for dead, snow, and wind loads as computed in the foregoing, and compute the greatest resultant stress in each member due to combined loads, assuming that the snow and wind do not act at the same time.

Stress Record.

Ans.

Member.	Dead.	Snow.	Wind Left.	Wind Right.	Resultant.
12	-3,250	-4,800	-3,450	-2,500	-8,050
23	-2,700	-4,000	-2,850	-3,100	-6,700
16	+2,600	+3,850	+3,750	+1,150	+6,450
26	0	0	-2,000	+1,250	-2,000 +1,250
36	0	0	+ 450	+ 450	+ 450
46	0	0	+1,250	-2,000	+1,250 -2,000
56	+2,600	+3,850	+1,150	+3,750	+6,450
43	-2,700	-4,000	-3,100	-2,850	-6,700
54	-3,250	-4,800	-2,500	-3,450	-8,050



EXCELLENT EXAMPLE OF ROOF CONSTRUCTION IN PENNSYLVANIA RAILROAD STATION, NEW YORK CITY

ROOF TRUSSES

1. Classes of Roof Trusses. Roof trusses may be divided into three classes according to the shape of their upper chord. These three classes are:

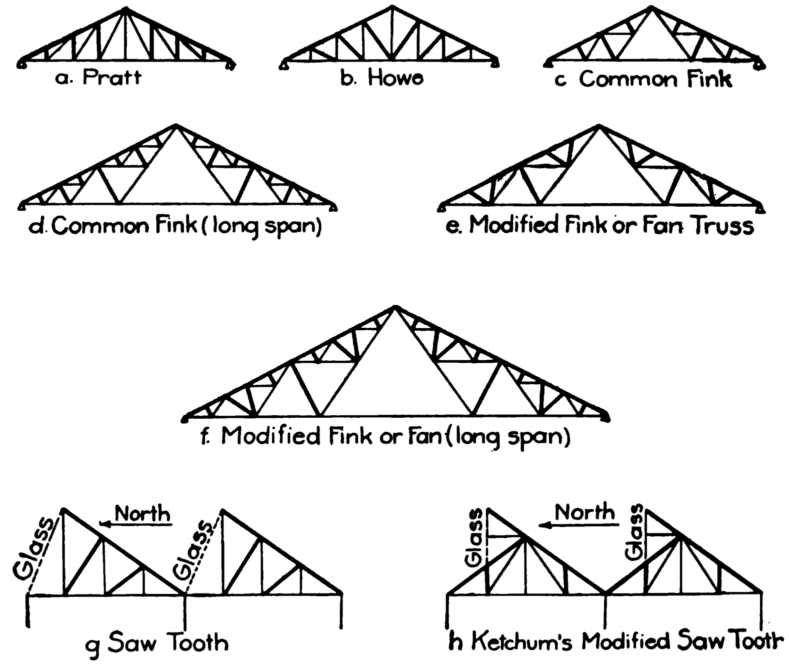
(1) Triangular roof trusses;
(2) Crescent roof trusses;
(3) Roof trusses other than these.

Fig. 1 shows various forms of triangular roof trusses. The *Pratt* and *Howe* trusses are shown respectively by *a* and *b*. These trusses obtain their name on account of their web bracing being of the Pratt or Howe type. The triangular truss in most common use is the *Fink*, next to which is the *Saw-tooth*. The Fink truss is built in a variety of forms, as shown in Fig. 1 (*c*, *d*, *e*, and *f*), *c* being for spans up to 60 feet; *e* for spans up to 70 feet, and *d* and *f* for spans up to 80 feet and over. The great advantage of this style of truss is that many of its members have the same stress, and therefore it can be constructed more cheaply on account of the fact that a large amount of the same sized material can be purchased at once.

When the top chord of a roof truss becomes bent as shown in Fig. 2, the truss is called a *crescent* roof truss. The bracing in the crescent roof trusses is not of any particular form, being as a usual thing built of members which can take either tension or compression. This is made necessary by the fact that the curved upper chord may cause either tension or compression in the webbing, according to the angle of its inclination with the horizontal.

Roof trusses which do not come under either of the above classes may be regarded in a class of their own. To this class belong those trusses which are somewhat like a bridge truss in that the two chords are horizontal or nearly so. The ends of these trusses may be rectangular or not. For various types of this class of truss, see Fig. 3.

In addition to the above classification, which is based on the form of the chords, roof trusses may be divided according to the manner in which their members are connected. This classification



a. Pratt b. Howe c Common Fink

d. Common Fink (long span) e. Modified Fink or Fan Truss

f. Modified Fink or Fan (long span)

g Saw Tooth h Ketchum's Modified Saw Tooth

Fig. 1. Triangular Roof Trusses.

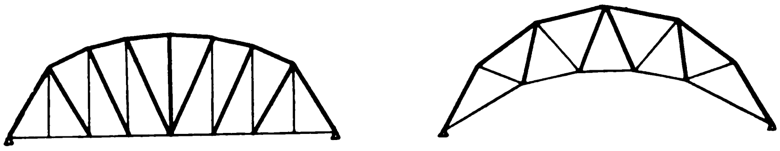


Fig. 2. Crescent Roof Trusses.

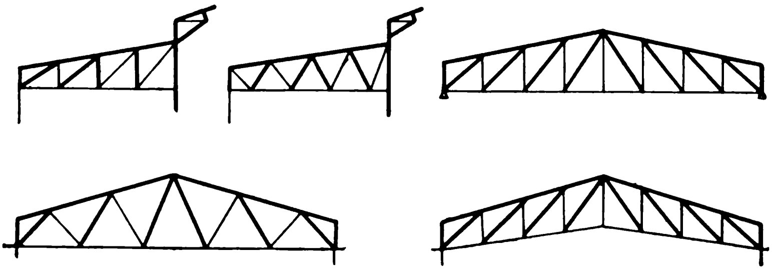


Fig. 3. Trusses with Chords Almost Parallel.

is that of *pin-connected* and *riveted*. For a definition of this, and for figures showing such joints, see "Statics," pp. 22 and 23.

Trusses are seldom built as pin-connected unless they are of long span, since roof trusses are comparatively light, and pin-connected trusses, unless of considerable weight, do not give very great stiffness.

Riveted roof trusses are used for nearly all practical purposes, since they give great rigidity under the action of wind and of moving loads, such as cranes, which may be attached to them.

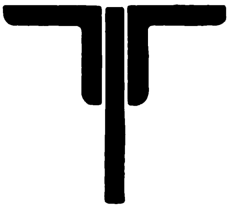


Fig. 4. Chord Section for Heavy Stresses.

2. **Physical Analysis of Roof Trusses.** In pin-connected roof trusses, the tension members consist of I-bars or rods; and the compression members are made of channels or angles and plates, either plain or latticed. In riveted trusses, both tension and compression members are made up of angles and plates or a combination of the two. The top chords of roof trusses of medium span usually consist of two angles placed back to back. If the stress becomes too great to be taken up by two angles larger than 5 by 3½ inches, then two angles and a plate are used (see Fig. 4). In case the roof truss is of great size and the stresses are exceedingly large, the chord member may be built up in a manner somewhat similar to a bridge truss, being constructed of two channels and a plate, or four angles and three plates. Figs. 5 and 6 show cross-sections of chords for long-span riveted trusses. These cross-sections may also be used for pin-connected trusses.

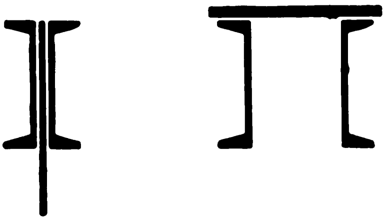


Fig. 5

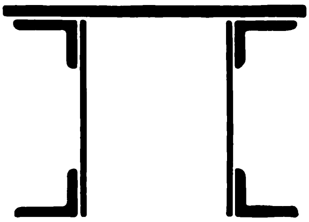


Fig. 6

Chord Sections for Trusses of Long Span

The web members of a truss usually consist of one angle; and if this is insufficient, two angles back to back are used. Fig. 80, page 65, gives a diagram of a roof, and shows not only the roof trusses but also various other parts which will be referred to in the succeeding articles.

3. **Wind Pressure and Snow Loads.** The wind pressure on a flat surface varies, of course, with the velocity of the wind, and is very closely given by the formula:

$$P = 0.004V^2$$

By substituting in this formula, the values shown in Table I are determined for given velocities in miles per hour.

TABLE I

Wind Pressure at Various Velocities

VELOCITY (Miles per hour)	PRESSURE (Lbs. per square foot)	REMARKS
10	0.4	Breeze
20	1.6	Strong breeze
30	3.6	Strong wind
40	6.4	High wind
50	10.0	Storm
60	14.4	Heavy storm
70	19.6	Hurricane
80	25.6	"
100	40.0	"

The pressures indicated in Table I are perpendicular to the direction of the wind. When the wind blows on an inclined surface, the wind is assumed to be acting horizontally, and the normal component on the inclined surface is determined. This component is not equal to the horizontal pressure times the sine of the angle of inclination, as one would suppose (see Fig. 7), but is greater by a small amount. Roofs are usually figured on a basis of 40 pounds pressure on a vertical surface. The value of the normal component for a horizontal wind pressure of 40 pounds per square foot, is given on page 24 of "Statics," and is here, for convenience, reduced to the normal pressure for any given pitch.

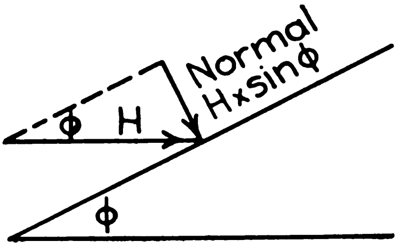


Fig. 7. Theoretical Determination of Normal Component.

Pitch	Normal Wind Pressure
½	34 pounds per square foot.
30°	32 " " " "
⅓	30 " " " "
¼	26 " " " "
⅙	22 " " " "

If the normal pressure on a roof making any other angle with the horizontal is desired, see "Statics," p. 24.

The determination of these values is based for the most part on data obtained by experiment. In the computations relative to the design of buildings, the wind is usually assumed to exert a pressure on the walls of 30 pounds per square foot.

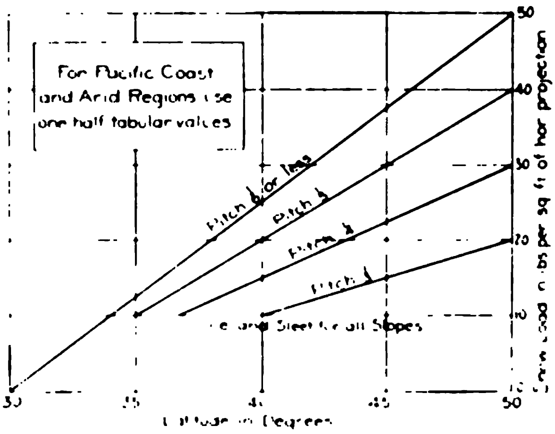


Fig. 8. Unit Snow Loads

The snowfall varies with the locality. The heaviest snow loads which come upon a roof are not always in the locality of the heaviest snowfall, since a comparatively light snowfall may occur, and if this is followed by wind and sleet, the result will be a load greatly in excess of the snowfall itself. The snow load per square foot of roof surface varies with the pitch of the roof, and will be greater the smaller the pitch. The ice and sleet will be comparatively constant. Fig. 8* gives values of snow and sleet loads which are recommended for use. It is customary to figure the snow load by taking it as so much per square foot of horizontal projection.

EXAMPLES FOR PRACTICE

1. Compute the wind panel load on a roof whose pitch is ¼, and whose panel length is 15 feet, the distance between trusses being 16 feet.

2. Compute the snow panel load for the truss of Problem 1, above.

---

*Ketchum's Steel Mill Buildings, p. 11

4. Weights of Roof Trusses. The weight of a roof truss varies with the material of which it is constructed, the span, the distance between trusses, the pitch, and the capacity of the truss. The actual weight, of course, cannot be determined until after the truss is designed; but an approximate weight may be obtained from any of the empirical formulæ which are now in use. Table II gives the most common and best of the empirical formulæ, together with the names of their authors.

TABLE II

Formulæ Giving Weights of Roof Trusses

Formula	Author
$W = \frac{3}{4}\, al \left(1 + \frac{l}{10}\right)$	Mansfield Merriman
$W = al \left(1 + \frac{l}{25}\right)$	E. R. Maurer, (p. 23, "Statics")
$W = al^2 \left(\frac{1}{25} + \frac{l}{6\,000}\right)$, wooden trusses.	N. C. Ricker
$W = \frac{Pal}{45} \left(1 + \frac{l}{5\sqrt{a}}\right)$	Milo S. Ketchum*
$W = 2a\left(4 + \frac{l}{25}\right) \sqrt{l^2 + r^2}$	C. W. Bryan
$W = al\,(0.06\,l + 0.6)$ for heavy loads $W = al\,(0.04\,l + 0.4)$ " light "	C. E. Fowler

In the above formulæ,

W = Weight of steel in truss, in pounds;
P = Capacity of truss in pounds per square foot of horizontal projection of roof;
r = Rise of peak, in feet;
a = Distance center to center of trusses, in feet;
l = Span of truss, in feet.

ROOF COVERINGS

5. The roof is covered with some material which will protect the interior of the building from the action of the elements. This covering may consist of any one or more of the materials which, together with their weights per square foot, are indicated in Table III. The weights here given for materials which must be laid upon sheathing, do not include the weight of the sheathing, which is given separately. A short description, together with necessary information for use in estimates, will now be given.

---

* "Steel Mill Buildings," p 5

TABLE III

Approximate Weights of Roof Coverings

MATERIAL	WEIGHT PER SQUARE FOOT
White pine sheathing 1 inch thick	3 lbs.
Yellow pine sheathing 1 inch thick	4 "
Batten sheathing, 4-in. by 1-in.	2½ "
Slate	10 "
Skylight glass, including frame	10 "
Tin	1 "
Shingles	3 "
Corrugated steel	2 "
Flat tiles	12 to 25 "
Corrugated tiles	10 "
Concrete slabs	35 to 50 "
Felt, asphalt and gravel	10 "
Felt and gravel	10 "
Patent roofings	½ to 1¼ "
Sheet steel	1½ "
Non-condensing base	1 "

Sheathing. Sheathing is generally laid directly upon the purlins (see Article 6); and upon this are laid the shingles, slate, tin, or tile. Sheathing is usually made of a single thickness of planks, 1 to 2½ inches thick, laid close together. In some cases, however, when batten sheathing is used, it is spaced from 2 to 4 inches apart. This has the advantage of being cheap and at the same time allowing good circulation of air beneath the roof covering, and consequently dampness due to any cause will soon dry out. Batten sheathing is much used where the roof covering consists of slate, shingles, or tile.

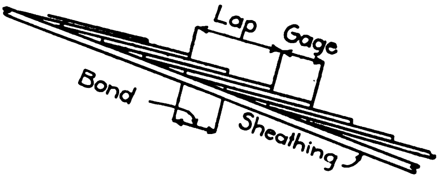


Fig. 9. Method of Laying Slate.

Slate. Roofing slate is generally of the characteristic slaty color, but may be obtained in nature in greens, purple, reds, and other colors. It is made in thicknesses of from ⅛ to ¼ inch, in widths from 6 to 24 inches, and in lengths from 12 to 44 inches. The 12 by 18 by $\frac{3}{16}$-in. slate is probably the most commonly used. Slate should be laid as shown in Fig. 9, and the pitch of the roof should not be less than ¼. If the pitch of the roof is less than this, the lap should be made greater than 3 inches, as is shown in Fig. 9. The lap should be increased at least ½ inch for every $\frac{1}{100}$ in pitch; and the minimum pitch should never be less than $\frac{1}{6}$, since it is practically impossible to prevent roofs with a smaller pitch than this from leaking, especially if a strong wind is blowing.

The number of different sizes of slate required to lay 100 square feet of surface, and also their weight, are given in the handbooks of the various slate companies. With a 3-inch lap, it takes 160 of the size and thickness mentioned above to lay 100 square feet, and the total weight of this square is 650 pounds. Slate is one of the most durable of roofing materials. Its first cost is high, being from 5 to 8 dollars per hundred square feet; but the cost of maintenance is almost nothing, since it is affected neither by the elements nor by the action of gases or acids. In case the roof would be subjected to the action of gases or acids, it is advisable to use copper slating nails.

Skylight Glass. Skylights usually consist of glass about $\frac{3}{16}$ to $\frac{3}{8}$ inch in thickness, supported on light members of iron or galvanized iron which act as a framework. The actual weight of glass of different kinds can be accurately obtained from manufacturers' catalogues, and the student is referred to these; they may be had by addressing the manufacturers (see Figs. 73 and 74).

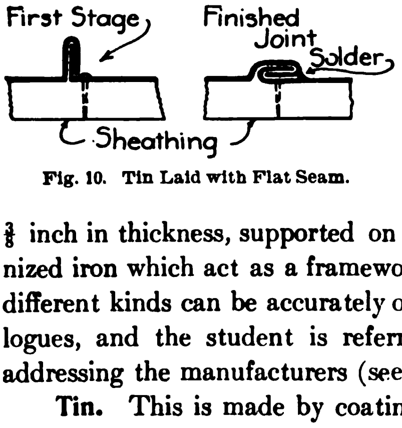


Fig. 10. Tin Laid with Flat Seam.

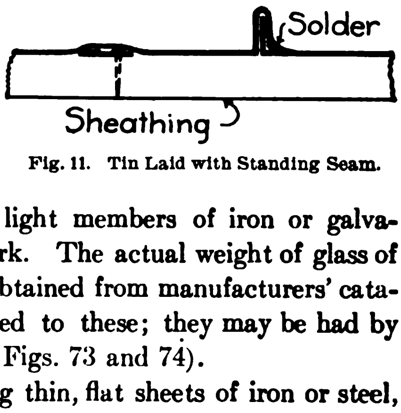


Fig. 11. Tin Laid with Standing Seam.

Tin. This is made by coating thin, flat sheets of iron or steel, either with tin alone or with a mixture of tin and lead. In the first case the product is called *bright tinplate*, and in the second case *terne plate*. Terne plate must not be used where it will be subjected to the action of acids or corrosive gases, since the lead coating is rapidly destroyed, and then of course the iron also.

Tin plates come in various sizes and thicknesses; but usually 112 come in one box. The most commonly used is a sheet 20 by 28 inches, and of sheet iron of No. 27 gauge, which weighs 10 ounces to the square foot. This is marked "IX." If the box were marked "IC," it would indicate that the sheets were of No. 29 gauge metal, which weighs 8 ounces to the square foot. The value of the roofing depends to a great extent upon the amount of tin used in the coating. This will vary from 8 to 50 pounds for a box of the 20 by 28-inch sheets.

A tin roof is formed by fixing together a number of these sheets. The sheets may be connected as shown in Fig. 10, or as shown in Fig. 11. In the first case, they are said to be laid with a *flat seam*, and in the second case they are said to be laid with a *standing seam*. Tin roofs rot out very readily unless they are kept painted. If a new coat is given them every couple of years, they will last from twenty to thirty years.

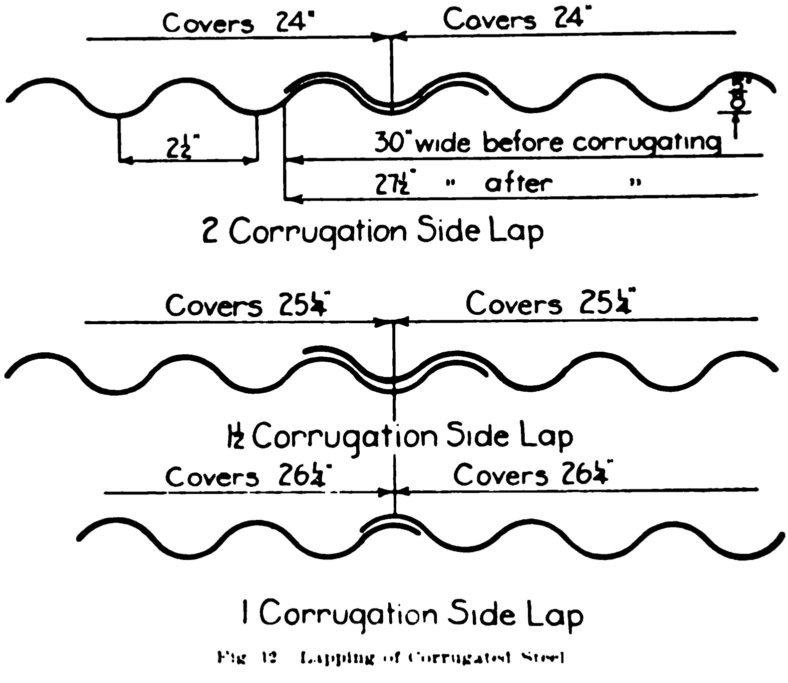


Fig. 12. Lapping of Corrugated Steel

Tin can be laid on roofs whose pitch is very small, say $\frac{1}{16}$. The first cost is about as much as that of slate, but the cost of maintenance is very high.

Shingles. Shingle roofs are very seldom used on buildings for manufacturing purposes, for the reason that they take fire quite readily, leak quite easily, and require renewal quite often. Shingles are from 18 to 24 inches long, and usually run from 2 to 8 inches in width, although they can be obtained of a uniform width of from 4 to 6 inches. They are laid like slate, the lap being made 4 inches or more. They should never be laid on roofs whose pitch is less than ⅓. It takes about from 800 to 1,000 shingles to lay 100 square feet of roof. The cost is about $5.00 per 100 square feet; but under the best conditions, the life of shingles is only about ten years.

Corrugated Steel. Corrugated steel is made from flat sheets of standard gauges, and may be either galvanized or left as it comes from the rolls. The corrugations are of different sizes and widths; the total width of the plates runs from 24 to 28 inches, and their length from 5 to 10 feet, varying by ½ foot. The sheet most used for roof construction has 2½-inch corrugations; is 26 inches wide after corrugating; and will cover a width of 24 inches with one lap and 21½ inches with two laps. This roofing should be laid with a pitch of not less than ¼, and should have from 6 to 8 inches lap at the ends. For further information regarding the method of lapping and the width covered, see Fig. 12.

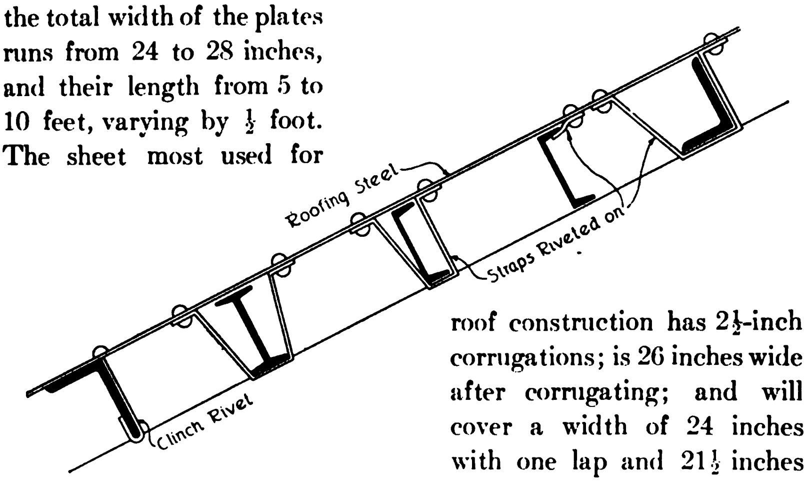


Fig. 13. Showing How Steel Roofing is Fastened to Purlins.

Corrugated steel is fastened either directly to wooden purlins by means of nails, or directly to iron purlins either by means of a bolt and clip or by a clinch nail (see Fig. 13).

It is often advisable to know the strength of corrugated steel when supported at certain distances apart by supports perpendicular to the corrugations. This unsupported length determines in many cases the spacing of the purlins. The load in pounds per square foot which can be carried by a plate of span l, parallel to the corrugation, is given by the formula:

$$W = \frac{330\,S\,d\,t}{l^2},$$

in which,

l = Unsupported length of sheet, in inches;
t = Thickness of sheet, in inches;
S = Allowable unit-stress;
d = Depth of corrugation, in inches.

Table IV, giving data relative to corrugated sheets, is taken from page 172 of the Pocket Companion of the Carnegie Steel Company (edition of 1902), where also other valuable information is given.

TABLE IV

Corrugated Steel Data

No. by Birmingham Gauge	Thickness (Inches)	Weight in lbs per 100 Sq Ft of Roof when Laid with 6-in. End Lap and One Corrugation, 2½-in. Side Lap, and Length of:					
		5 ft	6 ft	7 ft	8 ft.	9 ft.	10 ft.
16	0 065 in.	365	358	353	350	348	346
18	0 049 "	275	270	267	264	262	261
20	0 035 "	196	192	190	188	186	185
22	0 028 "	156	154	152	150	149	148
24	0 022 "	123	121	119	118	117	117
26	0 018 "	101	99	97	97	96	95

Tiles. One of the most common sizes of plain roofing tile is 10½ inches long by 6¼ inches wide and ⅝ inch thick. Tile of this size weigh about 2¼ pounds each. They are laid with a lap equal to one-half their length. They may be laid directly upon plank sheathing in a manner similar to shingles or slate, or they may be laid directly upon purlins (see Figs. 14, 15, and 16). In the first case they are nailed directly to the sheathing, and in the second case they are connected with the purlins either with copper wire or clinch nails. Flat tiles are usually laid in cement; corrugated tiles are made so as to interlock, and consequently in most cases require no cement. One convenience of the tile roof is that the skylights may be formed by laying glass tile in place of the other.

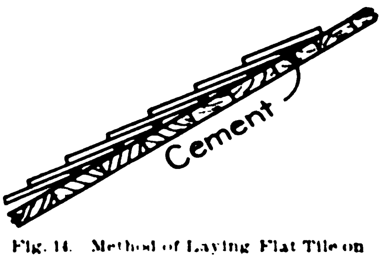


Fig. 14. Method of Laying Flat Tile on Plank Sheathing

Tile roofs are very substantial; but are very costly, in regard not only to the tiles themselves, but also in regard to the additional weight required in the trusses by reason of the great weight to be supported. Tile weigh from 700 to 1,000 pounds per 100 square feet of roof surface. They cost from $12.00 to $40.00 per 100 square feet on the roof.

Concrete Slabs. These are usually moulded directly in place by suspending forms from the roof trusses. They may or may not be reinforced, and in any case are usually not over 4 inches in thickness. Their weight is about 50 pounds per square foot. Their cost is from $16.00 to $30.00 per hundred square feet of roof surface. They are expensive, not only on their own account, but also from the fact that the weight of the roof trusses must be increased in order to carry the weight of the slabs. Concrete roofing may be used on roofs which are practically flat, $\frac{1}{2}$ inch to 1 foot being sufficient pitch.

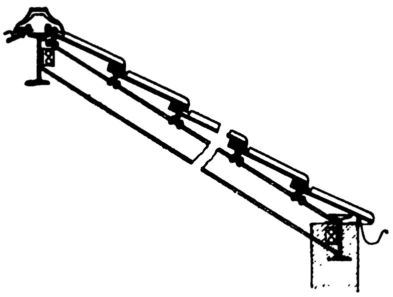


Fig. 15. Ludowici Tile on Steel Purlins.

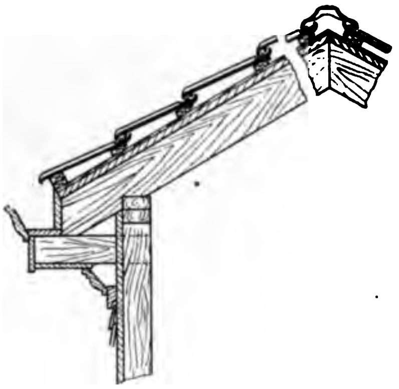
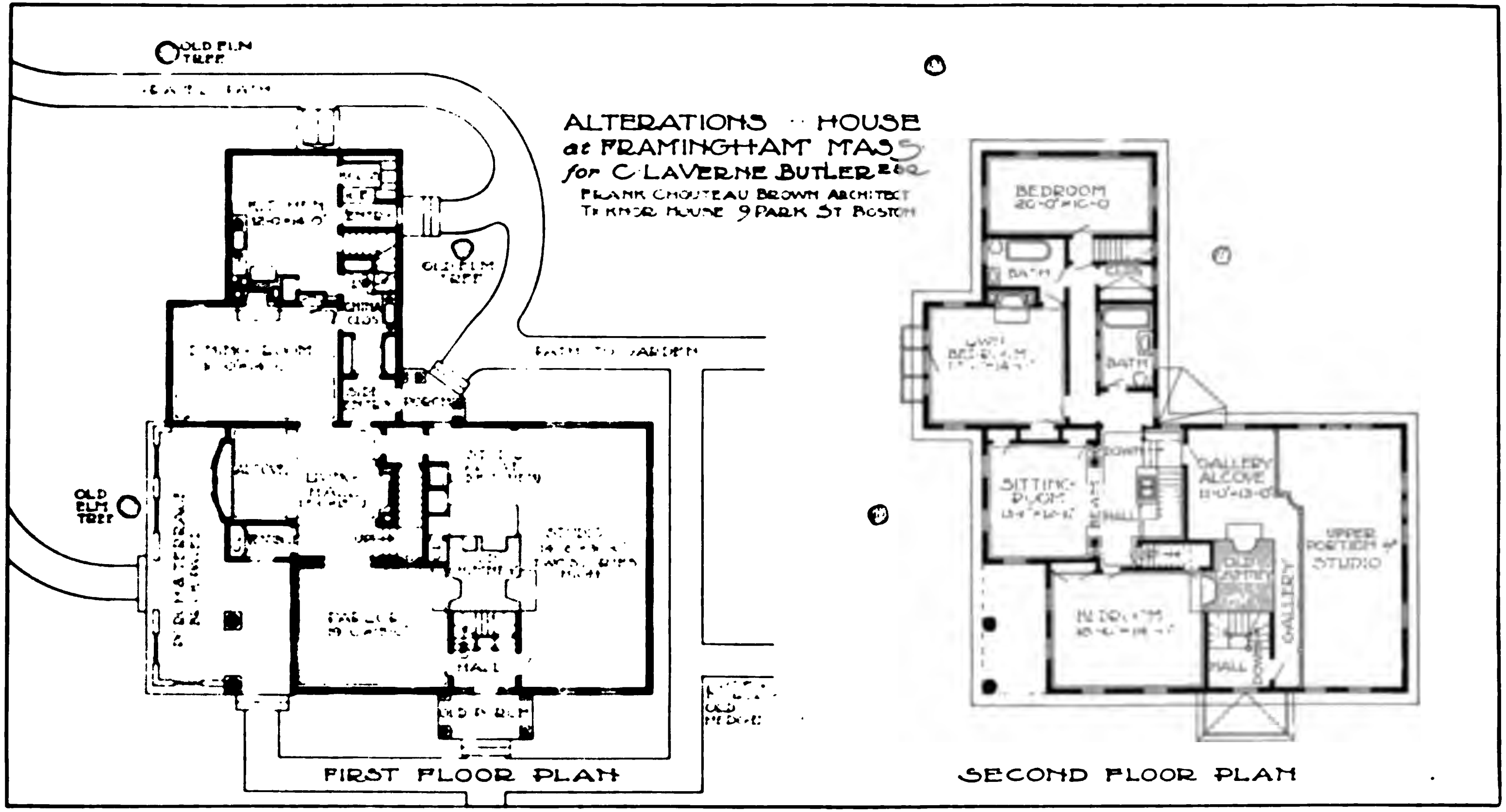


Fig. 16. Ludowici Tile on Sheathing.

Felt and Asphalt. This roofing is laid upon shingles, and consists of one thickness of dry felt, three or four thicknesses of roofing felt well cemented together with asphalt cement, and laid with good laps where they join, and a coating of from 100 to 200 pounds of asphalt per 100 square feet of roof surface. Upon this asphalt, while hot, gravel screened through a $\frac{5}{8}$-inch mesh is spread in the quantity of about $\frac{1}{6}$ of a cubic yard per 100 square feet. This class of roofing should never be laid on roofs whose pitch is greater than $\frac{1}{6}$, since, when heated by the rays of the sun, the asphalt will run and destroy the surface. It gives good satisfaction on roofs whose pitch is $\frac{1}{12}$. This class of roofing can be bought in rolls, and in this case the gravel is exceedingly fine, being screened through a $\frac{1}{8}$-inch mesh.



FLOOR PLANS OF HOUSE FOR C. LA VERNE BUTLER, AT FRAMINGHAM, MASSACHUSETTS

Frank Chouteau Brown, Architect, Boston, Massachusetts

Note the two different sizes of Doric columns used in this house. The large columns extend through two stories and form an integral structural part of the house, supporting a portion of the roof. The small columns of the integral supports serve as a decorative motif, and for supporting the protective hood over the entrance. For exterior see page 202.

Felt and Gravel Roofing. This roofing is similar to the above; only, in this case, tar instead of asphalt is used for the cementing constituent. This roofing does well on roofs of flat pitch, and should never be used on roofs whose pitch exceeds $\frac{1}{6}$. It can also be bought in rolls ready for laying, in which case the gravel is screened through a $\frac{1}{4}$-inch mesh. The prepared roofings are cheaper than those laid by hand; but they do not give good service unless great care is taken to fasten them down securely. In economy of first cost and maintenance, they are equal to or better than tin.

Sheet Steel. This should not be laid on a pitch less than $\frac{1}{4}$, unless the ends are cemented together where they lap. It comes in sheets 28 inches wide and from 4 to 12 feet long, or it may be purchased in rolls 26 inches wide and about 50 feet long. When used in sheets, it may be had with standing crimped edges, in which case it is laid as shown in Fig. 17. In case it comes in rolls, it may be laid in the same manner as tin, with either standing seams or horizontal flat ones as shown in Figs. 10 and 11. Like corrugated steel, it comes in different gauges, No. 28 being that most commonly used. It can be laid cheaper than tin, on account of the long lengths obtainable.

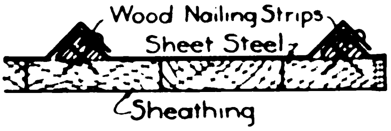


Fig. 17. Method of Laying Sheet Steel with Crimped Edges

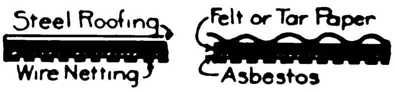


Fig. 18. Method of Laying Roofing on an Anti-Condensing Base

Patent roofings of many kinds are on the market. These come in rolls usually from 2 feet to 3 feet wide, and cover about 200 square feet of roof surface. The basis of most of these covers is asbestos, felt, magnesia, or rubber; and this is treated with either asphalt, tar, or some other preparation, and in some cases is covered with fine gravel.

Non-Condensing Roofing. In cases where a metal, slate, or tile roof is used without sheathing, moisture is liable to condense upon the under side and drip on the floor beneath. This can be prevented by laying the material upon an anti-condensing base consisting of a layer of wire netting on top of which are placed one or more sheets of asbestos paper about $\frac{1}{16}$ inch thick (see Fig. 18).

6. **Rafters and Purlins.** Roof trusses are usually connected by beams running from one to the other. These beams are called *purlins*. In case the purlins are spaced too far apart to lay the roof covering directly upon them, beams are placed upon the purlins, and on these beams the roof covering is placed. These beams are called *rafters*. Rafters are usually made of wood, while purlins are made of channels, I-beams, Z-bars, and, if the trusses are spaced sufficiently close together, *tees* or *angles*. Figs. 19 and 20 show how rafters and purlins are placed. Fig. 21 illustrates the use of purlins made of tees. As purlins are more rigid about an axis perpendicular to their webs, they are liable to sag toward the eaves at their center. In this case, *sag rods* are used, as shown in Fig. 22.

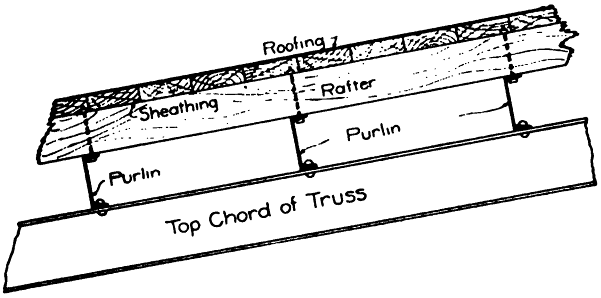


Fig. 19. Roof Construction in which Rafters Supporting the Sheathing are Laid on the Purlins which Connect the Trusses.

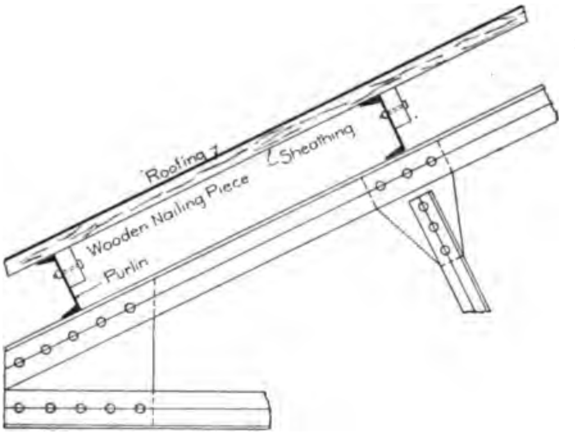


Fig. 20. Sheathing Laid Directly on Purlins.

EXAMPLES FOR PRACTICE

1. Compute the roof rafters if the purlins are spaced 10 feet apart, the roof covering weighs 10 pounds, the sheathing 4 pounds, and the snow load per square foot of roof surface 12 pounds.

This problem may be solved, either by assuming the size of the rafters and computing their spacing, or by assuming the spacing and computing the size of the rafters. The latter method is the one most commonly used. The spacing of rafters is from 18 inches to 4 feet.

The common spacing is 2 feet. The weight of the rafter itself is neglected in its design.

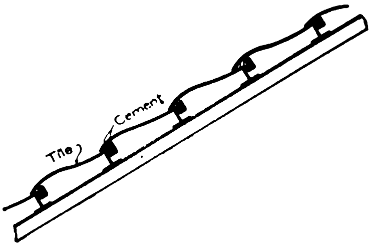


Fig. 21. Use of Purlins Made of Tees.

The total weight per square foot which comes on the rafter is $12 + 10 + 4 = 26$ pounds. Since each rafter carries a portion of the roof 10 by 2 feet, the total weight on one rafter is $10 \times 2 \times 26 = 520$ pounds. The moment created by this weight is $(520 \times 10 \times 12) \div 8 = 7\,800$ pound-inches. This should be equated to $\frac{S\,I}{c}$. Allowing 1 000 pounds to the square inch as the unit-stress on the extreme fibre, and noting that $I \div C = \frac{b\,d^3}{12} \div \frac{d}{2} = \frac{b\,d^2}{6}$ there results:

$$\frac{1\,000\,bd^2}{6} = 7\,800$$

$$d = \sqrt{46.8 \div b}$$

The market widths of rafters are 1½, 2, 3, and 4 inches, 2 inches being the size usually employed. Substituting in the above formula, we have:

$$d = \sqrt{46.8 \div 2} = 4.8 \text{ inches.}$$

The rafters will be made 2 by 6 inches, since this is the nearest market size. If a 3-foot spacing of rafters was used, the required depth would be 5.92 inches, and a 2 by 6-inch would still be used. This spacing and this size of rafter would be the one to employ in the solution of the above problem.

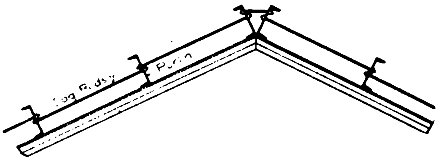


Fig. 22. Use of Sag Rods to Prevent Sagging of Purlins at their Center.

2. Design the purlin for the roof of Problem 1, above, if the trusses are spaced 16 feet apart.

The rafters are spaced so close together that their own weight, the weight of the roof covering, and the snow load may be considered as uniformly distributed over the purlin. The total weight which comes upon one purlin is the weight of snow and roof covering on a space 16 feet long and 10 feet wide. This weight is:

Snow load = 10 × 16 × 12	1 920 pounds
Roof covering = 10 × 16 × 14	2 240 "
16 rafters 6 by 2-in., 10 feet long, at 3 lbs. per 144 cu. in.	480 "
Total	4 640 pounds

The moment caused by this weight is:

$$(4\,640 \times 16 \times 12) \div 8 = 111\,360 \text{ pound-inches.}$$

The determination of the beam which will be used to withstand this bending moment is made by means of its section modulus. The formula $\frac{M}{S} = \frac{I}{c}$ is used in the design of beams. The values of I and c are constant for any given beam, and therefore the value of $I \div c$ for any particular beam is a constant, and this constant is called the *section modulus*. It is therefore evident that if we have a certain bending moment and a certain allowable unit-stress, we can obtain the value of the section modulus by dividing the moment by the allowable unit-stress. Then, looking into one of the steel handbooks, the beam can be determined which will have a section modulus equal to or slightly in excess of the value that has been obtained by dividing the bending moment by the unit-stress. This beam will be the beam which, with a unit-stress equal to the one assumed, will withstand the bending moment under consideration.

The handbooks issued by many of the steel companies are indispensable to the intelligent design of structural steel. That issued by the Carnegie Steel Company (edition of 1903) is one of the most convenient; and since it will be frequently referred to in this text, its purchase by the student is desired. This book may be obtained by addressing the Carnegie Steel Company at its offices in any of the larger cities. The cost to students has usually been 50 cents; to others, $2.00.

Assuming an allowable unit-stress of 18 000 pounds per square inch on the extreme fibre, the section modulus required to withstand the bending moment of 111 360 pound-inches is:

$$\frac{111\,360}{18\,000} = 6.19.$$

Looking in the Carnegie Handbook at column 11 on page 100, column 11 on page 102, and column 9 on page 104, it will be seen that any one of the following shapes will be sufficient:

One 5-inch 14.75-pound I-beam;
One 7-inch 9.75 " channel;
One $4\frac{1}{8}$ by $3\frac{3}{16}$ by $\frac{9}{16}$-inch Z-bar weighing 17.9 pounds per linear foot.

Instead of the 5-inch I-beam as given above, a 6-inch 12.25-pound I-beam with a section modulus of 7.3 could be used, and would be more economical, since it is less in weight; and it would also be stiffer, since its depth is greater and its section modulus is greater. A comparison of the above weights shows the channel to be the most economical, since its weight is considerably less than either of the other two shapes. Channels usually make the most economical purlins; and for this reason no other shapes are usually inspected, the channels being used in the first case without being compared with other sections. Inspection of column 11, page 110, Carnegie Handbook, shows that a 6 by 4 by ¾-inch angle could have been used for the purlin, since it gives a section modulus of 6.25. The weight of this angle, 23.6 pounds per linear foot, shows it to be far too uneconomical to employ.

EXAMPLES FOR PRACTICE

1. Design the rafters when the total weight of the snow and roof covering is 30 pounds per square foot, and the purlins are spaced 15 feet apart. Use 1 000 pounds per square inch as the allowable unit-stress.

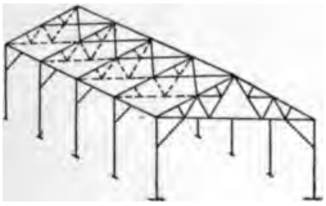
2. Design the purlins if the trusses are 12 feet center to center; the purlins are spaced 8 feet apart; the roof covering which weighs 6 pounds per square foot, is laid upon 1-inch yellow pine sheathing resting directly upon the purlins; and the snow load is 10 pounds to the square foot of roof surface. Use 18 000 pounds per square inch as the allowable unit-stress, and use a channel for the purlin section.

7. **Bracing.** In order to keep the roof trusses erect, bracing is employed to join together their top chords and also their bottom chords. This bracing may consist either of small round or square rods, or it may consist of angles. The latter is the best practice, since it gives great rigidity to the structure; and in fact it should be used in all cases where machinery of any kind is attached to the trusses. One disadvantage of the rod bracing is that good connections with the trusses are usually difficult. The bracing between the lower chords is lighter than that between the top chords, since its office is merely to prevent vibration, while that between the upper chords must take up the stresses caused by the wind blowing upon the ends of the building. The stresses in each of these classes of bracing can only be approximately determined; and for that reason it has become customary to determine their section by judgment rather than by computation. For lower chord bracing, single angles 3 by

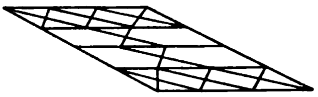
2 by $\frac{5}{16}$-inch are recommended; and for upper chord bracing, 3 by 3 by $\frac{5}{16}$-inch angles should be used.

It is not customary to place bracing between each pair of trusses, but to place them between each alternate pair or between every third pair of trusses. Fig. 23 shows several ways in which the bracing may be inserted.

8. Economical Spacing and Pitch of Trusses. The term pitch which has been used in the preceding pages is the fraction obtained by dividing the span into the height of the truss at the center of the span. For example, if a truss has a span of 60 feet, and a rise of 12 feet at the center, it would be said to have a pitch of $\frac{1}{5}$; if the rise were 15 feet, the pitch would be $\frac{1}{4}$; and if the rise were 20 feet, the pitch would be $\frac{1}{3}$ The pitch of a truss is seldom expressed in degrees by giving the angle that the top chord makes with the horizontal. One exception is very common. It is to use the 30° pitch. This has the advantage of making the height of the center equal to one-half the length of one side of the top chord—a fact which lends itself to ease in making the shop drawings.



Upper Chord Bracing



Looking Upward - The Lower Chord Bracing

Fig. 23. Methods of Inserting Bracing between Trusses.

The maximum or minimum allowable pitch for any given roof depends to a great extent upon the class of roof covering employed. For pitches required for any given class of roof covering, see Article 5, p. 6. It might be noted that most of the patent roofings, or any roofing in which tar or asphalt is an ingredient, should not be laid upon roofs with a pitch greater than $\frac{1}{5}$ or $\frac{1}{6}$; while most of the coverings which consist of steel or clay products require pitches of $\frac{1}{5}$ or over.

Pitches varying from $\frac{1}{6}$ to $\frac{1}{3}$ have very little effect upon the weight of the trusses. This is true only for trusses with horizontal lower chords. If the lower chord is *cambered*—that is, raised above the horizontal position—it greatly increases the stresses in the truss, and consequently the weight of the truss. The greater the camber, the greater the weight of the truss, the pitch remaining the same. If the camber is constant, then the greater the stresses (and consequently the weight of the truss), the smaller the pitch. It is advisable not to camber the lower chord unless it is positively necessary. A camber of 5 per cent of the span will increase the weight of the truss from 10 to 40 per cent, according to the pitch.

Taking all things into consideration, a pitch of $\frac{1}{3}$ or $\frac{1}{4}$ is to be preferred over that of $\frac{1}{6}$ or less, since, after the pitch becomes less than $\frac{1}{6}$, the weight of the truss increases quite rapidly, the span being constant.

For any given roof, there is an economical spacing of the trusses. As the spacing of the trusses increases, the weight of the purlins and bracing per square foot of area increases, while the weights of the trusses, the columns that support them, and the girts, or members which run from one column to the other and on which the siding of the building is placed, decrease. The most economical spacing of the trusses is such as will make the *cost* of the above quantities a minimum. It is evident that this spacing for trusses which rest upon masonry supports will be different from the spacing in case they rest upon steel columns. Attention is called to the statement that the sum of the costs, instead of the sum of the weights of the above-mentioned quantities, should be a minimum. This is due to the fact that the unit-cost of the purlins is considerably less than that of the trusses, it being in some cases only about one-half.

The spacing of trusses is sometimes governed by local conditions, such as the placing of the machinery in the building and the probable position of future additions. Considering the spacing from a purely economical standpoint, it is probably well to space trusses about as indicated in Table V.

TABLE V

Spacing of Trusses

SPAN IN FEET	SPACING IN FEET
10 to 30	12
30 to 60	15
60 to 75	20
75 to 150	21 to 25

The spacing indicated in Table V is for triangular roof trusses of equal size and span. For other conditions, such as when the main roof consists of one span, and the side roofs consist of different spans and different classes of trusses—the economical spacing may be somewhat different, and is usually less.

The best method of determining the economical spacing is either to make a comparative design or to consult the back volumes of *The Engineering Record, Engineering News,* or some other good engineering periodicals. Designs of buildings which have been constructed are frequently given in these periodicals; and from these the student may, in addition to the spacing of the trusses, obtain much other valuable information regarding roof construction.

Bulletin No. 16 of the University of Illinois Experiment Station gives a systematic study of roof trusses, and shows the effect on the variation in the weights of rafters and purlins due to a variation in the length of span. This bulletin, which can be had free for the asking, should be in the hands of the student. It may be had by addressing "The Director," Engineering Experiment Station, University of Illinois, Urbana, Illinois. A most valuable book giving a systematic and extensive study of roof trusses and mill buildings, is "Steel Mill Buildings," by M. S. Ketchum, Engineering News Publishing Company, New York, N. Y.

9. **Stresses in Roof Trusses, and Sizes of Members.** Stresses in roof trusses of any form can be computed by the methods of "Statics" (pp. 23 to 73). On account of the ease and economy of manufacture, some form of truss is usually used in which there are many members with equal stresses. The Fink truss, or some modification of it, is almost universally used (see Fig. 1, *c, d, e, f*). On pages 21 and 22 are shown some forms of trusses, together with the pitches which are commonly used.

The stresses in the various members due to a vertical panel load of one pound are given. To obtain the stress in that member due to any other vertical panel load, multiply the stress here given by the vertical panel load.

For example, if the stresses in U_2L_2 (Fig. 24) or L_0L_1 (Fig. 31) due to a panel load of 3 000 pounds, were required, they would be determined as follows:

$U_2 L_2$ (Fig. 24) $3\,000 \times -1.73 = -5\,190$ pounds.
$L_0 L_1$ (Fig 31) $3\,000 \times +5.00 = +15\,000$ pounds

These diagrams are especially useful, since it is the custom of many engineers not to compute the stresses due to wind, snow, and

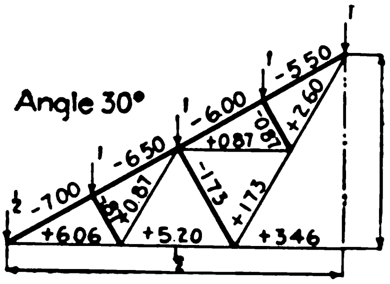


Fig. 24.

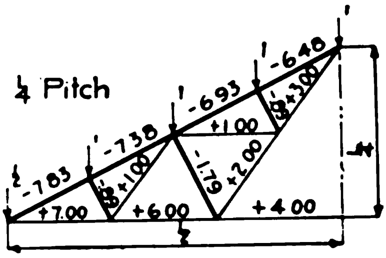


Fig. 25.

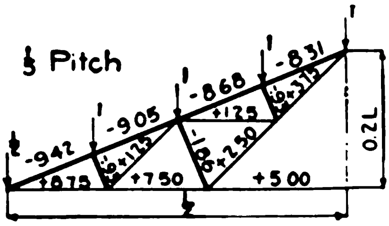


Fig. 26.

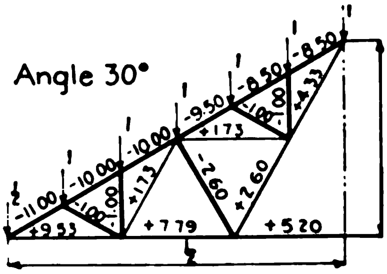


Fig. 27.

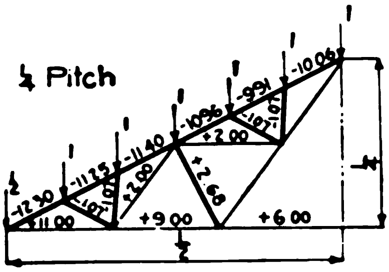


Fig. 28.

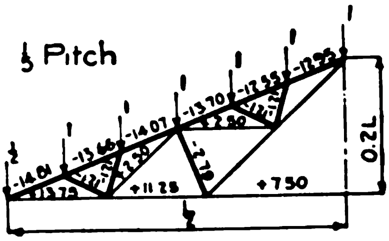


Fig. 29.

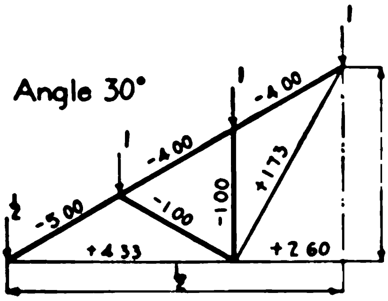


Fig. 30.

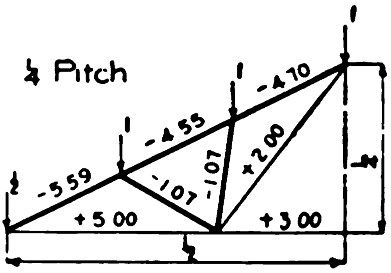


Fig. 31.

Analysis of Stresses in Various Members of Fink Truss Due to Unit-Loads.

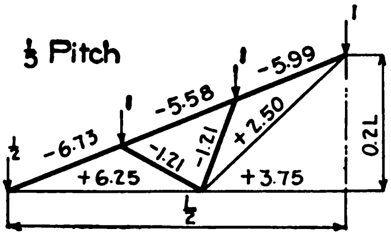


Fig. 32.

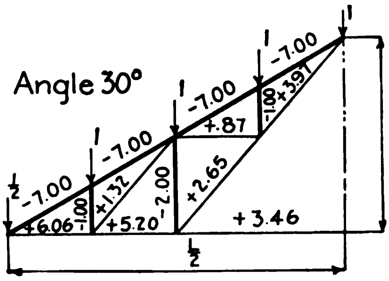


Fig. 33.

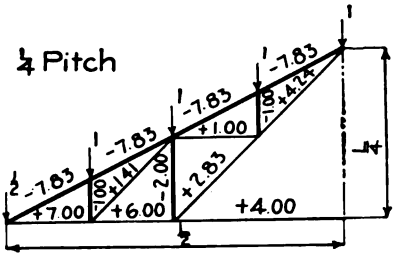


Fig. 34.

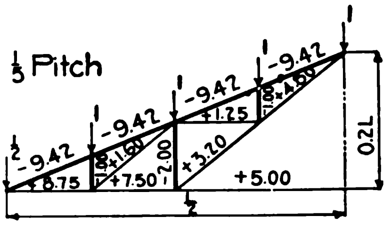


Fig. 35.

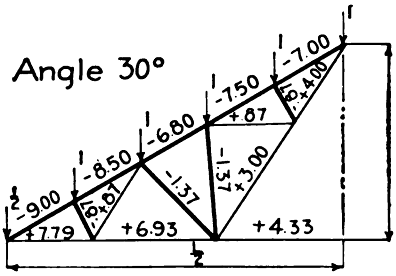
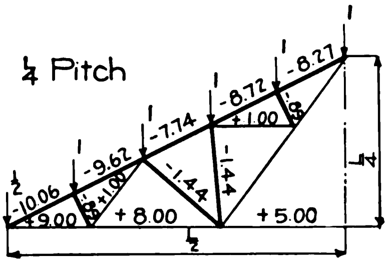


Fig. 36.



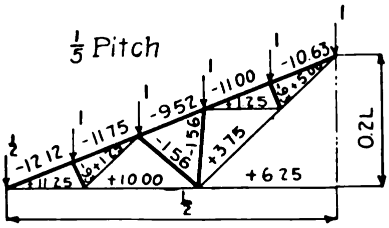


Fig. 37. Fig. 38.

Analysis of Stresses in Various Members of Fink Truss Due to Unit-Loads.

dead weight of roof trusses and coverings, but to compute the stresses due to a dead panel load caused by 40 pounds per square foot of horizontal projection. The stresses resulting from this procedure are very nearly equal to those produced by considering the various loads—as snow, dead load, and wind—separately or together. Whenever differences occur, they are on the safe side, except as noted below, and in the next article, in case of the stresses produced by the use of knee-bracing.

The panel load to be used when 40 pounds per square foot of horizontal projection is considered, may be computed from the formula:

$$P = \frac{40 \times a \times l}{n}$$

in which,

a = Distance between trusses, in feet;
l = Span of truss, in feet;
n = Number of panels in top chord of truss.

For example, let it be required to compute the panel load P for the truss of Fig. 24 when the span is 70 feet and the distance between trusses is 16 feet. Here $a = 16$; $l = 70$; and $n = 8$.

$$P = \frac{40 \times 16 \times 70}{8} = 5{,}600 \text{ pounds.}$$

The truss would then be computed for a vertical panel load of 5,600 pounds, and the members designed to withstand the stresses thus obtained.

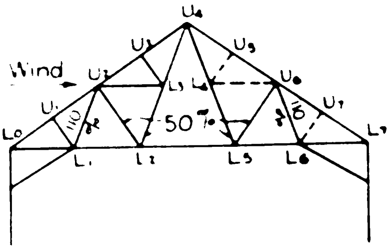


Fig. 25. Allowance for Stresses Due to Wind

This method is applicable to all spans up to 100 feet when the truss is set on masonry walls or steel columns built in masonry walls and the roof covering is of corrugated steel or any of the ordinary materials. Where clay tile or slate are used, 50 pounds should be taken; and in case of concrete slabs, 65 pounds would be about right. It is better practice to compute the stresses due to wind, snow, and dead loads when clay tile, slate, or concrete are used.

In cases where the roof truss is placed on steel columns and is connected with the column by a knee-brace at the first joint (see Fig. 39), stresses caused by the overturning action of the wind take place in those members shown by heavy lines. In this case the stresses caused by 40 pounds per square foot of horizontal projection are not large enough; but the truss will be safe enough if the stresses as determined by the 40 pounds are increased by the amounts indicated in Fig. 39.

For example, let the truss of Fig. 24 be supported by steel columns and knee-bracing. Let the span be 60 feet, and the distance between trusses 16 feet; and let it be required to compute the stresses in $L_1 U_2$ and $L_3 U_4$. Here $P = (40 \times 16 \times 60) \div 8 = 4\,800$, and the stresses will be:

$L_1 U_2$ $(0.87 \times 4\,800) \times 2.10 = +\,8\,770$ pounds.
$L_3 U_4$ $(2.60 \times 4\,800) \times 1.50 = +18\,700$ pounds.

In addition to the above conditions, shafting, heating apparatus, small cranes, and electric wiring and other conductors are often attached to the lower chord of the truss. These cause additional stresses. The case is that of a concentrated load or loads at the lower chord, and the stresses may be computed by the methods given in "Statics."

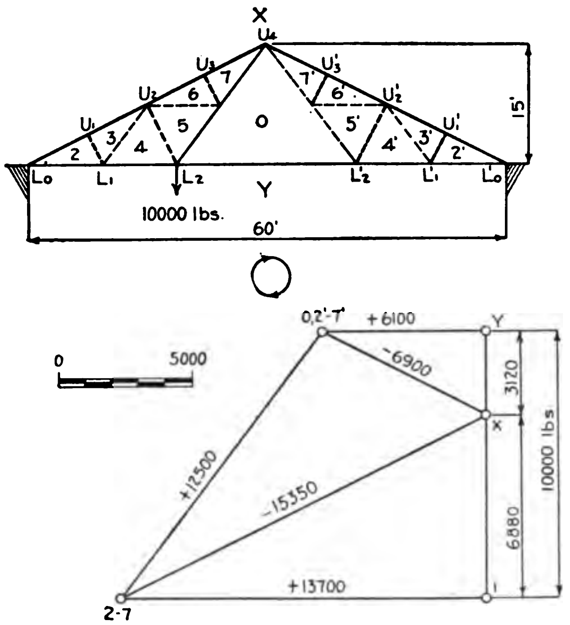


Fig. 40. Fink Truss Loaded with 5-Ton Hoist; also Stress Diagram of Same.

For example, let a 5-ton hoist be connected as shown in Fig. 40 This hoist runs longitudinally of the shop, or perpendicularly to the plane of the roof truss. The maximum stress in the truss due to this cause will occur when the hoist is directly beneath the truss. The stresses will be those caused by a load of 10 000 pounds at the second panel point of the lower chord. Fig. 40 gives the stress diagram for

TABLE VI

Hoist Stresses in Fink Truss

Member	Stress	Member	Stress
L_0U_4	−15 350	L_2U_4	+12 500
L_0L_2	+13 700	$U_4L'_0$	− 6 900
$L'_2\ L'_0$	+ 6 100	All Others	0

this condition, and Table VI gives the stress record. From this it is seen that the hoist does not affect all members of the truss. The stresses due to the hoist should be added to those caused by the 40 pounds per square foot of horizontal projection, and the member designed accordingly. Of course, if the stress caused by the hoist decreases the stress caused by the 40 pounds, the member must be designed for the stress due to the 40 pounds.

Note that concentrated loads, as in the case of the hoist, cause different stresses in symmetrical members on the two sides of the truss. In the final design, the members are made the same, being designed for the greatest stress. This is done for the sake of economy in manufacture; and besides, it might be desirable to change the hoist to the other side of the truss.

For Fink trusses with pitches of from ¼ to ½, and spans of less than 100 feet, very light angles are usually required for the members, unless heavy, concentrated loads are placed on the lower chord. The thickness of the connection plates is seldom more than ⅜ inch, the top chord angle seldom greater than 5 by 3½-inch, the lower chord angle seldom greater than 3 by 3-inch; and the web members are usually composed of angles either 2 by 2-inch or 2½ by 2½inch. It appears to be the rule, in present practice, to make the sizes such that the thickness shall be ¼ or 5/16 inch. Connection plates for spans up to 60 or 70 feet are usually ¼ inch thick, except in the case of that at point L_0.

The stresses in knee-braces depend upon the height and also the width of the building. The stresses may be computed according to the methods of the next article, and the knee-bracing should be designed accordingly. The inspection of a number of plans seems to indicate that the sizes of knee-braces vary from two angles 2½ by 2½ by ¼-inch for spans of 30 feet and a height of building of 35 feet, to two angles 4 by 3 by $\frac{5}{16}$-inch for a span of 70 feet and a height of building to the top of the truss of 75 feet.

In case of roof trusses with the chords nearly parallel (see Fig. 3, p. 2), the stresses, on account of the small depth, are usually quite large, and much heavier members than above mentioned are required. In some cases, 6 by 6-inch angles with 8-inch plates are used, and connection plates of ⅜ to ½ inch are common.

In cases where the trusses are subjected to the action of corrosive gases, the thickness of the members should be made greater than that required by the design alone, since corrosion will decrease the section considerably, and this should be allowed for.

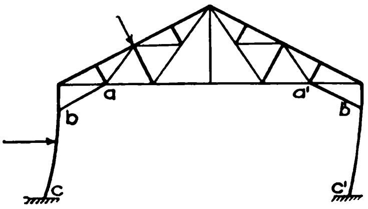


Fig. 41. Bending Tendency, Ends Free.

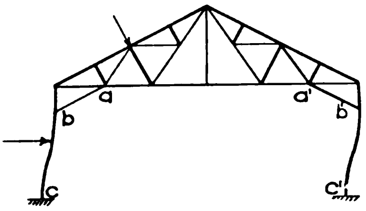


Fig. 42. Bending Tendency, Ends Fixed.

10. **The Steel Truss-Bent.** When a truss is connected to steel columns at its ends and by means of knee-bracing (see Fig. 39), it forms what is called a *steel truss-bent*. The stresses in the truss due to the roof covering and snow loads are the same as when it is supported by a masonry wall; but the wind stresses are different. The wind blowing on the roof and also on the sides of the building, causes stresses in the truss. The wind on the building is transferred to the columns, which, by means of the knee-braces, cause stresses in the truss. The whole bent tends to bend as shown in Fig. 41 if the ends of the columns rest on masonry pedestals. If the ends of the columns are securely bolted to heavy masonry pedestals so that the ends of the post will remain vertical, they will tend to bend as shown in Fig. 42. In the first case, the overturning is resisted by the bending of the post as shown at *b* and *b′* (Fig. 41); in the second case, by bending as at *b*, *c*, *b′*, and *c′* (Fig. 42). Since the post is the same size throughout, and the bending caused by the wind the same in both cases, the bend-

ing moment in the post at *b* and *b'* (Fig. 42) is less than it is at *b* and *b'* (Fig. 41), as in the first case there are only one-half the number of points to withstand the total bending that there are in the second case.

The wind blowing on one side of the building causes a compressive stress in the column on the *leeward* side (the side opposite that on which the wind blows) and a tensile stress in the column on the *windward* side (the same side on which the wind blows). It also creates a bending moment as mentioned above; and this, as well as the direct stresses, must be taken into account when the post is designed. The case is that of a member under direct compression and bending at the same time.

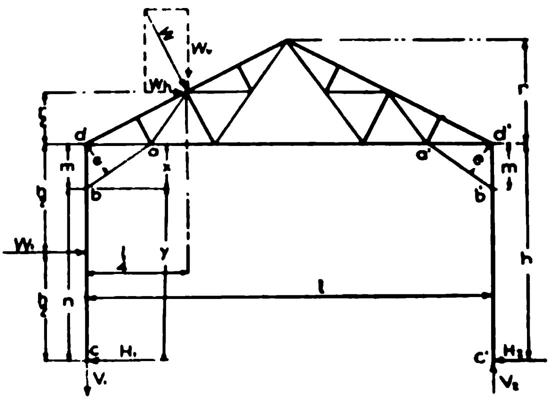


Fig. 43. Notation for Formulæ, Ends Free.

The stresses in the knee-braces and the columns, and the bending in the columns when the ends of the posts are not fixed, may be computed from the following formulæ, in which,

W = Total wind load perpendicular to the roof;
W_h = Horizontal component of W;
W_v = Vertical component of W;
W_1 = Total wind load on the side of the building;
w = Unit wind load normal to the roof;
w_1 = Unit wind load normal to the side of the building;
a = Distance between trusses, in feet

These and other characters are shown in Fig. 43.

$$W = wa\sqrt{r^2 + \left(\frac{l}{2}\right)^2}$$

$$W_1 = w_1 ah$$

$$H_1 = H_2 = \frac{W_h + W_1}{2}$$

$$S_{b'c'} = -\frac{W_1 \frac{h}{2} + W_h \left(h + \frac{r}{2}\right) + W_v \frac{l}{4}}{l} = V_2$$

$$S_{bc} = +\frac{W_1 \frac{h}{2} + W_h \left(h + \frac{r}{2}\right) - W_v \frac{3l}{4}}{l} = V_1$$

$$S_{a'b'} = -\frac{H_2 h}{e}$$

$$S_{ab} = +\frac{H_1 h - W_1 \frac{h}{2}}{e}$$

Bending moment at $b = H_1 n - W_1 \left(\frac{h}{2} - m\right)$

Bending moment at $b' = H_2 n$.

The stresses in the truss caused by the wind are the same as if it were under the action of the normal wind load W, and in addition two concentrated loads equal in intensity and direction to the stresses in the knee-braces and at the same point of application, and two forces E_1 and E_2, which may be computed as follows:

$$E_1 = \frac{H_2 n}{m}$$

$$E_2 = \frac{H_1 n - W_1 \left(\frac{h}{2} - m\right)}{m}$$

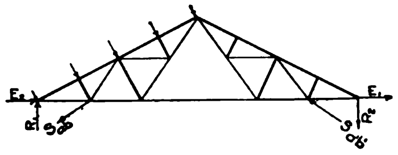


Fig. 44. Application and Direction of the Exterior forces.

For the points of application for these loads and for their direction, see Fig. 44. The stresses can now be computed by the method of Statics.

The diagram for such a truss-bent is given in Fig. 45. The span is 60 feet, the rise ¼, the distance between trusses 16 feet; and the wind pressure is taken as 18 pounds per square foot normal to the roof surface, and 20 pounds per square foot normal to the sides. In this case, w = 18 pounds; a = 16 feet; r = 60 ÷ 4 = 15 feet; w = 20 pounds; h = 20 feet; n = 14 feet; m = 6 feet; and l = 60 feet. The length of L_0U_4 is readily computed to be 33.5 feet; $L_0 L_1$, 9.1 feet; and e = 5 feet. The values of the quantities and stresses are computed as follows (see Fig. 46):

$W = 18 \times 16 \sqrt{30^2 + 15^2} = 9\,650$ pounds.

$W_1 = 16 \times 20 \times 20 = 6\,400$ pounds.

$W_h = (9\,650 \div 33.5) \times 15 = 4\,320$ pounds.

$W_v = (9\,650 \div 33.5) \times 30 = 8\,650$ pounds.

$H_1 = H_2 = (4\,320 + 6\,400) \div 2 = 5\,360$ pounds.

$$S_{ab} = +\frac{5\,360 \times 20 - 6\,400 \times 10}{5} = +8\,640 \text{ pounds.}$$

$$S_{a'b'} = -\frac{5\,360 \times 20}{5} = -21\,440 \text{ pounds.}$$

$$E_1 = \frac{5\,360 \times 14}{6} = 12\,520 \text{ pounds.}$$

$$E_2 = \frac{5\,360 \times 14 - 6\,400 \times 4}{6} = 8\,240 \text{ pounds.}$$

The horizontal and vertical components of the stresses in the knee-bracing should now be computed. They are:

For *ab*: horizontal, 7 240; vertical, 4 760 pounds.

For *a'b'*: horizontal, 17 900; vertical, 11 800 pounds.

As a check upon the computations, the sum of the values of E_1, E_2, and W'_h should be equal to the sum of the horizontal components of the knee-braces. By summing up the above values, it will be seen that they check by 80 pounds, which is less than 0.4 of one per cent and is a close enough check (see Figs. 46 and 47).

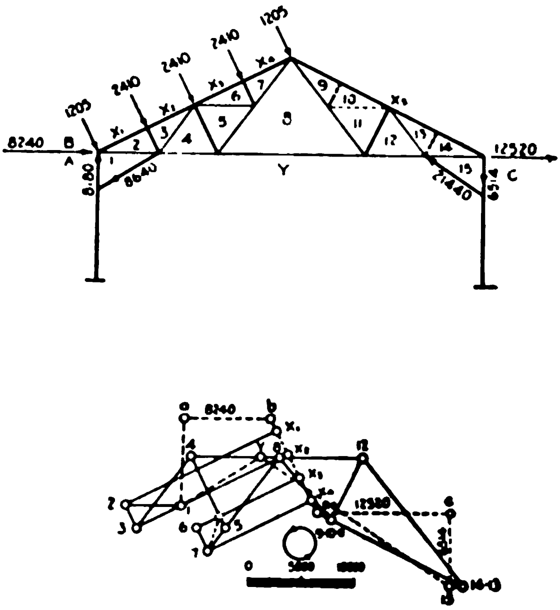


Fig. 45. Stress Diagram of Truss Bent under Wind Load.

To obtain the vertical reactions, proceed as with a simple truss. For R_2, take the center of moments at L_0 (see Fig. 47). Then:

$$R_2 = \frac{1}{60}\left\{8\,650 \times 15 + 4\,320 \times 7.5 + 4\,760 \times 9.1 - 11\,800 \times (60 - 9.1)\right\}$$
$$= -6\,514 \text{ pounds.}$$

The negative sign indicates that the reaction acts downward; that is, the truss must be riveted to the post at L_0, or the end of the post would be lifted off the top of the column.

For R_1, the center of moments is at L_8, and the resulting equation is:

$$R_1 = \frac{1}{60}\left\{-11\,800 \times 9.1 + 8\,650\,(60 - 15) - 4\,320 \times 7.5 + 4\,760\,(60 - 9.1)\right\}$$

$$= +8\,180 \text{ pounds.}$$

The bending moment at b is:

$$M_b = 14 \times 5\,360 - 4 \times 6\,400 = 49\,440 \text{ pound-feet;}$$

and the bending moment at b' is:

$$M_{b'} = 5\,360 \times 14 = 75\,040 \text{ pound-feet.}$$

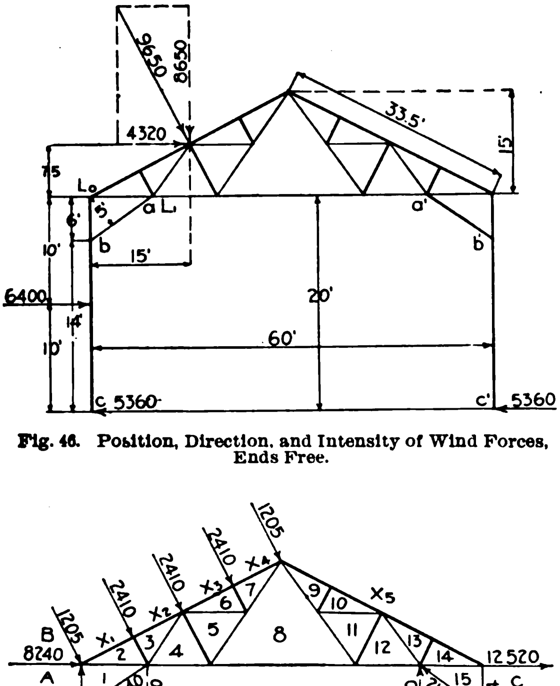


Fig. 46. Position, Direction, and Intensity of Wind Forces, Ends Free.

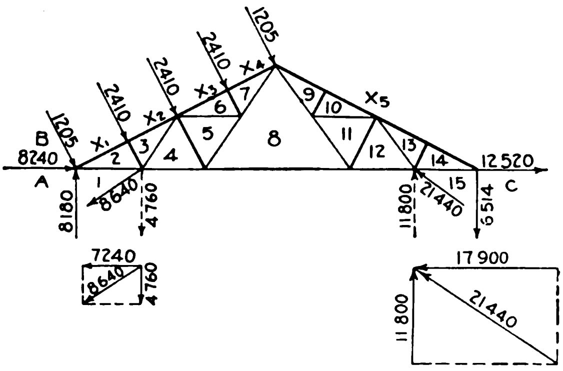


Fig. 47. Positi n, Direction, and Intensity of Exterior Forces.

The forces in their proper direction are now placed on a diagram of the truss (Fig. 47), and the stresses are solved by the method of Statics. The stress diagram is given in Fig. 45, and the stress record in Table VII.

The above formulæ are for cases when the columns are free at the lower end. When the columns are not free, they are called *fixed*; that is, they are supposed to be so tightly connected that they cannot move when the post bends as shown in Fig. 42. In such cases the result is the same as if the columns were shortened by an amonut $n \div 2$, and the following formulæ result (see Fig. 48):

TABLE VII

Stress Record of Truss-Bent under Wind Load

Member	Stress	Member	Stress
X-2	−15 700	9-10, 11-12	+ 6 500
X-3	−15 700	12-13, 14	−15 300
X-6	−10 700	Y-4	+ 6 500
X-7	−10 700	Y-8	− 1 900
X-9	+ 1 500	Y-12	− 9 600
X-13	+15 400	13, 14-15	− 1 300
1-2	+ 5 200	1-Y	+ 8 640
2-3	− 2 410	15-Y	−21 440
3-4	+ 8 500	A-1	− 8 180
4-5	− 7 500	c-15	+ 6 514
5-6	+ 2 700	b-c	− 3 457
6-7	− 2 410	b'-c'	− 5 193
7-8	+11 000	9-10-11	0
8-9	− 7 600	13-14	0

$$W = wm\sqrt{r^2 + \left(\frac{l}{2}\right)^2}$$

$$W_1 = w_1 a\left(m + \frac{n}{2}\right)$$

$$H_1 = H_2 = \frac{W_h + W_1}{2}$$

$$S_{ab} = +\frac{H_1\left(m + \frac{n}{2}\right) + W_1\left(\frac{m + \frac{n}{2}}{2}\right)}{c}$$

$$S_{a'b'} = -\frac{H_2\left(\frac{m}{2} + \frac{n}{4}\right)}{c}$$

$$S_{bc} = +\frac{W_1\left(\frac{m}{2} + \frac{n}{4}\right) + W_1\left(m + \frac{n}{2} + \frac{r}{2}\right) + W\frac{3l}{4}}{l}$$

$$S_{b'c'} = +\frac{W_1\left(\frac{m}{2} + \frac{n}{4}\right) + W_1\left(m + \frac{n}{2} + \frac{r}{2}\right) + W\frac{l}{4}}{l}$$

$$E_1 = H_2\frac{n}{2m}$$

$$E_2 = \frac{H_1 \frac{n}{2} - W_1 (\frac{n}{4} - \frac{m}{2})}{m}$$

Bending moment at $b = M_b = H_1 \frac{n}{2} - W_1 (\frac{n}{4} - \frac{m}{2})$

Bending moment at $b' = M_{b'} = H_2 \frac{n}{2}$.

For the truss-bent of Fig. 45, when the columns are fixed at the base, the stresses are the same as if the columns were shortened by an amount $n \div 2$, as above mentioned. The bent would then appear as in Fig. 49, and the values of the various stresses and the quantities, together with their points of application, are:

$W = 18 \times 16\sqrt{30^2 + 15^2} = 9\ 650$ pounds, as before.

$W_1 = 13 \times 16 \times 20 = 4\ 160$ pounds.

$H_1 = H_2 = \frac{4\ 160 + 4\ 320}{2} = 4\ 240$ pounds.

$S_{ab} = + \frac{4\ 240 \times 13 - 4\ 160 \times 6.5}{5} = +5\ 616$ pounds.

$S_{a'b'} = - \frac{4\ 240 \times 13}{5} = -\ 11\ 024$ pounds.

$S_{bc} = + \frac{4\ 160 \times 7 + 4\ 320 \times 20.5 - 8\ 650 \times 45}{60} = -4\ 526$ pounds.

$S_{b'c'} = - \frac{4\ 160 \times 7 + 4\ 320 \times 20.5 + 8\ 650 \times 15}{60} = -4\ 124$ pounds.

$E_1 = \frac{4\ 240 \times 7}{6} = 4\ 947$ pounds.

$E_2 = \frac{4\ 240 \times 7 - 4\ 160 \times 0.5}{6} = 4\ 600$ pounds.

$M_b = 4\ 240 \times 7 - 4\ 160 \times 0.5 = 27\ 600$ pound-feet.

$M_{b'} = 4\ 240 \times 7 = 29\ 680$ pound-feet.

The stresses in the bent are then computed in a manner similar to that used when the columns are fixed, E_1, E_2, and the stresses in the knee-braces being attached to the truss as concentrated loads.

Since in this case, E_1, E_2, and the stresses in the knee-braces are less than they are when the columns are free at the base, the wind stresses throughout the truss will be less when the columns are fixed than when they are free.

On account of the difficulty of fixing the ends rigidly, it is advisable always to consider the ends free and to compute the stresses accordingly.

The student is advised not to take the trouble of determining the wind stresses in trusses of steel truss-bents by the method given above, but to use the 40 pounds per square foot of horizontal projection and to correct the stresses as previously mentioned (see Fig. 39).

The formulæ of this article giving the stresses in the knee-bracing and the bending moment in the columns, should be used in all cases, and the posts and knee-braces designed according to the stresses so determined.

In cases where the 40 pounds per square foot is used, the direct stress in each column is:

$$S = \frac{40 \times a \times l}{2};$$

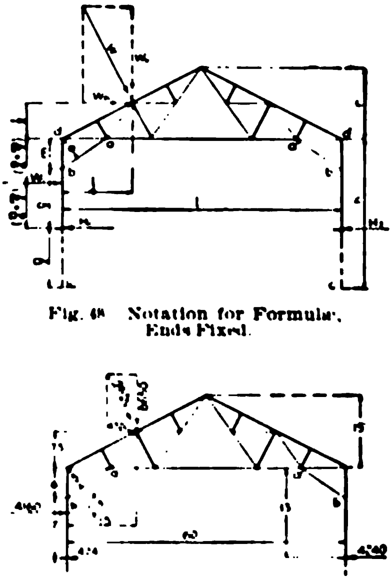


Fig. 48 Notation for Formulæ, Ends Fixed.

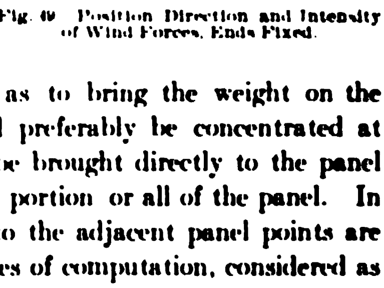


Fig. 49 Position Direction and Intensity of Wind Forces, Ends Fixed.

and the column should be designed for this stress, together with the stress due to the bending at the point where the knee-brace joins the column. See "Strength of Materials," pp. 85 and 86.

In case a crane is attached to either the truss or the column, the stresses due to its action must be considered in the design.

11. Suspended Loads. Under this head come any loads which may be suspended from the lower chord of the truss. The load may not be actually suspended from the underneath part, but may be placed above, and the connections so arranged as to bring the weight on the lower chord. This weight should preferably be concentrated at a panel point. In case it cannot be brought directly to the panel point, it may be distributed over a portion or all of the panel. In this case the portions distributed to the adjacent panel points are computed, and they are, for purposes of computation, considered as concentrated loads at the panel points. The sections of the chord over which these loads are distributed are in the condition of direct tension and bending, and must be designed for such stresses (see "Strength of Materials," pp. 85 and 86).

The suspended loads may consist of small hand cranes; shafting for transmission of power; heating apparatus, such as steam or hot-air pipes; water or compressed-air tanks; or platforms on which stand the operators for the cranes or hydraulic lifts. Figs. 50 and 51 show trusses with various forms of suspended loads attached.

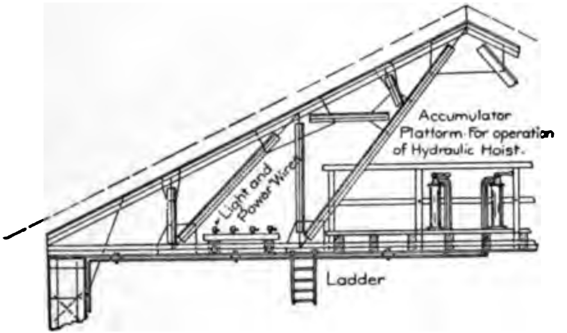


Fig. 50.

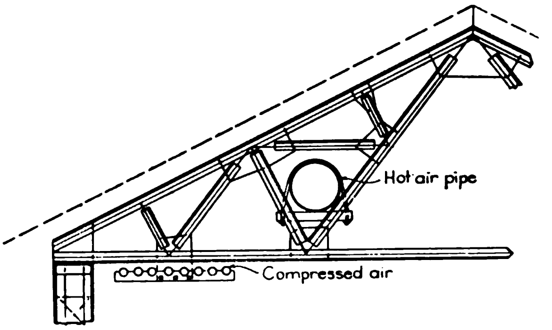


Fig. 51.

Various Kinds of Suspended Loads.

12. **Details of Roof Trusses.** The spans of triangular roof trusses of the Fink type are usually less than 100 feet, and the spans of roof trusses with chords nearly horizontal are seldom greater than 50 feet. For trusses of such spans the details are almost standard. Since these spans and trusses constitute a large majority of those built, only the details of such trusses will be considered in this text.

Where trusses rest on masonry walls or on light columns in masonry walls, provision is made for expansion due to temperature. For trusses up to 75 or 80 feet, slotted holes are placed in the end-bearing, and the bearings rest directly upon another plate. Bolts are fastened to the masonry, and extend upward through the slotted holes and have nuts on their ends. The nuts hold the truss securely to the wall, while the slotted holes allow the bearing to move backward and forward when the temperature falls or rises. *The slotted holes* *should be $\frac{1}{8}$ inch in length for every ten feet of span.* The bolts should not be less than $\frac{1}{2}$ inch in diameter, and should be buried in the masonry at least 6 inches. Fig. 52 shows details of an expansion bearing of this character. In case the span of the truss is greater than 75 or 80 feet, a roller or a rocker bearing is used. Figs. 53 and 54 show details of this class of bearings.

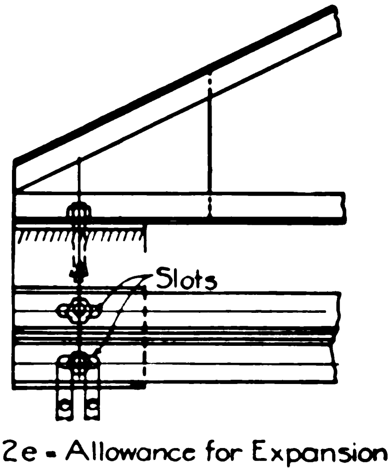


Fig. 52. Slotted Hole Expansion Bearing.

For convenience in references to the common Fink truss, the following notation will be used: the points in the upper chord are given the letter U, with a subscript corresponding to the number of the joint from the left end. The lower chord and interior joints are given the letter L, with a subscript corresponding to the number of the joint from the left end (see Figs. 24 to 38). The advantage of this system of notation is that it enables one to refer to any particular joint by the use of the letter and its subscript, and its position will at once be apparent to the mind without the use of a figure.

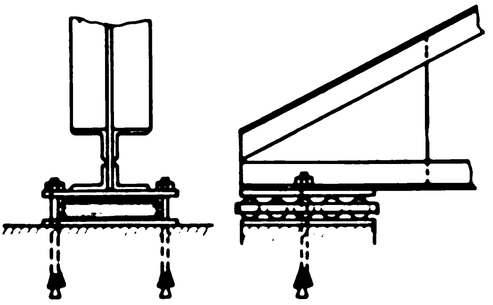


Fig. 53. Roller Expansion Bearing

If a truss rests on masonry walls, three methods of making the details at L_0 are in common use. These are shown in Figs. 55, 56, and 57. The detail shown in Fig. 55 is the most commonly used; but its use is not advised unless a sufficient number of rivets are placed in the members to take up both the direct stress and that due to the fact that the point of application of the reaction does not coincide with the intersection of the center lines of the chord members.

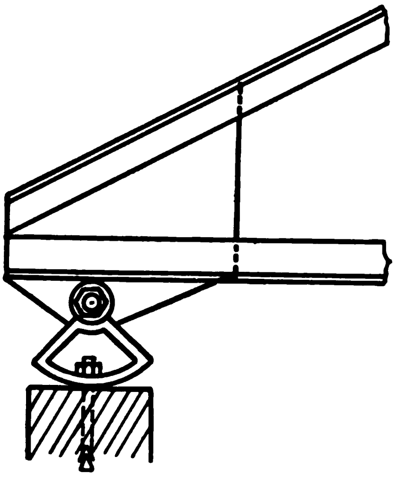


Fig. 54. Rocker Expansion Bearing.

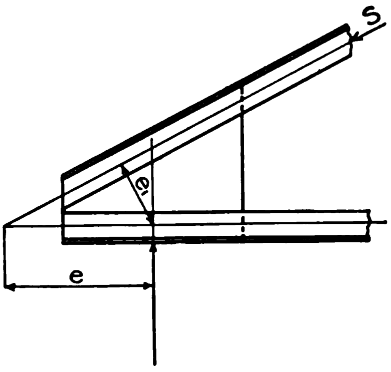


Fig. 55.

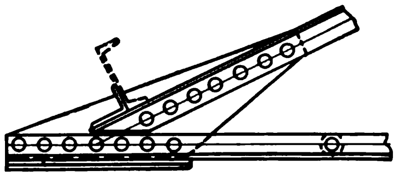


Fig. 56.

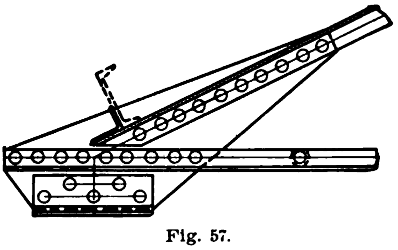


Fig. 57.
Details of Ends of Roof Trusses.

In case the bearing shown in Fig. 55 is used, the number of rivets in $L_0\,L_1$ may be calculated from the equation:

$$n^2v - Rn = \frac{6\,Re}{p},$$

in which,

n = Number of rivets required;
v = Allowable stress on one rivet;
R = Vertical reaction;
p = Rivet spacing, in inches;
e = Distance as shown in Fig. 55.

If the number of rivets in $L_0\,U_1$ is desired, it may be calculated from the equation:

$$n^2v - Sn = \frac{6\,S\,e_1}{p}.$$

in which S is the stress in $L_0\,U_1$, e the distance shown in Fig. 55, and the remaining notation as above.

If the point of application of the reaction coincides with the intersection of the center lines of the top and bottom chords, the number of rivets required to withstand the direct stress, which is the only stress would be equal to the stress in that member divided by the allowable stress in one rivet.

In order to illustrate the use of the above equation, and to bring out the fact that more rivets are required when the point of application of the reaction does not coincide with the intersection of the upper and lower chords than when it does coincide, an example will be solved. The stresses, the thickness of the connection plate, and the distance of the point of application of the reaction from the intersection of the chord, are as shown in Fig. 58.

It will be assumed that the chords consist of two angles each; and since this is the case, the allowable unit-stress in one rivet will be

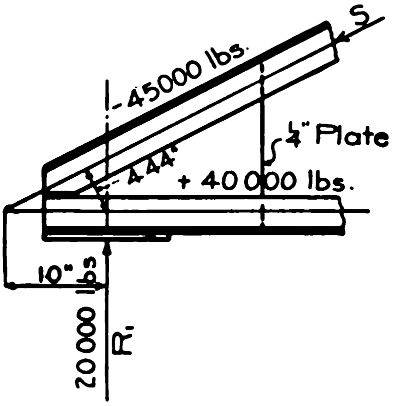


Fig. 58. Data for Example on Page 37.

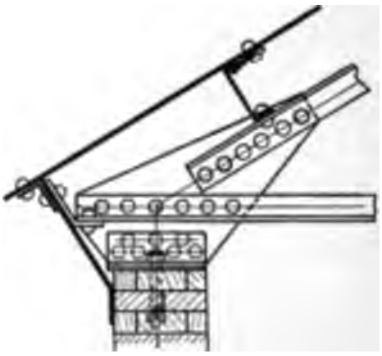


Fig. 59. Eave Detail for Fig. 57.

3 750 pounds, the value of a $\frac{3}{4}$-inch rivet in bearing in a $\frac{1}{4}$-inch plate when the allowable unit bearing stress is 20 000 pounds per square inch. If the point of application of the reaction coincides with the intersection of the two chords, the number of rivets required will be:

$$\text{For } L_0 U_1 = \frac{45\,000}{3\,750} = 12.00 \text{ rivets.}$$

$$\text{For } L_0 L_1 = \frac{40\,000}{3\,750} = 10.67 \text{ rivets.}$$

Since the point of application of the reaction does not coincide with the intersection of the chord, the number of rivets required in $L_0 U_1$ is:

$$3\,750\, n^2 - 45\,000\, n = \frac{6 \times 45\,000 \times 4.44}{3},$$

the spacing being 3 inches; dividing by 3 750, we have:

$$n^2 - 12n = 106.56.$$

Completing the square and solving for n, there is obtained:

$$n = 6 + \sqrt{142.56} = 17.9, \text{ say 18 rivets.}$$

The number of rivets required in $L_0 L_1$ is:

$$3\,750\,n^2 - 20\,000\,n = \frac{6 \times 20\,000 \times 10}{3};$$

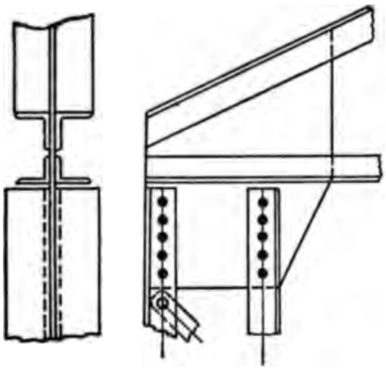


Fig. 60.

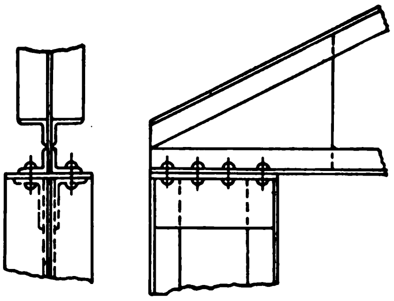


Fig. 61.

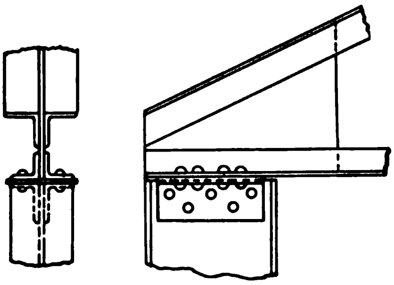


Fig. 62.

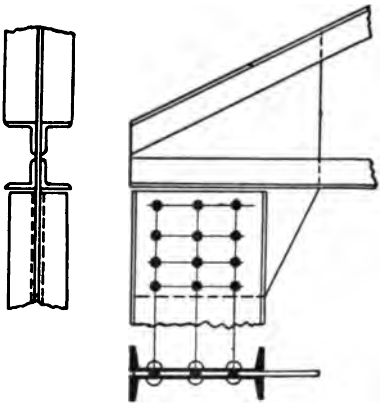


Fig. 63.

Details of Tops of Columns.

and dividing by 3 750 and completing the square, there results:

$$n = 2.67 + \sqrt{113.75} = 13.32, \text{ say 14 rivets.}$$

Inspection of the above results shows that when the point of application of the reaction is placed 10 inches from the intersection of the chords, it requires 6 more rivets in the upper chord and 3 more rivets in the lower chord than would be required if the point of application of the reaction coincided with the intersection of the chords.

The detail just discussed is a very convenient one, and is very commonly used; but in most cases no allowance is made for the additional rivets required because of the fact that the reaction does not coincide with the intersection of the chord members. The student should always compute the rivets by the formulæ given above, since it is very evident that neglect to do so causes the joint to be exceedingly weak, in some cases as much as 50 per cent, as is shown in the case of $L_0\,L_1$ in the above problem.

Fig. 56 is excellent, but the length of the bearing plate, which should be as long as the connection plate, is liable to become greater than the width of the wall. In such cases the detail shown in Fig. 57 is to be used. The objection raised to these details is that the end connection plate prevents the placing of a purlin near the end of the roof truss. In case sheathing is used, this objection does not hold good, since the overhanging sheathing will reach to the end of the truss and form a good eave detail, as shown in Fig. 59.

When the roof truss rests on steel columns which are composed of latticed angles, the connections may be made as shown in Figs. 60 and 61. Fig. 60 is preferable, because it gives a more rigid connection than is given by Fig. 61. If the columns consist of two panels placed close together, back to back, the same details may be used. If the column consists of one I-beam or of two channels placed back to back at some distance apart, then details shown in Figs. 62 and 63 may be used.

Where one member is joined to another and makes an angle or is perpendicular to it, then details as shown in Figs. 64 and 65 may be used. It is not good practice to cut the angles as shown at *b* in Fig. 65; *a* is a better detail. No joints should have less than two rivets.

In places where three members meet, and two make the same angle with one of the others, the details should be made as shown in Fig. 66. The leg of the angle which is not joined to the plate should always be upward. This prevents the dust and dirt from becoming mixed with the moisture and running or jarring down into joints at the lower ends of the members.

At L_1 and L_2, square plates (see at left, Fig. 67) should be used where possible. If the stresses are such that more rivets are required in one member than in the other, then the plate should be cut as shown at right in Fig. 67.

At L_2 the splice occurs, since Fink trusses are usually shipped in two parts. In addition to the vertical connection plate, which also acts as a splice plate, the bottom plate is used (see Fig. 68). Rivets shown in black indicate that the holes are left open, the pieces in which they occur are shipped separately, and then are riveted together at the place where the truss is put up.

In some cases where the member L_2 L_5 is long enough to sag considerably, or where it is desired to connect a load (such as a hand hoist) at its center, a vertical U_4 M is run from U_4 and connected to the lower chord. No stress is caused in this member by any load except the load at M, in which case the stress is equal to that load. If a load is at M, it will cause stresses in other members of the truss, the stresses in the truss being the same as if the dead panel load at U_4 were increased by an amount equal to the load at M.

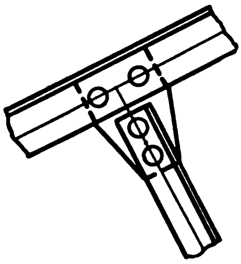


Fig. 64.

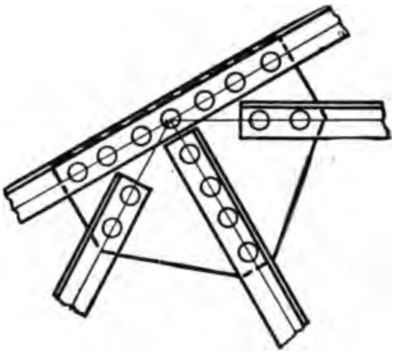
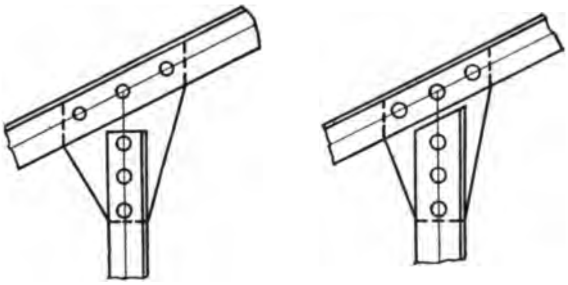


Fig. 66.



a

b

Fig. 65.

Details of Roof Truss Connections. See also Figs. 67 and 68.

The general details of a Fink truss are shown in Plate I (p. 43), Plate II (p. 60), and Plate III (p. 61).

In case the building is devoted to some purpose wherein no smoke or noxious gases are produced, some form of *patent ventilator* may be used. One very excellent make is shown in Fig. 69—called the *Star* ventilator (Merchant & Co., Philadelphia, Pa.). These ventilators are made from 2 to 60 inches in diameter at the lower portion, where they fit to the ridge of the roof. Fig. 70 shows one of them in position on a roof. The number and size of these ventilators depend of course upon the number of times per hour it is desirable to change the air in the shop.

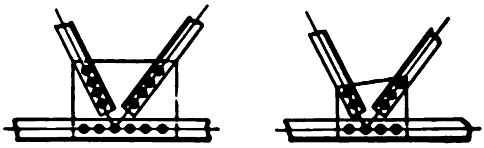


Fig. 67.

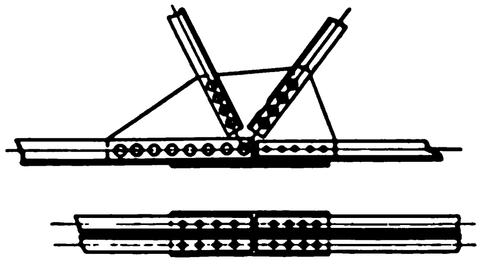


Fig. 68.

Details of Roof Truss Connections. See also Figs. 64, 65, and 66.

In case the shop is for such purposes that smoke, gases, or noxious fumes of any kind are produced, it is desirable to have some channel for ventilation which is considerably larger than those given by the patent ventilators. In such cases the ventilation is usually obtained by a small house-shaped construction called a *lantern*, *monitor*, or *ventilator* (see Fig. 71). The sides of these ventilators may be fitted with *louvres* or windows, or left open. Louvres may be made either of wood or of corrugated or plain bars. For details of monitors and louvres, see Figs. 124, 125, and 126.

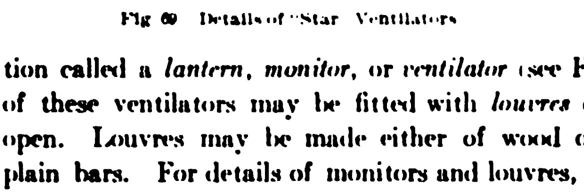


Fig 69 Details of "Star" Ventilators

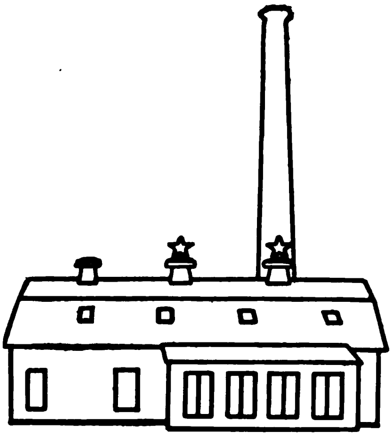


Fig. 70. "Star" Ventilators on a Roof.

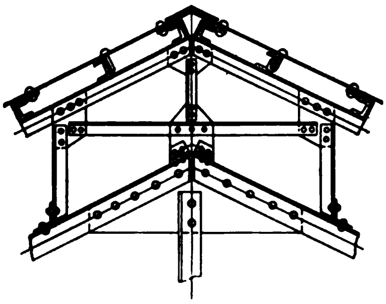


Fig. 71. Detail of a Monitor Ventilator.

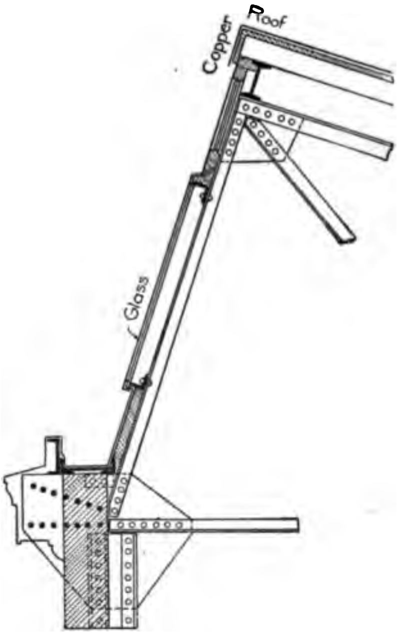


Fig. 72. Detail of Window in Saw-Tooth Roof.

In order to admit sufficient light into the building, part of the roof of buildings over 80 feet wide must be made of glass, since the amount of light admitted from the sides of the building is not sufficient to light up those parts of the shop near the center of the trusses. In some cases the saw-toothed truss is used, in which case the entire surface of the short rafter is covered with glass. In case the ordinary triangular roof truss is used, a portion of the roof covering must be made of glass, so put on as to prevent leakage and also to prevent the moisture which forms on the under side of the glass from dropping in the shop. Fig. 72 shows the glass in place on a saw-toothed roof; and Figs. 73 and 74 give the details of several methods of securing glass on the roof so that no leakage or condensation will get onto the shop floor. The glass area should be from $\frac{1}{10}$ to $\frac{1}{4}$ of the floor area.

13. Specifications for Roof Trusses and Steel Buildings. In case of an important structure, special specifications are written, embody-

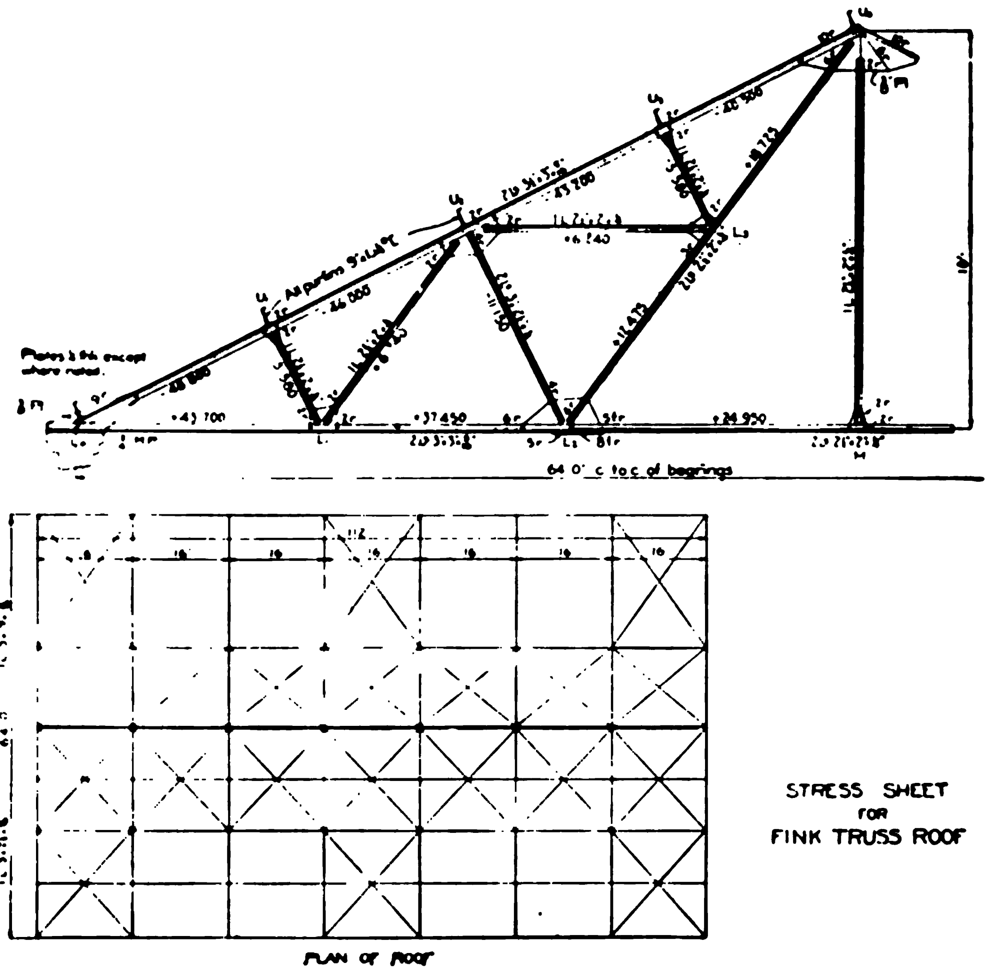


Plate I. Stress Sheet for a Fink Truss Roof.

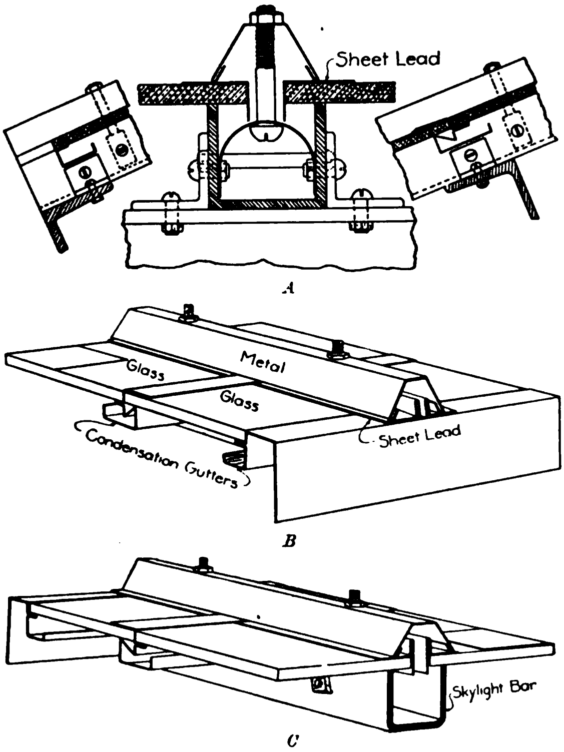


Fig. 73. "Paradigm" Method of Glazing.

ing certain features which the experience of the engineer in charge indicates as necessary. For ordinary structures, however, several very satisfactory specifications are on the market. These consist of from 15 to 20 pages, bound in paper, and may be had for twenty-five cents a copy. Two very satisfactory specifications are those of Charles Evan Fowler and Milo S. Ketchum. Either may be had by addressing the Engineering News Publishing Company, New York City. Fowler's specifications, in addition to giving specifications for load stresses and workmanship, give much valuable information regarding the stresses in different kinds of trusses, besides various details showing the use of corrugated steel.

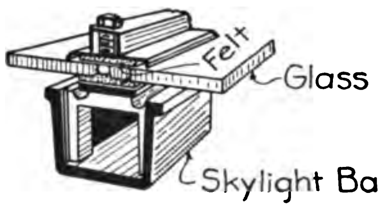


Fig. 74. "Anti-Pluvius" Method of Glazing.

An extended set of specifications is not required for the design of ordinary roof trusses. In addition to the information regarding the weight of trusses, the weight of roof covering, the snow load, and the wind load, the use of Table VIII will be found to be all that is necessary in order to design cross-sections of the various members, once the stresses are determined.

TABLE VIII

Allowable Unit-Stresses, Medium Steel

For Shear	10 000	pounds per square inch.
For Bearing	20 000	" " " "
For Tension	15 000	" " " "
For Bearing of Steel on Masonry.	250 to 400	" " " "
For Compression	$P = 24\,000 - 110\,\frac{l}{r}$.	

In case the stresses are those due to crane loads, the unit-stresses in tension and compression indicated in Table VIII should be reduced ¼ and ⅓ respectively. Members of the lateral bracing and their connections may be allowed an increase of 25 per cent over the unit-stresses there indicated.

In the equation above given "For Compression," l is the length of the member in inches, and r the least radius of gyration. The ratio of $\frac{l}{r}$ should never be greater than 120.

The *gauge line* or *gauge* is the line on the flange of a shape, on which the rivets are placed. In angles and channels it is located by giving its distance from the back of the shape; in the case of I-beams the distance between two gauge lines on opposite sides of the web is indicated. Some angles have double gauge lines, in which case the rivets are placed first on one and then on the other; this is called *staggering*. Fig. 75 shows gauge lines for various shapes.

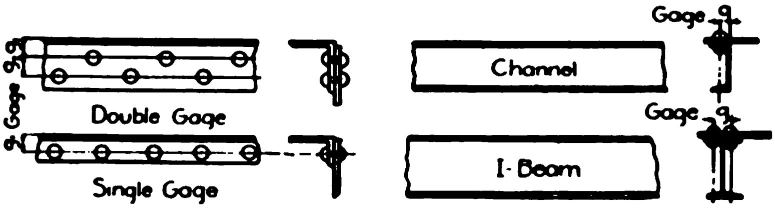


Fig. 75. Gauges for Angles, Channels, and I Beams.

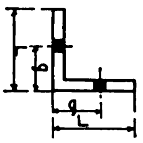
Rivets ¾ inch in diameter are generally used in legs of angles 3 to 4 inches long or greater. For the gauge lines and the maximum sizes of rivets to be used in angles, see Table IX. For similar data for channels and I-beams, see Carnegie Handbook, pp. 177–185.

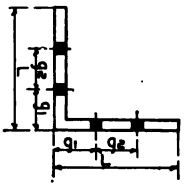
It is often desirable to express the length in feet instead of inches, in which case the formula becomes:

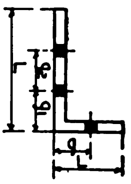
$$P = 24\,000 - 1\,320\,\frac{L}{r}.$$

TABLE IX

Gauges and Maximum Allowable Rivets for Angles







L	g	Maximum Rivet or Bolt	L	g	Maximum Rivet or Bolt	L	g	Maximum Rivet or Bolt
8	4½	⅞	3½	2	⅞	2	1⅛	½
7	4	⅞	3	1¾	⅞	1¾	1	½
6	3½	⅞	2¾	1⅝	¾	1½	⅞	⅜
5	3	⅞	2½	1⅜	⅝	1¼	¾	⅜
4	2¼	⅞	2¼	1¼	⅝	1	$\frac{9}{16}$	¼

L	g_1	g_2	L	g_1	g_2
8	3	3	6*	2½	2¼
7	2½	3	5	2	1¾
6	2¼	2½			

* When thickness is ¾ inch or over.

For convenience in designing, the values of $L \div r$ should be plotted as ordinates, and the resulting values of P as abscissæ, on cross-section paper, and the curve drawn in. Then the value of P for any given value of $L \div r$ may be taken at once from the diagram without the labor of substituting in the above formula.

The bearing value of a rivet in a plate of given thickness is equal to the thickness of the plate, *times* the diameter of the rivet, *times* the allowable unit bearing stress. The value of a rivet in single shear is equal to the area of the cross-section of the rivet, *times* the allowable unit shearing stress. The bearing values of rivets of different diameter in plates of different thickness, and the shearing values of rivets of different diameter, are given in Table X, the unit-stresses being as given above.

TABLE X

Bearing and Shearing Values of Rivets

Diameter of Rivet (Inches)	Single Shear (at 10 000 lbs per sq in)	Bearing in Different Thicknesses of Plates (at 20 000 lbs per sq in)							
		¼ in	$\frac{5}{16}$ in	⅜ in	$\frac{7}{16}$ in	½ in	$\frac{9}{16}$ in	⅝ in	$\frac{11}{16}$ in
½	1 960	2 500	3 130	3 750					
$\frac{9}{16}$	2 480	2 810	3 520	4 210	4 920				
⅝	3 070	3 130	3 910	4 690	5 470				
$\frac{11}{16}$	3 710	3 440	4 290	5 160	6 010	6 880			
¾	4 420	3 750	4 690	5 630	6 560	7 500	8 440		
$\frac{13}{16}$	5 180	4 070	5 080	6 090	7 110	8 120	9 150	10 160	
⅞	6 010	4 380	5 470	6 570	7 660	8 750	9 840	10 940	12 040

DESIGN OF A RIVETED ROOF TRUSS

14. Let it be required to design a Fink roof truss of 64 feet span and ¼ pitch, the distance between trusses being 16 feet. The roof covering is taken as 12 pounds per square foot of roof surface, and the total snow and wind load will be taken as 30 pounds per square foot of horizontal projection. The weight of the steel in the roof truss will be computed from Merriman's formula (see Art. 4, p. 6). The total weight is now found to be:

Weight of truss, $\frac{3}{4} \times 16 \times 64 \left(1 + \frac{l}{10}\right) = 5\ 580$ pounds.

Weight of roof cover, $35.6 \times 2 \times 16 \times 12 = 13\ 650$ pounds.

Weight of wind and snow $64 \times 16 \times 30 = 30\ 700$ pounds.

Total 49 930 pounds.

Each apex load is therefore $49\ 930 \div 8 = 6\ 240$ pounds. By multiplying this value by each of the stresses as given in Fig. 25, the stress in each member is computed as follows:

$L_0U_1 = 7.83 \times 6\ 240 = 48\ 800$ pounds

$L_0L_1 = 7.00 \times 6\ 240 = 43\ 700$ "

$U_1L_1 = 0.89 \times 6\ 240 = 5\ 580$ "

$U_1U_2 = 7.38 \times 6\ 240 = 46\ 000$ "

L_1U_2 and $U_3L_2 = 1.00 \times 6\ 240 = 6\ 240$ "

$L_1L_2 = 6.00 \times 6\ 240 = 37\ 450$ "

$U_2L_2 = 1.79 \times 6\ 240 = 11\ 150$ "

$U_2U_3 = 6.93 \times 6\ 240 = 43\ 200$ "

$L_2L_3 = 4.00 \times 6\ 240 = 24\ 950$ "

$L_2L_4 = 2.00 \times 6\ 240 = 12\ 475$ "

$U_3L_3 = 0.89 \times 6\ 240 = 5\ 580$ "

$L_3U_4 = 3.00 \times 6\ 240 = 18\ 725$ "

$U_3U_4 = 6.48 \times 6\ 240 = 40\ 500$ "

In the design of this truss, no material thinner than ¼-inch, and no angles smaller than 2½ by 2-inch, will be allowed.

Fig. 76 shows an outline diagram of the truss, with the stresses placed upon it. A positive sign signifies a tensile stress, and a negative sign signifies a compressive stress. The length of the top chords is $\sqrt{32^2 + 16^2} = 35.6$ feet; and the length of each panel is ¼ of this, or 8.9 feet. The horizontal projection of one panel is ¼ of half the span, or 32 ÷ 4 = 8 feet.

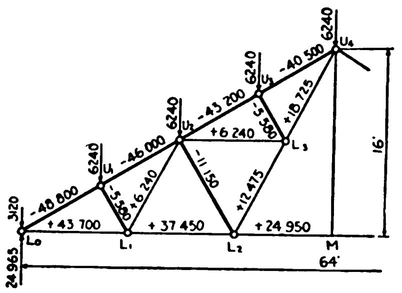


Fig. 76. Stresses in a Fink Truss.

Design of the Purlins. The distance between the trusses is 16 feet, and the distance between the purlins is 8.9 feet; therefore the load coming on one purlin is:

Roof covering, 8.9 × 16 × 12	=	1 710 pounds
Snow and wind, 8 × 16 × 30	=	3 840 "
Total	=	5 550 pounds

This should be resolved in two components, V and H, perpendicular and parallel to the truss chord. These are determined by the proportions of similar triangles, as follows:

$$V : 5\,550 = 32 : 35.6$$
$$V = 4\,990 \text{ pounds.}$$
$$H : 5\,550 = 16 : 35.6$$
$$H = 2\,495 \text{ pounds.}$$

The bending moment caused by V is $M_v = (4\,990 \times 16) \div 8 = 9\,980$ pound-feet. The bending moment caused by H is $M_H = (2\,495 \times 16) \div 8 = 4\,990$ pound-feet. The stress caused by V is $= \frac{M_v c}{I}$; and the stress caused by H is $\frac{M_H c^1}{I'}$; and there is also the condition that the sum of these two stresses shall not be greater than 15 000 pounds. Since the above formula involves the moment of inertia and half the depth of the beam, a beam must be chosen, and its moment of inertia and half-depth substituted in the above equation, and the equation solved. In case the sum of the stresses is in excess of 15 000 pounds, or very much smaller, a re-computation must be made, using a larger or a smaller beam.

A 15-inch 42-pound I-beam will be assumed, and will be examined to see if it fulfils the necessary conditions. The value of I and I' are taken from the Carnegie Handbook, p. 97. The value of c is $\frac{15}{2} = 7\frac{1}{2}$ in the first case, and $\frac{5.50}{2} = 2.75$ in the second case. The quantity 5.50 is the width of the flange of the I-beam. Substituting in the above formula, there results:

$$\frac{9\,980 \times 12 \times 7\frac{1}{2}}{411.8} + \frac{4\,990 \times 12 \times 2.75}{14.62} = 13\,330 \text{ pounds}$$

The above I-beam could be used; but in case the sheathing is laid closely and nailed tightly, we may consider it acting as a beam of a span of 16 feet, 8.9 feet deep, and of a thickness equal to that of the sheathing, which in this case will be assumed as 1½ inches. The sheathing will then take up the moment caused by the force H; and the purlin will take up the vertical bending moment alone. The stress in the sheathing due to the force H is $\frac{M_H c}{I}$. Here $M = 4\,990 \times 12$; $c = 8.9 \times 12 \div 2$; and $I = \frac{1.5\,(8.9 \times 12)^3}{12}$. Therefore,

$$S = \frac{4\,990 \times 12 \times 8.9 \times 12 \times 12}{2 \times 1.5\,(8.9 \times 12)^3}$$
$$= 20.95 \text{ pounds per square inch, which is insignificant.}$$

The vertical bending moment taken up by the purlin is $9\,980 \times 12 = 119\,760$ pound-inches, and this requires a section modulus of $119\,760 \div 15\,000 = 7.98$. By consulting pages 101 and 102 of the Carnegie Handbook, the following is found to be true:

An 8-inch 11.25-pound channel is just too small.
A 7-inch 17.25-pound channel gives the nearest section modulus.
An 8-inch 13.75-pound channel would be lighter and stiffer.
A 9-inch 13.25-pound channel would be still lighter and stiffer; and since it weighs less than any of the others, it will be more economical.

A 9-inch 13.25-pound channel will accordingly be used for the purlins.

On account of one half-panel load coming on the purlin at the ends and ridge of the truss, these purlins must theoretically be only one-half as strong as the other; but, on account of the fact that all purlins must be of the same height, these purlins are made of the lightest weight channel of the same height as the others. In this case it happens that the lightest weight 9-inch channel is required for the intermediate purlins as well as for the end ones. To illustrate the above, suppose that the purlins were required to be 10-inch 25-pound channels, then the end purlins would be made of 10-inch 15-pound channels.

In case sheathing is not used, then some other method must be employed to take up the bending moment due to the force H. The usual method of doing this is to bore holes in the center of the purlins at the middle point of their span, and to connect them with rods which run from one eave up over the ridge and down to the other eave (see Fig. 22).

Design of Tension Members. *For Member* $L_0\ L_1$: The required net area is 43 700 ÷ 15 000 = 2.92 square inches. By consulting the Carnegie Handbook, p. 118, it is seen that two 3 by 3 by $\frac{5}{16}$-inch angles give a gross area of $1.78 \times 2 = 3.56$ square inches. From this must be subtracted the rivet-hole made by a $\frac{3}{4}$-inch rivet. Since all rivet-holes are punched $\frac{1}{8}$ inch larger in diameter than the rivet, the amount to be substracted from the above gross area is $\frac{5}{16} \times (\frac{3}{4} + \frac{1}{8}) \times 2 = 0.54$, there being two rivet-holes taken out of the section. This gives a total net area of $3.56 - 0.54 = 3.02$ square inches. As this is but slightly larger than the required net area, these angles will be used for this member. Since the stress in this member is the greatest stress in the bottom chord, and since the bottom chord is made of the same section up to the splice at L_2, on account of economical construction, it being cheaper to run the same sized angle throughout than it would be to change the size of each panel and make a splice at each panel point, the size of angle as determined above will be used for the first two panels of the bottom chord at each end.

For Member $L_2\ L_5$: The required net area is 24 950 ÷ 15 000 = 1.67 square inches. From Carnegie Handbook, p. 115, two angles $2\frac{1}{2}$ by 2 by $\frac{1}{4}$-inch give a gross area of $2 \times 1.06 = 2.12$ square inches; and taking out two $\frac{5}{8}$-inch rivets, the net area is $2.12 - \frac{1}{4}\,(\frac{5}{8} + \frac{1}{8}) \times 2 = 1.74$ square inches. This coincides very closely with the required area, and this angle will be used. Even if this angle should have been in excess of the required area, it would still be necessary to use it, since it is the smallest angle and of the least thickness allowed.

For Member $L_3 U_4$: The required net area is 18 725 ÷ 15 000 = 1.25 square inches. Two angles 2½ by 2 by ¼-inch give a gross area of 2.12 square inches, and a net area of 1.74 square inches, as above computed. Although they give an area considerably larger than that required, nevertheless they must be used, since they are the smallest allowed.

For Members $L_1 U_2$ and $U_2 L_3$: The required net area is 6 240 ÷ 15 000 = 0.42 square inch. One angle 2½ by 2 by ¼-inch gives a gross area of 1.06 square inches. The amount to deduct from this is ¼ × (⅝ + ⅛) = 0.19 square inch, one ⅝-inch rivet-hole being taken from the section. This gives a net area of 1.06 − 0.19 = 0.87 square inch, which shows this angle to be sufficient.

Since the member U_4M has no other use than to prevent the bottom chord from sagging, it will be made of the lightest angle allowed. It will therefore be made of one angle 2½ by 2 by ¼-inch.

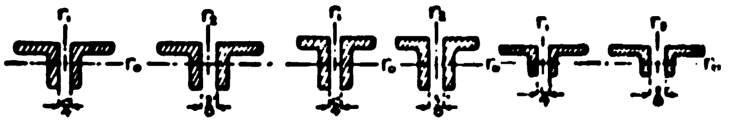
The member $L_2 L_3$ is made of the same section as the member $L_3 U_4$, since this is more economical than to change the section and to make a splice at L_3. It will be made of two angles 2½ by 2 by ¼-inch.

Design of the Compression Members. The general method of procedure in the design of compression members is, first, to assume a cross-section, and then to determine the unit compressive stress allowable by inserting the length of the member and the radius of gyration of the assumed section in the formula given for the unit allowable compressive stress; then divide the stress in the member by the unit allowable compressive stress determined as above. This will give the required area. If this required area is equal to, or slightly less than, the area of the cross-section assumed, the section assumed will be the correct one. If the required area as computed above is greater than the area of the section, then a larger section must be assumed and the operation repeated. Usually only two operations are required in order to obtain a section whose area is correct. It should be noted that the area of the rivet-holes is not deducted from the section in compression members, since the rivet fills up the rivet-hole and makes a section as strong in compression as it was in the first place. Care should be taken to assume a section whose radius of gyration is equal to or greater than the length of the member divided by 120. This is due to the fact that $l : r$ should not be greater than 120. Compression members of roof trusses for the usual spans are made of two angles placed back to back. The radius of gyration of such a section is equal to the radius of gyration of one angle, if it is referred to an axis perpendicular to the legs which are placed together. If it is referred to an axis through the center of the section and parallel to the legs which are placed together, it is equal to some value other than the radius of gyration of one angle. The radii of gyration for pairs of angles placed either directly back to back or a small distance apart, are given on pages 144 to 146 of the Carnegie Handbook and in Table XI, and should be used in the design. The value of the radius of gyration for sizes of angles other than those given, may be obtained by interpolation.

For example, let it be required to determine the radius of gyration of two 5 by $3\frac{1}{2}$ by $\frac{1}{2}$-inch angles placed $\frac{1}{2}$ inch apart and back to back, the 5-inch legs being horizontal (see p. 146, Carnegie Handbook). Since this value is not given in the tables, it must be interpolated from the values given for r_2 for the above sized angle, which are $\frac{5}{16}$ inch and $\frac{7}{8}$ inch thick. The difference between the two thicknesses given is $\frac{7}{8} - \frac{5}{16} = \frac{9}{16}$ inch. The difference between the two values given for the radius of gyration is $2.55 - 2.44 = 0.11$. This gives a difference of $.11 \div 9 = 0.0122$ for each $\frac{1}{16}$ inch difference in thickness in the angle. The difference between the thickest angle and the angle under consideration is $\frac{7}{8} - \frac{1}{2} = \frac{3}{8}$, or $\frac{6}{16}$. Therefore the amount to be subtracted from the radius of gyration of the thickest angle is $6 \times 0.0122 = 0.0732$; and the radius of gyration for two angles placed back to back as above stated is $2.55 - 0.07 = 2.48$. In case one angle is used for a member in compression, the least rectangular radius of gyration must be used; and if two angles are employed, placed back to back, care should be exercised to use the least radius of gyration; and if the angles have unequal legs, those legs should be placed back to back, which will make the rectangular radii of gyration as nearly equal as possible. The values of the radii of gyration will indicate whether the short legs or the long legs should be placed together. The tables given in the Carnegie Handbook give the radii of gyration for angles spaced at distances $\frac{1}{2}$ inch and $\frac{3}{4}$ inch apart; but since the connection plates of roof trusses are usually $\frac{1}{4}$ inch or $\frac{3}{8}$ inch thick, the values of the radii of gyration should be given for angles spaced $\frac{1}{4}$ inch and $\frac{3}{8}$ inch apart. Such values are given in Table XI.

TABLE XI

Radii of Gyration of Angles Placed Back to Back



EQUAL LEGS Size (Inches)	r_1	r_0	UNEQUAL LEGS Size (Inches)	r_0	r_0	r_1	r_1
2 × 2 × 3/16	0.93	0.98	2½ × 2 × 3/16	0.88	0.92	1.19	1.24
2 × 2 × 7/16	0.98	1.03	2½ × 2 × ½	0.91	0.96	1.25	1.30
2½ × 2½ × ¼	1.14	1.19	3 × 2½ × ¼	1.09	1.13	1.40	1.45
2½ × 2½ × ½	1.19	1.24	3 × 2½ × 9/16	1.15	1.20	1.46	1.51
3 × 3 × ¼	1.34	1.39	3½ × 2½ × ¼	1.04	1.09	1.67	1.72
3 × 3 × ⅝	1.41	1.46	3½ × 2½ × 11/16	1.13	1.18	1.75	1.80
3½ × 3½ × ⅜	1.56	1.61	3½ × 3 × 5/16	1.30	1.35	1.61	1.66
3½ × 3½ × 11/16	1.65	1.70	3½ × 3 × 13/16	1.40	1.45	1.71	1.76
4 × 4 × 5/16	1.76	1.80	4 × 3 × 5/16	1.25	1.30	1.88	1.93
4 × 4 × 13/16	1.85	1.89	4 × 3 × 13/16	1.35	1.40	1.97	2.02
6 × 6 × 7/16	2.58	2.63	5 × 3 × 5/16	1.17	1.22	2.42	2.47
6 × 6 × ⅞	2.66	2.70	5 × 3 × 13/16	1.27	1.32	2.52	2.57
			5 × 3½ × ⅜	1.42	1.46	2.36	2.41
			5 × 3½ × ¾	1.51	1.56	2.45	2.50
			6 × 3½ × ⅜	1.34	1.39	2.90	2.95
			6 × 3½ × ¾	1.44	1.49	3.00	3.05
			6 × 4 × ⅜	1.58	1.62	2.83	2.87
			6 × 4 × ¾	1.67	1.71	2.92	2.97

---

r_0 = In all cases the radius of gyration of one angle referred to neutral axis parallel to the horizontal leg as shown above.

For Member L_1U_1. Two angles 3½ by 3 by 5/16-inch, long legs spaced back to back, and ¾ inch apart, will be assumed. The least radius of gyration is 1.10, and the length is 8.9 feet. The area of this section is 2 × 1.93 = 3.86 square inches. The unit allowable compressive stress is:

$$P = 24{,}000 - \frac{110 \times 12 \times 8.9}{1.10} = 13{,}400 \text{ pounds}$$

The required area is 48,900 ÷ 13,400 = 3.65 square inches. Since the angles given are of somewhat larger area than that required, it might be well to examine the next smallest angle.

Two angles 3½ by 2½ by 5/16-inch, with a radius of gyration

1.11 and a total area of 3.56 square inches, will be assumed. The unit allowable compressive stress is:

$$P = 24\,000 - \frac{110 \times 12 \times 8.9}{1.11} = 13\,510 \text{ pounds.}$$

The required area is 48 800 ÷ 13 510 = 3.61 square inches. Since the required area is greater than the given area, it shows that these angles are too small. Two angles 3½ by 3 by $\frac{5}{16}$-inch will therefore be used for this member, and also for all the members of the top chord, since it is more economical to run the same size throughout than to change the section and make splices at all the upper chord panel points.

For Member $U_2\ L_2$: The length of this member is easily computed from similar triangles, and is found to be 8.9 feet. Two angles 2½ by 2 by $\frac{5}{16}$-inch, with the long legs back to back, give a total area of 1.62 square inches and a radius of gyration of 0.78. The unit-stress is computed and found to be 8 950 pounds. The required area is 11 150 ÷ 8 950 = 1.25 square inches. These two angles would be used, but the least allowable radius of gyration is 8.9 × 12 ÷ 120 = 0.89. This is seen to be considerably greater than the radius of gyration given above, and therefore these angles cannot be used, according to Specifications. By consulting the tables, it is seen that two angles 3 by 2½ by ¼-inch are the smallest angles that will give a radius of gyration nearest to the required amount (0.89) and still be standard size angles. Angles marked with a star in the tables are special angles, and can be procured only at a cost greatly in excess of the others, and then only with great delay in delivering except when large quantities are ordered. It may be said that special angles should never be used.

For Members $U_1\ L_1$ and $U_3\ L_3$: The length of these members is 4.45 feet. The radius of gyration must therefore not be less than 4.45 × 12 ÷ 120 = 0.45. One angle 2½ by 2 by ¼-inch, with an area of 1.06 square inches and a least rectangular radius of gyration of 0.59, will be assumed. The allowable unit compressive stress is:

$$P = 24\,000 - \frac{110 \times 12 \times \frac{8.9}{2}}{0.59} = 14\,050 \text{ pounds.}$$

The required area is 5 580 ÷ 14 050 = 0.40 square inch. The angle chosen gives a much larger area than that required; but since it is the smallest one allowed by the Specifications, it must be used.

Many designers do not place a limit on the value of the radius of gyration, but simply use the compressive formula, and any section whose radius of gyration will bring the required area near to its own area. This should not be the case, since the formula here given is not applicable when the value of $l \div r$ is greater than 120.

Top and Bottom Lateral Bracing. Since the stresses in the lateral bracing are not susceptible of a well-defined mathematical analysis, it cannot be rationally designed. Experience indicates that it should be as in Article 7. The lower chord bracing will therefore consist of single angles 3 by 2½ by $\frac{5}{16}$-inch; and the upper chord bracing, of 3 by 3 by $\frac{5}{16}$-inch angles. This bracing should not be placed between every truss, but should be placed as indicated on the stress sheet, Plate I. If one ¾-inch rivet is taken out of the section of the bottom lateral bracing, it will give a net area of $1.62 - 0.27 = 1.35$ square inches; this could withstand a stress of $1.35 \times 15\,000 \times 1.25 = 27\,000$ pounds, which is the stress the bracing is assumed to carry, and which is to be used in determining the number of rivets for the connection. The stress in the top lateral bracing may be assumed to be the same.

Determination of Number of Rivets Required. It is to be remembered that ⅝-inch rivets are to be used in the 2½ and 2-inch legs of the angles, and ¾-inch rivets in all larger legs. Field rivets are to have a value equal to ⅘ of a shop rivet. Connection plates $\frac{5}{16}$ inch thick are to be used in all cases, except where the number of rivets required will be greater than 10. In such cases, use a ⅜-inch connection plate. The correct number of field rivets may be determined by multiplying the required number of shop rivets by $\frac{5}{4}$.

Whenever two angles back to back join on a plate, the number of rivets is governed by the bearing on the connection plate; and when one angle is joined to a plate, the number of rivets is governed by single shear if the rivet is ⅝ inch in diameter, and by single shear if the rivet is ¾ inch in diameter and the plate is over $\frac{5}{16}$ inch thick. The bearing and shearing value of the rivets are taken from Table X, p. 47.

Lower End of $L_0 U_1$: Rivets ¾-inch. Plate ⅜-inch.
$48\,800 \div 5\,630 = 9$ shop rivets required.
Upper End of $U_3 U_4$: Rivets ¾-inch. Plate ⅜-inch.
$40\,500 \div 5\,630 = 8$ shop or 10 field rivets
Upper End of $U_3 L_3$: Rivets ¾-inch. Plate $\frac{5}{16}$-inch.
$18\,725 \div 4\,690 = 4$ shop or 6 field rivets.

Lower End of L_2 L_3: Rivets ⅝-inch. Plate ¼-inch.
12 474 ÷ 3 130 = 4 shop rivets.
Each End of U_2 L_2: Rivets ⅝-inch. Plate ¼-inch.
11 150 ÷ 3 130 = 4 shop rivets.
Each End of L_1 U_2 *and* U_2 L_3: Rivets ⅝-inch. Plate ¼-inch.
6 240 ÷ 3 070 = 2 shop rivets.
Each End of U_1 L_1 *and* U_3 L_3: Rivets ⅝-inch. Plate ¼-inch.
5 580 ÷ 3 070 = 2 shop rivets.

Where U_1L_1 and U_3L_3 join the top chord, two rivets will be required in the top chord.

Since the components of the two diagonals meeting at U_2 are parallel and equal, and opposite to the stress in U_2 L_2, no rivets will be required, theoretically, to hold the plate to the top chord. A sufficient number, however, must be put in to take up the vertical reaction of the purlin. This number is 5 550 ÷ 3 130 = 2 shop rivets. In practice a greater number are usually put in to prevent vibration and to fill out the plate.

At L_3 a sufficient number of rivets must be placed in L_2 U_4 to take up the difference in stress between L_3 U_4 and L_2 L_3. The number required is (18 725 − 12 475) ÷ 3 130 = 3 shop rivets.

At the end L_0 of the member L_0 L_1, there is a horizontal stress of 43 700 pounds, and a vertical force equal to the reaction, which is 49 930 ÷ 2 = 24 965 pounds (see Fig. 76). The force acting on the rivets in this member is the resultant of these two forces, and is:

$$\sqrt{43\ 700^2 + 24\ 965^2} = 50\ 300 \text{ pounds.}$$

Since the rivets are ¾-inch and the plates ⅜-inch, the number of rivets required is 50 300 ÷ 5 630 = 9 shop rivets. This number should be placed symmetrically with respect to the intersection of the two chords. In case the point of application of the reaction had not coincided with the intersection of the chords, the number of rivets must be computed according to the formula on page 36.

For the joint at L_1, a sufficient number of rivets must be put in, in order to take up the difference in stress between the members L_0 L_1 and L_1 L_2. The number required is (43 700 − 37 450) ÷ 3 750 = 2 shop rivets.

The purlins have a horizontal shear at each end, of H ÷ 2 = 2 490 ÷ 2 = 1 245 pounds. This requires 1 245 ÷ 4 420 = 1 shop rivet or 1 field rivet, to keep them from sliding down on the top chord. Clip angles 5 by 3½ by ⅜-inch will be used as shown in Plate I.

These help in the erecting of the purlins, since they are shop-riveted to the truss and therefore hold the purlins in place while they are being field-riveted to the truss and to the clip angles (see Fig. 77).

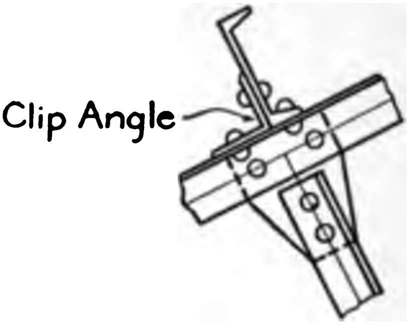


Fig. 77. Detail Showing Clip Angle Connection.

Rivets in Lateral Bracing. The plates of the lateral bracing should be made ⅜ inch thick. The 3-inch leg of the angle will be placed against the plate. Rivets ¾ inch in diameter can then be used, and the strength of the joint will be governed by bearing in the ⅜-inch plate. The stress for which the rivets are to be determined is given on p. 55. It is 27 000 pounds. The number of field rivets in bearing in ⅜-inch plate, required to withstand the stress, is (27 000 ÷ 4 420) × $\frac{3}{2}$ = 9. The size and shape of the plate can be determined only while making the detailed drawing (see Plate III, p. 61).

Design of the Splice. The general details of the splice will be as shown in Fig. 68. The plate underneath will be made ⅜ inch thick, the same thickness as the vertical connection plate at this point. Note that the member on the left-hand side of the splice must have ¾-inch shop rivets, and the member on the right-hand side must have ¾-inch field rivets. The total number of rivets on either side of the splice must be sufficient to take up the entire stress of the member through which they are driven. If eight ¾-inch field rivets are driven through the horizontal legs and the bottom splice plate, and five ¾-inch field rivets are driven through the vertical plate and legs of the angles (see Fig. 78), the total strength of the joint, remembering that the rivets are ¾-inch, will be:

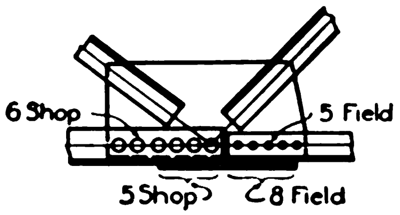


Fig 78. Detail of Lower Chord Splice.

$$8 \times \tfrac{2}{3} \times 3\,070 = 16\,370 \text{ pounds.}$$
$$5 \times \tfrac{2}{3} \times 3\,130 = 10\,430 \text{ pounds.}$$
$$\text{Total} \quad 26\,800 \text{ pounds.}$$

Note that the rivets through the bottom splice plates are governed by single shear; and those through the vertical plate, by bearing in the plate. Since 16 370 pounds is the value of the rivets through the bottom splice plate, this amount will be transmitted to the other side, where it must be taken up by shop rivets. Bearing in the plate governs the number of $\frac{3}{4}$-inch shop rivets required. This number is 16 370 ÷ 3 750 = 5. Since 16 370 pounds of the stress in the member $L_1 L_2$ is taken up by these 5 shop rivets, the remainder, 37 450 − 16 370 = 21 080 pounds, must be taken up by the rivets through the vertical connection plate. This requires 21 080 ÷ 3 750 = 6 shop rivets.

Since 16 370 pounds is transmitted from one side of the splice to the other by means of the bottom splice plate, this plate should be 16 370 ÷ 15 000 = 1.09 square inches in net section. The net width, the plate being $\frac{1}{4}$ inch thick, is 1.09 ÷ 0.25 = 4.36 inches. If two $\frac{3}{4}$-inch rivet-holes are taken out of the section, the entire width of the plate must be $4.36 + 2\left(\frac{3}{4} + \frac{1}{8}\right) = 6.11$, say 7 inches wide. The length of the plate must be sufficient to get in the number of rivets, and this length is determined in detailing.

Design of the Masonry Plate. If this truss rested upon a masonry wall, it would require a bearing of (49 930 ÷ 2) ÷ 250 = 100 square inches. The width of the plate cannot be less than twice the width of the legs of the bottom chord angle, nor should it extend outside the legs of the chord angle more than 3 inches on each side. The masonry plate will be assumed as 12 inches wide, in which case it must be 100 ÷ 12 = 8.34, say $8\frac{1}{2}$ inches long. The thickness should be $\frac{1}{2}$ inch.

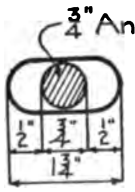


Fig. 79. Slotted Hole for Truss of Fig. 76.

Temperature Allowance. Slotted holes must be put in one end of the truss, to allow for a variation of 150 degrees in temperature. A common rule is to allow $\frac{1}{8}$-inch expansion for every ten feet of span. The total allowance for expansion is 64 × $\frac{1}{8}$ = say, 1 inch. Since the bolts which go through this hole are $\frac{3}{4}$ inch in diameter, the hole must be long enough to allow for $\frac{1}{2}$ the expansion on each side. The width of the hole should be $\frac{1}{4}$ inch greater than the diameter of the bolt (see Fig. 79).

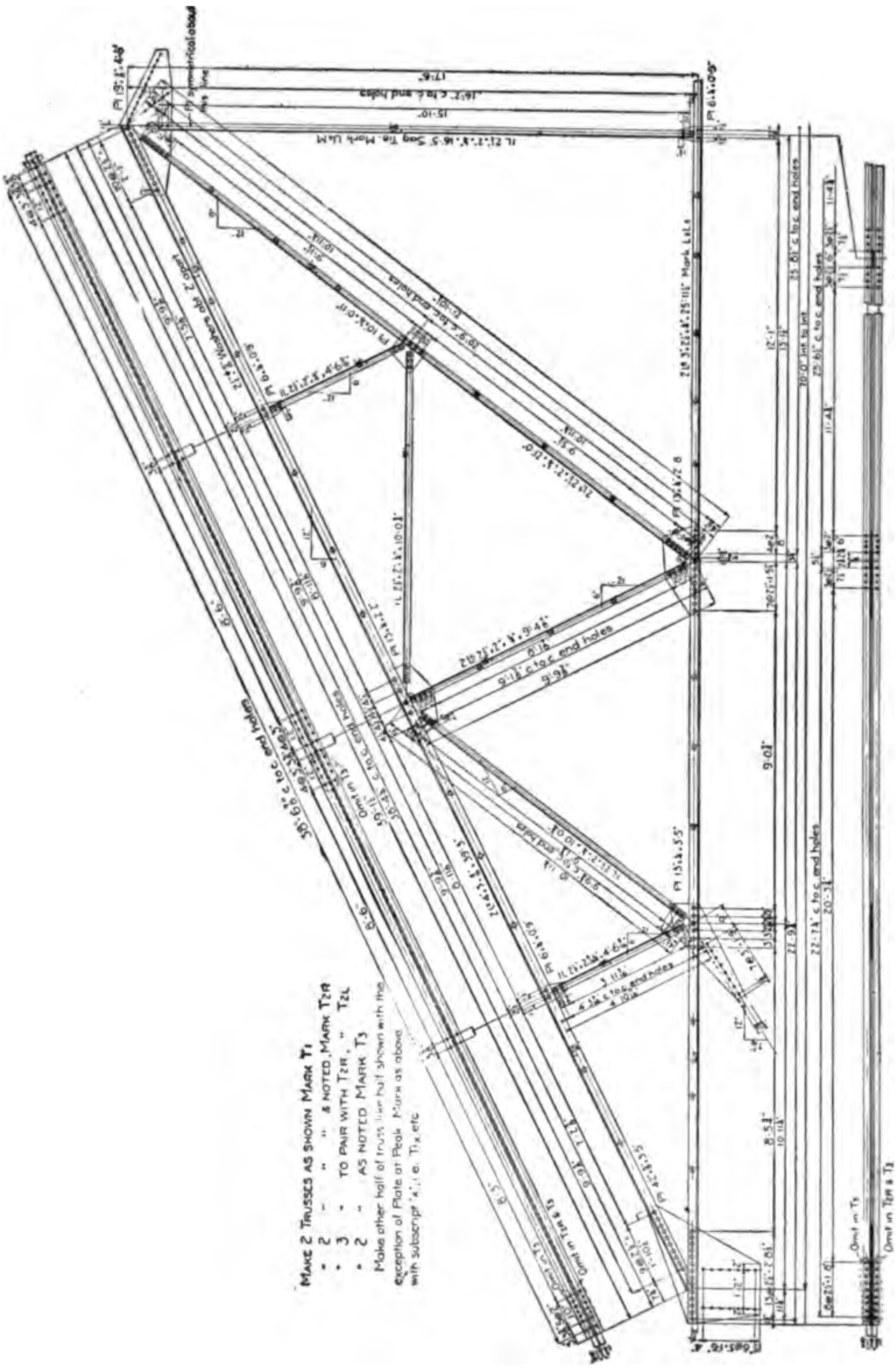
Connections to the Posts. If the truss rests upon posts at the end, sufficient rivets must be driven through the posts and the end connection plates to take up the end reaction, which (see page 38) is 24 970 pounds. Since the rivets are field rivets, this will require 24 970 ÷ 3 750 = 7. This number is to be used in case the posts are built in masonry walls. In case the truss has knee-braces and the walls of the building consist of steel framework, the reaction due to the wind must be added to the above.

15. **The Stress Sheet.** This should also be somewhat of a general drawing, showing the details. It should give an outline sketch of the building, showing bays, the distances between trusses, and the bracing in the plane of the top and bottom chords. See Plate I, p. 43, which is a stress sheet of the truss designed in Article 14. While not necessary, it is very convenient to have the required number of rivets noted upon the stress sheet.

16. **The Detail Drawing.** The stress sheet, in the matter of sizes, gives general dimensions only. It would be impossible for the shop men to make a truss from the stress sheet.

The shop or detailed drawings must be prepared by the draftsman. These drawings must show the exact number of rivets, and their positions, the dimensions of every plate, member, and purlin. The placing of the dimensions so that it will be unnecessary to add or subtract in order to get another desired dimension, is quite an art, and can be attained only through experience or from the study of correctly detailed work. Plates II and III give the shop drawings for a roof truss and the bracing. These are made according to the latest and best practice, and a thorough study of them will be a help to an intelligent design of the trusses.

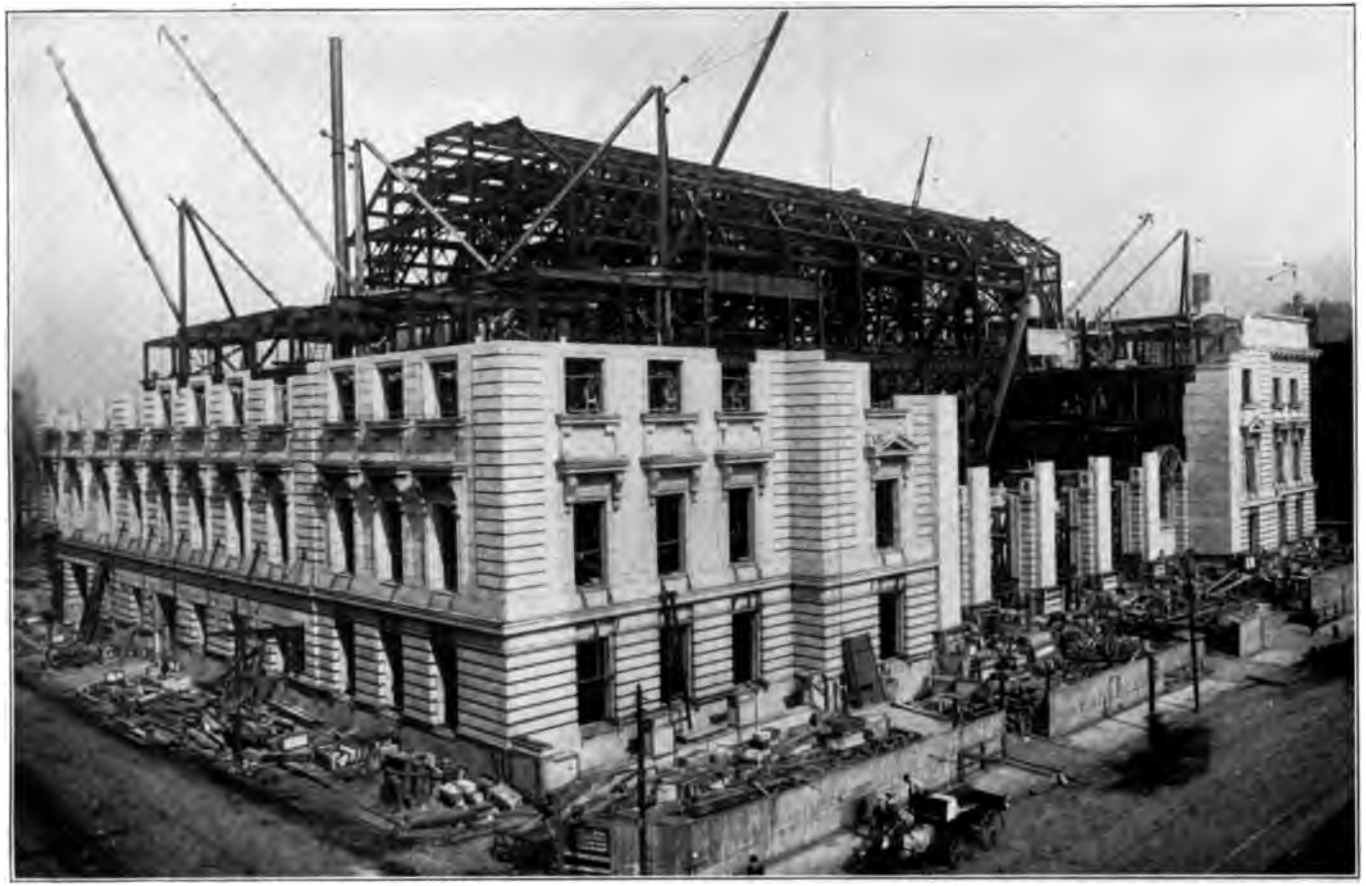
All members and plates which are to be riveted together in the field should be given a mark. This mark should be painted on the member or plate, and also marked on the *Marking* or *Erection Diagram* (see Plate IV). This diagram is a sketch, with the pieces in their proper position and the correct mark placed upon them. For example, if it is desired to rivet into place the first panel of the lower lateral system, the men look on the marking diagram and see that plates Pl_7, Pl_8, and Pl_9, and the laterals $BL1$, $BL2$, and $BL3$ are required. They would then go to the place where all the trusses are piled up, and pick out the plates and members with these marks upon them. They would then rivet Pl_7 at L_0, Pl_8 or Pl_9 at L_2, then $BL1$,



MAKE 2 TRUSSES AS SHOWN MARK T1
" 2 " " " & NOTED, MARK T2R
" 3 " TO PAIR WITH T2R, " T2L
" 2 " AS NOTED MARK T3

Make other half of truss like half shown with the exception of Plate at Peak. Mark as above with subscript 'x', i.e. T1x, etc.

Plate II. Working Shop Drawing of a Roof Truss.



CHICAGO AND NORTHWESTERN RAILWAY DEPOT IN PROCESS OF CONSTRUCTION

Fine Example of Fireproof Construction of Stone, Protected Steel, and Concrete.

Architect, Chas. S. Frost, Chicago. General Contractors, Geo. A. Fuller, Co.

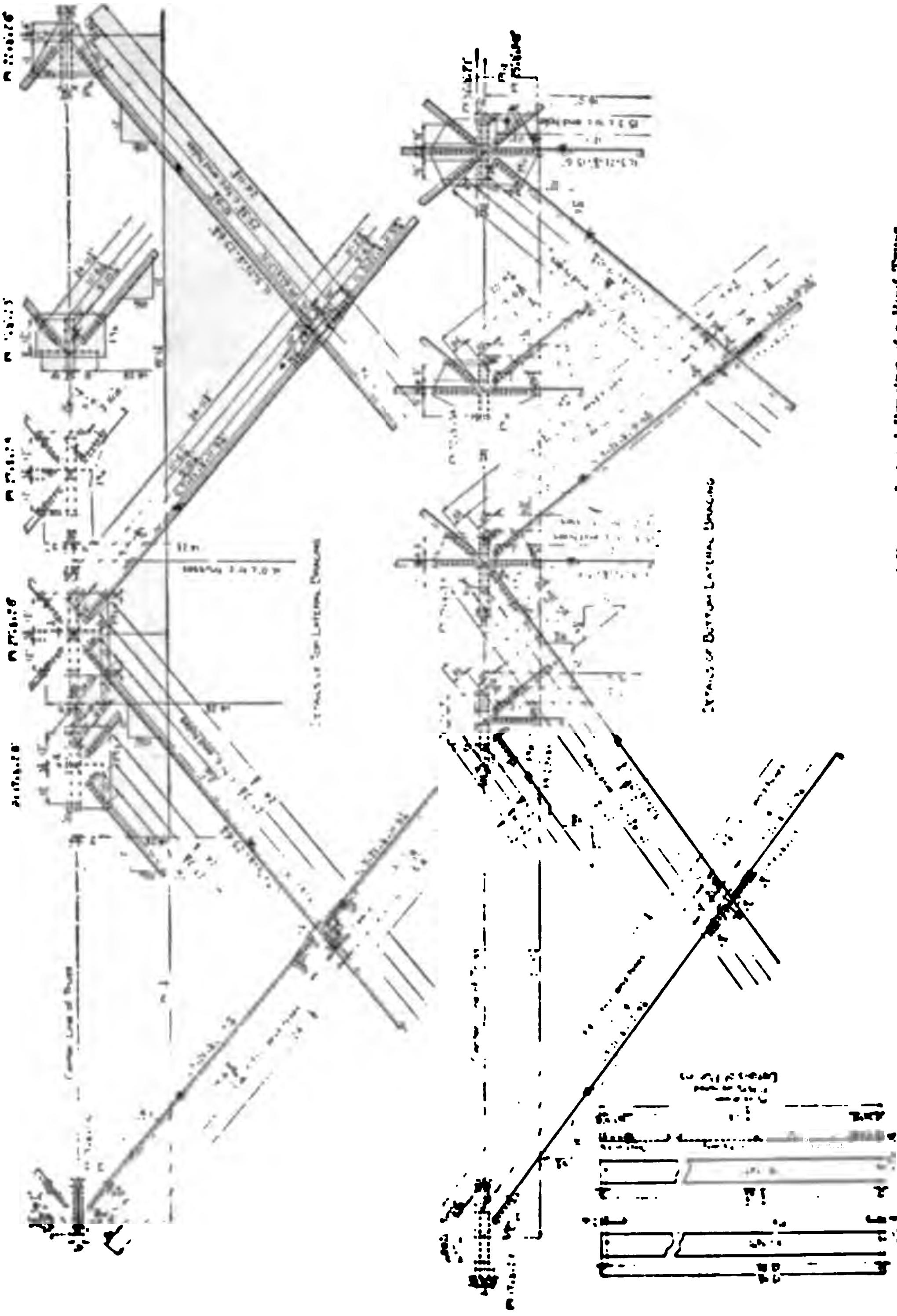

Details of Top Lateral Bracing
Details of Bottom Lateral Bracing

Plate III Working Shop Drawing Showing Detail of Top and Bottom Lateral Bracing of a Roof Truss.

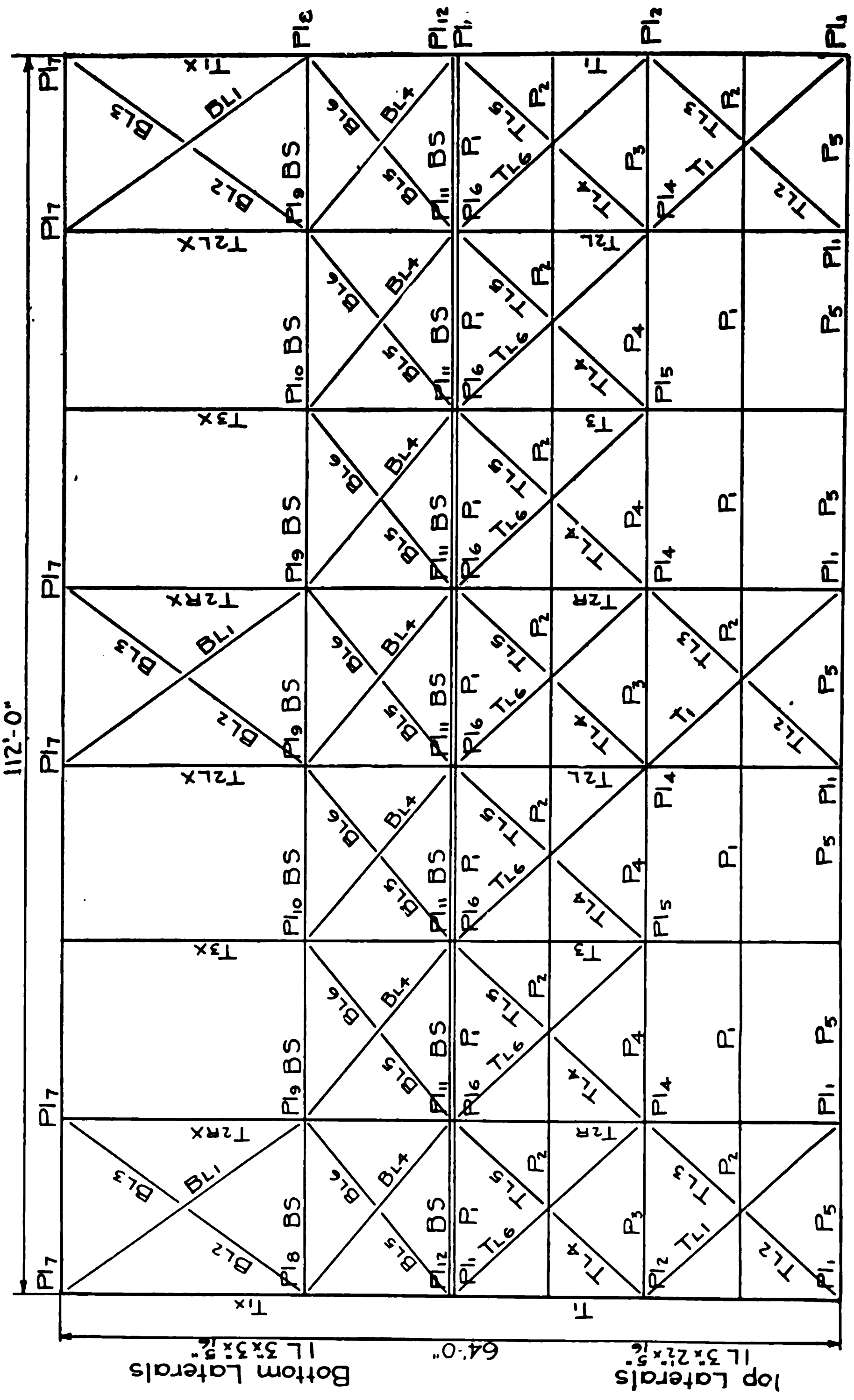


Plate IV. Marking or Erection Diagram.

then $BL3$, and finally $BL2$, all of which are shown on the Marking or Erection Diagram.

Care should be taken to give each piece that is different from others in any way whatsoever, a different mark. For instance, the purlins are the same size, and differ only in length and on account of the fact that one has holes in the bottom flange (see Plate III).

Plate IV gives the roof marking and erection diagram for the roof trusses of Plates II and III. Note that the roof truss on Plates II and III is not the same as that of which Plate I is a stress sheet.

17. **Estimate of Cost.** A rough estimate of the cost of steel in the roof may be obtained by multiplying the weight of the purlins, in pounds, by 2½ cents; then adding to this the result obtained by multiplying the weight of all the steel in the trusses and the bracing by 3½ cents. This will give the cost of the steel work in place with two coats of paint. This will give the cost closely enough for an Engineer's estimate; but should a Contractor desire to bid, a detailed estimate should be made as indicated in the remainder of this article.

The cost of the roof covering may be approximately determined according to the prices given in Article 5, but may be more accurately obtained by asking a Contractor for a figure which his experience will indicate as correct.

Paint of various kinds may be bought in open market. Table XII gives some of the kinds used in painting structural steel, together with the amount of surface one gallon will cover.

TABLE XII

Surface Covered per Gallon of Paint*

Paint	Square Feet	
	1 coat	2 coats
Iron Oxide (powdered)	600	350
" " (ground in oil)	630	375
Red Lead (powdered)	630	375
White Lead (ground in oil)	500	300
Graphite " " "	360	215
Black Asphalt	515	310
Linseed Oil	875	

*Pencoyd Handbook, 1898, p. [illegible]

One gallon of paint will give two tons of structural steel the first coat, or 2½ tons the second coat. The cost of one coat of paint in the shop is 45 cents, and two coats after erection $1.80 per ton of structural steel.*

The detailed estimate of the cost of steel includes several items which are given in Table XIII. In each case the weight of the steel on which the work is done must be multiplied by the unit-cost, and the sum total of all the costs will be the total cost of the entire roof or building. Table XIII gives the various operations which go to make up the cost, and also the unit-costs. Note that the costs vary considerably. This table is given as a rough guide. In order to analyze intelligently the cost in this manner, great experience or access to the cost records of some structural steel company is necessary.

TABLE XIII

Analysis of Cost of Roof Trusses and Mill Buildings†

Operation		Cost per Ton
Raw Material		$37.00 to $40.00
Work done at Rolling Mills (mill work)		3.00 " 7.00
Work done in Bridge Shops	Columns	14.00 " 20.00
	Trusses	12.00 " 25.00
	Girders	12.00 " 25.00
Work done in Drafting Room	Purlins	.30 " 1.00
	Trusses and Buildings	2.00 " 8.00
Painting		1.50 " 3.00
Shipping (depends upon freight rates)		....
Erection		5.00 " 15.00

It is not to be supposed that all of the operations indicated in Table XIII are made on one piece. Usually pieces which have mill work done on them require no shop work. In such cases a saving of freight is effected, since the material may be shipped directly from the mill to the place of erection.

MILL BUILDING

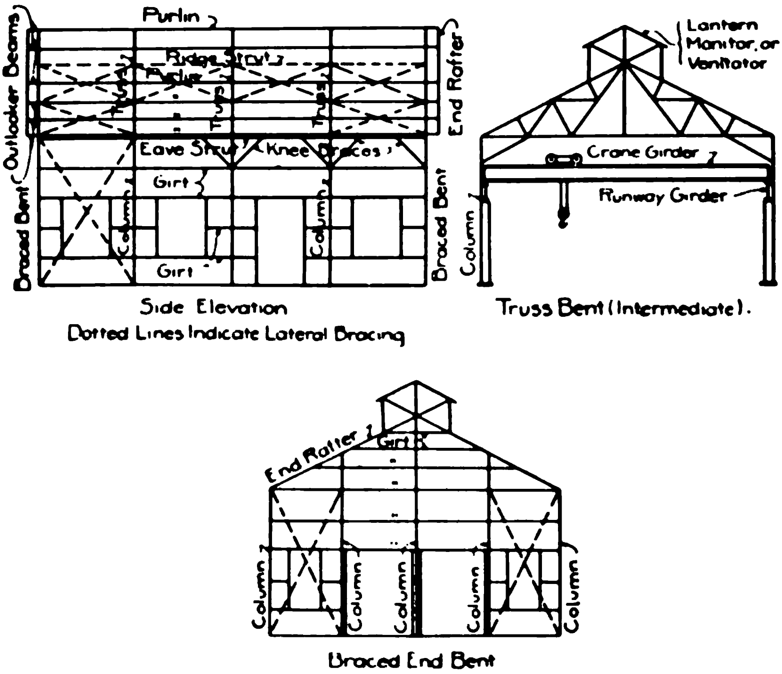
18. **Definitions and Description.** A *mill building* consists of a roof supported either on steel columns, on steel columns built in

---

*Pencoyd Handbook, 1898, p. 293.

†Compiled from Ketchum's "Steel Mill Buildings."

masonry walls, or on masonry walls alone. The roof may consist of any of the forms of roof trusses that have previously been mentioned; and the roof covering, which may rest on purlins, or on rafters and purlins, may consist of any of the roof coverings mentioned in Article 5. In case the roof is supported on steel columns, the columns are connected at their tops by a strut called the *eave-strut*; and they are also connected at certain distances throughout their height by horizontal members called *girts*. The building may or may not have a monitor ventilator on top. See Fig. 80 for general form of mill buildings, together with the names of the various parts.



Side Elevation
Dotted Lines Indicate Lateral Bracing

Truss Bent (Intermediate).

Braced End Bent

Fig. 80 Physical Analysis of a Mill Building

The eave-struts and the girts are used as a framework on which to place the covering for the walls of the building. This covering may be of wood, of wire lath and plaster, or of corrugated steel. The eave-strut may also act as the end purlin.

Since the majority of mill buildings have their roofs and sides covered with corrugated steel, the remainder of this text will be devoted to mill buildings with this kind of covering.

19. **Types of Buildings.** Mill buildings may be classified according to their width and the number of bays which they have. A building may consist of one center bay (see Fig. 81). In this case the span may vary from 30 to 100 feet. Usually side windows give sufficient light, no skylights being required in the roofs or monitor if the span is less than 80 feet.

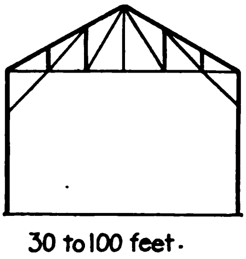


Fig. 81.

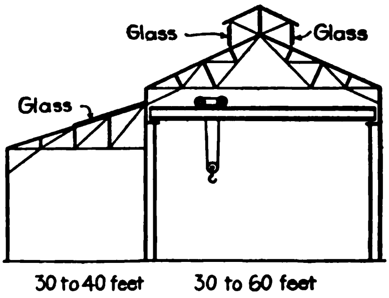


Fig. 82.

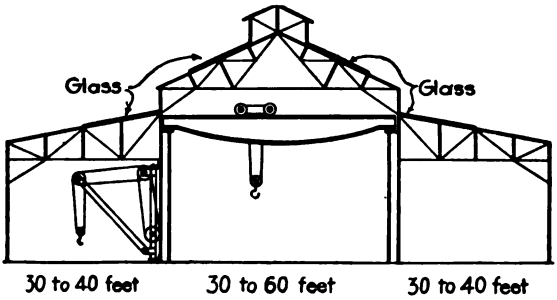


Fig. 83.

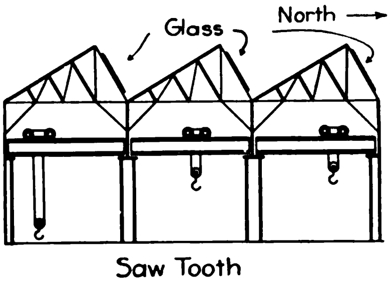


Fig. 84.

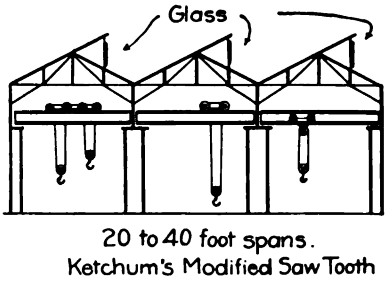


Fig. 85.

Cross-Sections of Mill Buildings.

The building may consist of one center bay and one or two side bays, as shown in Figs. 82 and 83. The truss of the center bay is usually of the Fink type, and in most cases is supplied with skylights and lights in the monitor. The side trusses for the most part consist of that type in which the chords are nearly parallel. The center bay is generally not more than 60 feet in span. This is due to the fact that the crane girder would be unnecessarily heavy if a longer span were used. The side spans are usually from 30 to 40 feet.

In case it is desirable to have the building wider than 150 feet, and still have it lighted by natural light, the common saw-tooth roof (Fig. 84) or Ketchum's modified saw-tooth (Fig. 85) is used. In such cases the bays are seldom greater than 40 feet. Cranes may be placed in one or all of the bays. One great advantage of this type of roof is that it gives a good light uniformly throughout the entire shop; and at any time it is desired to widen the shop, additional bays may be added. The shop may also be lengthened by adding additional trusses at the end. Of course, shops of the first two types mentioned may be widened by addition of extra bays; but the connection to the old work will be unsatisfactory, and skylights will have to be placed in the roofs both of the old bays and of the new ones. For views showing the interior of shops, see pages 77 to 84.

20. **General Requirements.** The general requirements of a mill building depend in detail on the purpose for which it is intended. The requirements which are common to all classes of buildings are ventilation, good light, and transportation facilities both inside and outside the shop. The question of light and ventilation is discussed on pp. 42 and 66. In regard to transportation facilities, it may be said that either the building should be placed so close to a railway track that the material may be unloaded by means of a crane and hauled into the shop, or the track must run into the shop so that the material may be unloaded and placed on the stock floor by means of a crane in the center bay and wall jib-cranes (see Fig. 109, p. 83) or by means of hand trucks.

21. **Layout.** The purpose for which the building is intended, and its relative location in regard to transportation facilities, will determine its layout. For manufacturing purposes, it should be so laid out that the materials will always pass *forward* in going from the raw material to the finished product. In general it may be said that the engines, machines (lathes, milling machines, drill presses, shears,

punches, etc.), and stock room, should be in the side bays; and the laying out, erection, and shipping floors should be placed in the center bay in the order mentioned. Fig. 86 gives a layout of a concern manufacturing frogs and switches.

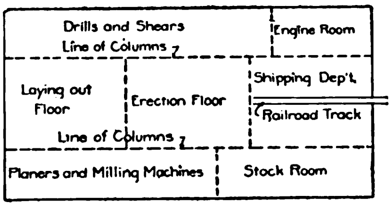


Fig. 86. Layout of a Frog and Switch Company's Building.

22. **Framing.** The framing of a mill building consists of the *roof framing*, which has been discussed in the preceding articles; the *columns*, which will be discussed in the next article; and the *girts* and *eave-struts*.

Eave-struts are a detail of cornice design. Various forms and methods of connections are shown in Article 29, p. 95, and the student is referred to this article.

Girts may be made of wood, angles, or channels. They should be designed for a pressure of from 20 to 30 pounds to the square foot on the side of the building. The spacing of the purlins depends upon the thickness of the corrugated steel used. On account of the fact that corrugated steel can be procured in lengths up to 10 feet and for spans of 5 feet, the stress per square inch due to 30 pounds per square foot is about 25 000 pounds. In No. 24 gauge corrugated steel, the spacing of the girts is limited to 5 feet or less.

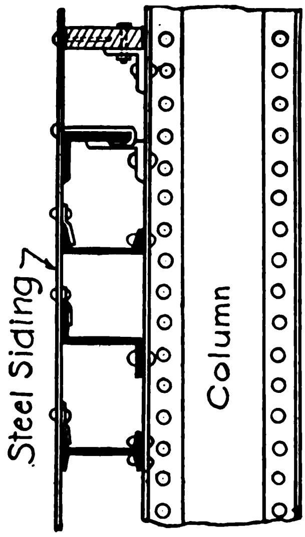


Fig. 87. Methods of Connecting Corrugated Steel to Girts.

Corrugated steel may be fastened to the girts by barbed roofing nails in case the girts are wood, or by clinch nails in case the girts are angles, or by clips fastened with rivets or $\frac{3}{16}$-inch stove bolts $\frac{3}{8}$ inch long. Nails and clinch nails should be spaced about 8 to 12 inches apart. Clips are made of No. 16 gauge steel from $\frac{5}{8}$ inch to $2\frac{1}{2}$ inches wide, and are spaced 8 to 12 inches apart. Fig. 87 shows girts, together with the method of attaching the corrugated steel. The number of nails, rivets, and stove bolts in a pound is to be found in the handbooks of various manufacturers.

Window-frames in mill buildings are, in general, similar to those placed in frame or brick buildings. These frames are fastened either directly to an iron framing or to wood nailing-pieces placed on the iron framing. The windows may be glazed in the usual fashion by means of putty, or may have the glass held in place by some of the methods shown in Fig. 73, p. 44. Windows in the side of the shop may be so fixed that they may be raised and lowered as the ordinary dwelling window; or they may slide horizontally; or, again, they may be fixed so that they cannot be moved. The windows in the monitors are usually fixed with a swinging sash which can be operated from the floor of the shop (see Fig. 89).

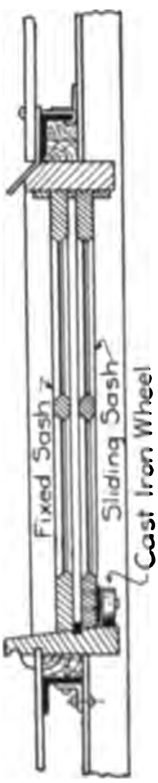


Fig. 88. Section of a Sliding Sash Window.

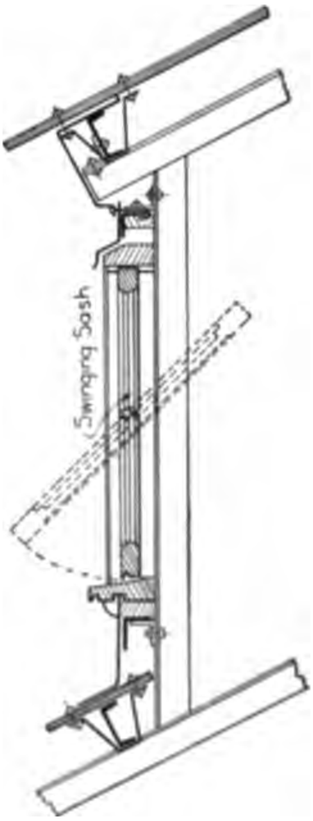


Fig. 89. Section of a Monitor Swing Window.

The glass in the windows may be the common window glass, common plate glass, ribbed or corrugated glass, wire glass, or prisms. Of these varieties, the prisms and the ribbed or corrugated are the best, since they give a more uniform light and are not easily broken. Wire glass, which is made of wire netting moulded in the middle of the sheet of glass, gives a very good light, and has the additional advantage that it does not crack and fall out under the action of fire and water. It is considered fireproof. Common window glass does not diffuse the light so well as most of the other glasses. It is very liable to fracture, and for this reason the inside of the window should be covered with wire netting. Prisms are made by the American Luxfer Prism Company, of Chicago. They may be obtained up to 84 inches in width and 36 inches in height. The width is parallel to the saw teeth. Figs. 88 and 89 give sections of windows, showing the framing. Attention is called to the fact that the roof on the monitor should overhang sufficiently to prevent the water from dropping upon the swinging window when it is fully opened.

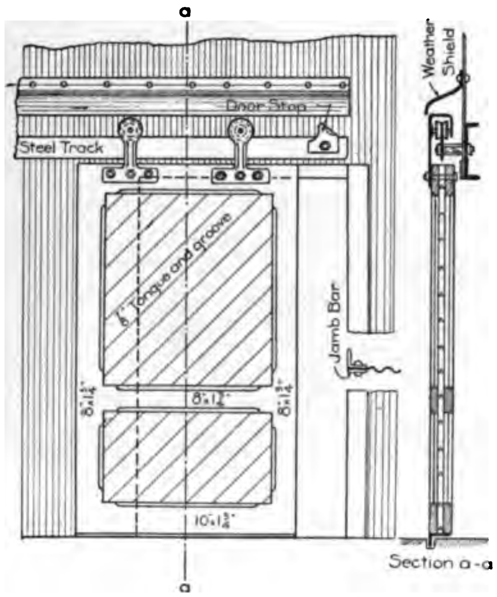


Fig. 90. Detail of a Wooden Door.

Doors may consist entirely of wood, of a frame of angles covered with corrugated steel, or of corrugated steel alone. The first two classes may be so fixed that they will slide, as in the folding doors of residences; open outward like a common door; lift vertically; or, in case they are made entirely of corrugated steel, roll up like a window-shade. This latter door is a patented one. Shop doors are seldom made to open outward or inward, on account of the space required—a space which can be devoted to better purposes. Figs. 90, 91, and 92 show details of the above doors.

Let it be required to design the girt when the trusses are 16 feet apart and the girts are 5 feet center to center. The moment is $\frac{5 \times 16 \times 30 \times 16 \times 12}{8} = 57\,600$ pound-inches; and the required section modulus is $\frac{57\,600}{15\,000} = 3.84$. By inspecting the tables in the Carnegie Handbook, pp. 97 to 119, it is found that the following shapes will be sufficient:

Shape	Section Modulus
One 5-inch 9.75-pound I-beam	4.80
One 6-inch 8.00-pound channel	4.30
One 4$\frac{1}{16}$ by 3¼-inch 10.3-pound zee-bar	3.91
One 6 by 4 by $\frac{7}{16}$-inch 14.3-pound angle	3.83

From this it is evident that the channel is the most efficient and economical.

23. **Columns.** Columns may consist of almost any combination of shapes, either latticed or connected by plates. Some of the most common cross-sections are shown in Fig. 93, those illustrated in *b* and *c* being used to a great extent. The advantage of these forms is that they give a small radius of gyration about the axis *b-b*, and a larger one about the axis *a-a* (see Fig. 94). This is especially desirable, since, in addition to the direct stress due to the weight of the cranes, roof truss, and covering, the column must withstand the moment due to the wind and to the eccentricity of the runway girder. Both of these moments tend to bend the column around the axis *a-a*. The bending moment due to the eccentricity of the runway girder is equal to the reaction of the girder, *times* the distance from the center of the column (see Figs. 95 and 96). In case the details of the column are as given in Fig. 96, the direct load due to the reaction of the truss and its covering produces a moment due to its eccentricity. This moment is $R_1 \times e_1$. Since R_1 acts on the opposite side of the center of the column from the point of action of R_e, it tends to counteract the effect of the moment due to the eccentricity of the runway girder. The total moment due to eccentricity is $M_e = R_1 \times e_1 - R_e \times e$. If

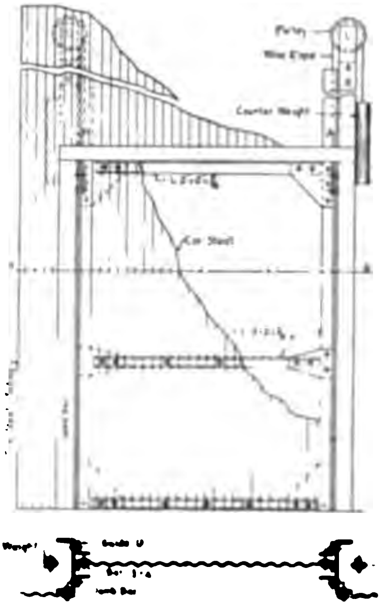


Fig 91 Detail of Vertical Hoist Door.

the first term of this equation is less than the last term, the compressive stress on the side of the column with the runway girder is increased, and *vice versâ*. The stress in the column from the runway girder to the roof is that due only to the vertical reaction of the roof and the bending due to the wind. In that part of the column below the crane girder, the stress is that due to the direct action of the weight of the roof; its eccentricity, if there be any; the direct action and eccentricity of the runway girder; and the bending moment due to the wind. The bending moment due to the wind is less in this part of the column than it is at the foot of the knee-bracing, but it is customary to consider it the same.

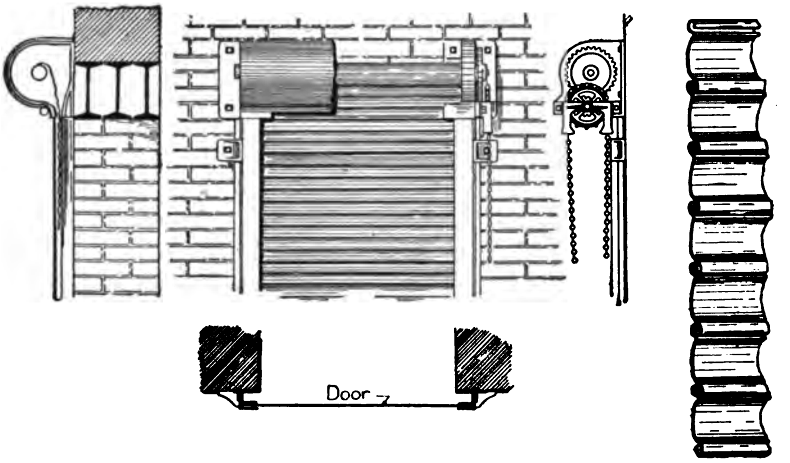


Fig. 92. Detail of Rolling Door of Corrugated Steel.
Courtesy of Kinnear Mfg. Co., Columbus, Ohio.

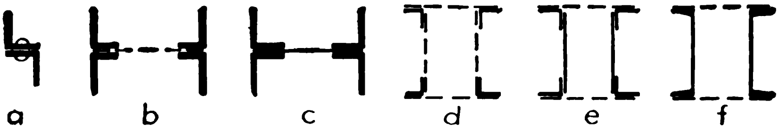


Fig. 93. Cross-Sections of Columns.

In order to prevent eccentric stresses due to the reaction of the runway girder, an extra column to carry the crane girder is placed alongside the roof column (see Figs. 100 and 117, pp. 75 and 88). This is much used by A. F. Robinson, Bridge Engineer of the Atchison, Topeka & Santa Fé Railroad System, who claims it to be a very

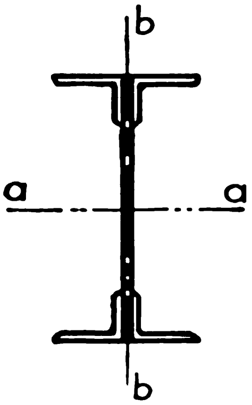


Fig. 94. Illustrating the Two Radii of Gyration.

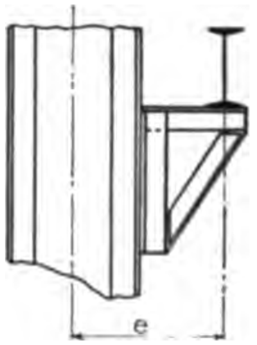


Fig. 95. Runway-Girder Eccentricity.

economical and efficient detail. One advantage of this is, that, if at any time it is desirable to use a heavier crane, this column can be removed and a stronger one put in its place, without in any way affecting the remainder of the building.

In order to illustrate the design of a column, let it be required to design a column with detail as shown in Fig. 95, the height being 20 feet, the distance of the runway girder from the face of the column 8 inches, the direct stress 15 600 pounds, and the bending moment due to the wind 924 000 pound-inches. The reaction of the runway girder is 20 000 pounds. The stress due to the bending moment caused by the wind and the eccentricity of the runway girder must be worked out by formula 8, "Strength of Materials," p. 86; and to this must be added the direct stress caused by the weight of the roof and the crane-girder reaction.

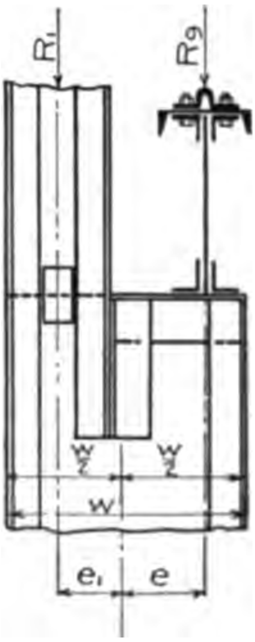


Fig. 96. Runway Girder and Roof Truss Eccentricity.

Since, according to Article 13, the unit-stress must be reduced one-half in determining the section to withstand stresses due to crane loads, the moment due to the crane loads and also its direct action must be

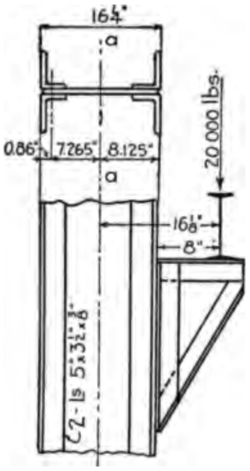


Fig. 97. Illustrating Problem on Page 73.

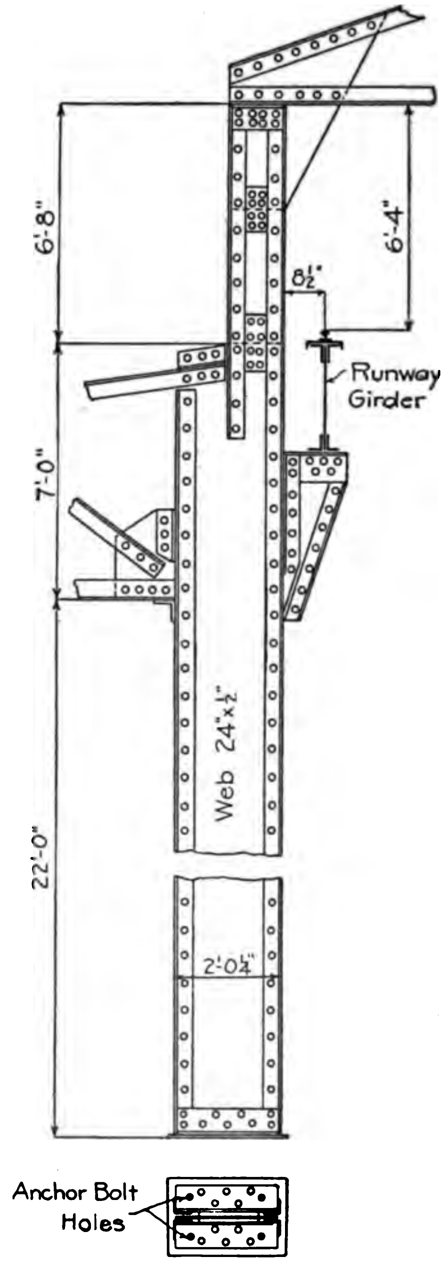


Fig. 98. Detail of Column.

multiplied by 2 in order that the same formula for the unit-stress may be used throughout in the design of the column.

Let four 5 by 3½ by ⅜-inch angles with a 16 by ⅜-inch web plate be assumed, and placed as shown in Fig. 97. These angles have an area of 3.05 square inches each; and a moment of inertia parallel to the long leg, of 3 18. Then (see "Strength of Materials," pp. 48–53), the moment due to crane reaction is 20 000 × (8.125 + 8) = 322 500 pound-inches. Accordingly, in using this in the formula, it will be 2 × 322 500 = 645 000 pound-inches; and this, added to the 924 000 pound-inches due to the wind, will make a total bending moment of 1,569,000 pound-inches.

$$I_{a-a} = 4 \times 3.18 + 4 \times 3.05 \times 7.265^2 + \frac{\frac{3}{8} \times 16^3}{12} = 784.72,$$

$$r_{a-a} = \sqrt{\frac{784.72}{4 \times 3.05 + \frac{3}{8} \times 16}} = 6.56$$

The allowable unit-stress is:

$$P = 24\,000 - 110 \times \frac{20 \times 12}{6.56} = 19\,975 \text{ pounds per square inch.}$$

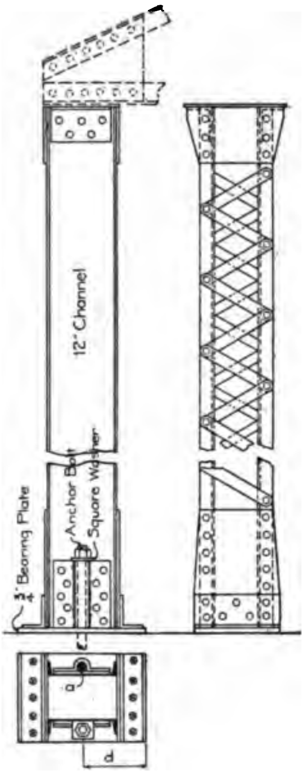


Fig. 99. Detail of Column.

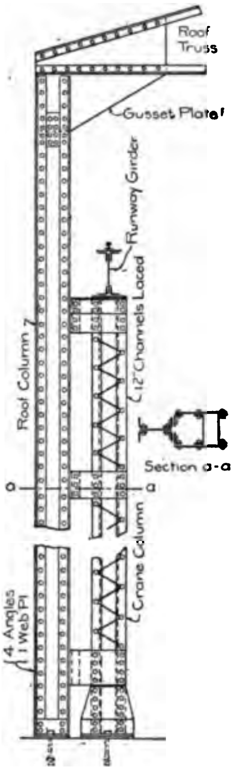


Fig. 100. Detail of Column.

The actual unit-stress (see "Strength of Materials," p. 86), is:

$$S = \frac{15\,600 + 2 \times 20\,000}{4 \times 3.05 + \frac{3}{8} \times 16} + \frac{1\,569\,000 \times \frac{16.25}{2}}{784.72 - \frac{(15\,600 + 2 \times 20\,000)(20 \times 12)^2}{10 \times 28\,000\,000}}$$

$$= 3\,024 + 16\,420$$

$$= 19\,444 \text{ pounds per square inch.}$$

Since this is slightly less than the allowable stress, 19 975 pounds per square inch, this section is the correct one.

Details of columns are shown in Figs. 98, 99, and 100. In case the column is considered fixed at its base, the base of the column is usually made as shown in Fig. 99. Long bolts deeply imbedded in the masonry are run up through the holes *a*; a heavy washer is placed over the bolt, and the nut screwed down tightly. Each bolt must be designed to withstand a stress of $H_2 \times n \div 2d$ (see Figs. 43 and 99).

24. **Knee-Braces.** The determination of the stresses in knee-bracing has been made in Article 9. Knee-braces consist of two angles placed back to back, and are joined to the column and roof truss as shown in Fig. 101 and in the figures showing cranes. They must be designed to withstand the greatest compressive stress; and must also be examined to see if they are safe in tension, since they are under either tensile or compressive stresses according to the direction in which the wind blows.

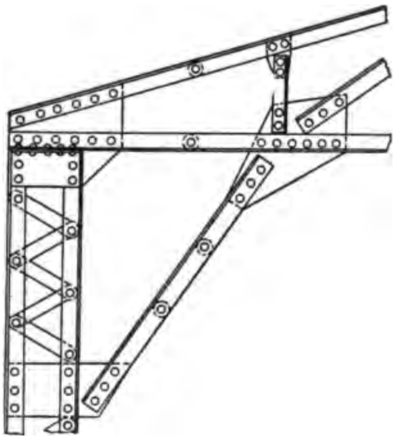
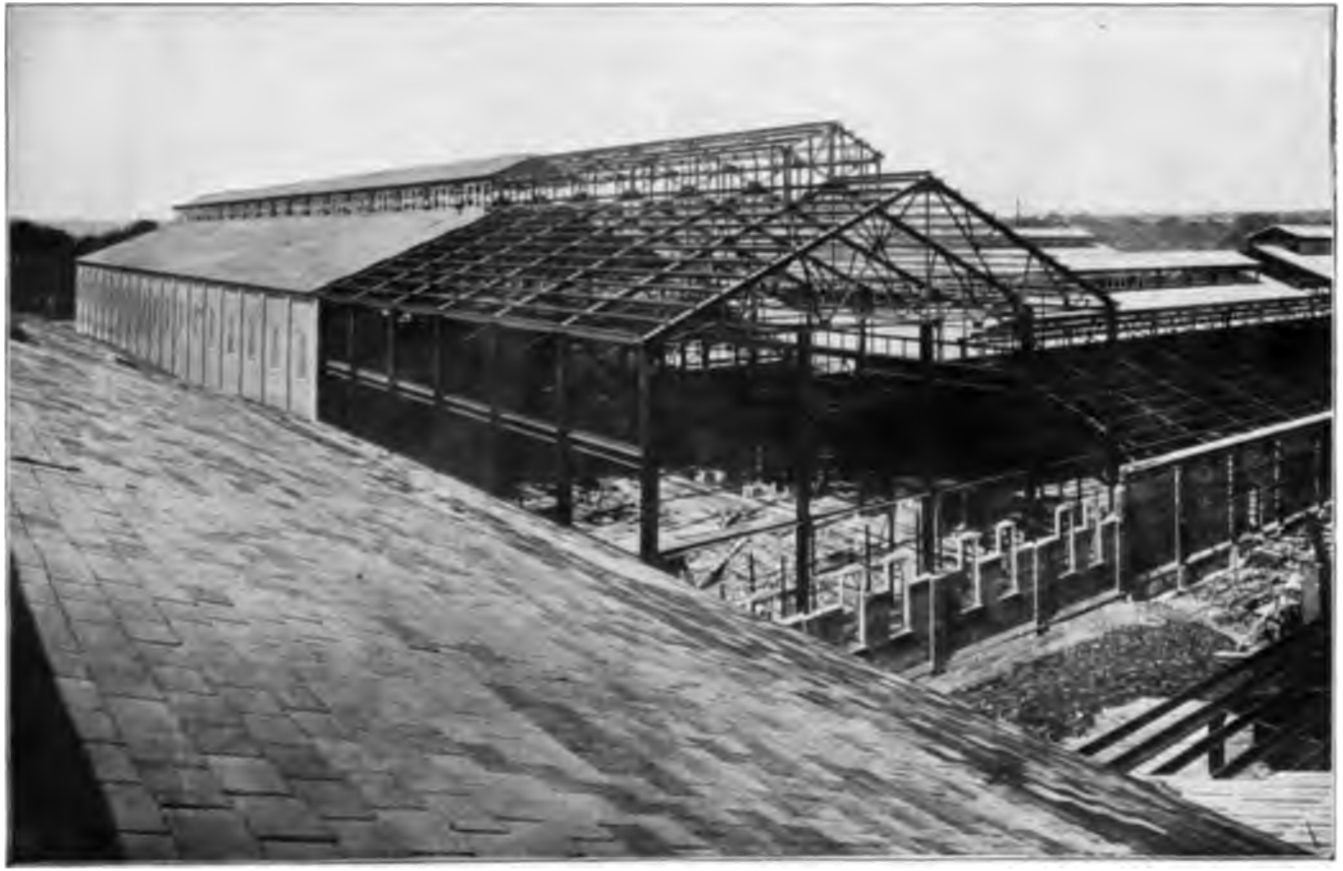


Fig. 101. Detail of Knee-Brace and Connections.

The knee-brace for the truss-bent of Article 9 will now be designed. The maximum compressive stress is 21 440 pounds. The radius of gyration must be at least 131 ÷ 120 = 1.09. Two angles 3½ by 3 by $\frac{5}{16}$-inch, placed back to back with their longer legs ¼ inch apart, will be assumed, since they are the smallest size to be used with r = 1.09 or greater. The radius of gyration about an axis perpendicular to the longer legs is 1.10; and the allowable unit-stress is $P = 24\,000 - \frac{110 \times 131}{110} = 10\,900$ pounds per square inch, the length being 131 inches. The required area is $\frac{21\,440}{10\,900}$ = 1.97 square inches. Since this is less than the given area, and since the angles are the smallest allowed, these angles are sufficient. The maximum tensile stress is 8 640 pounds, and the required net area is 8 640 ÷ 15 000 = 0.58 square inch. The net area given



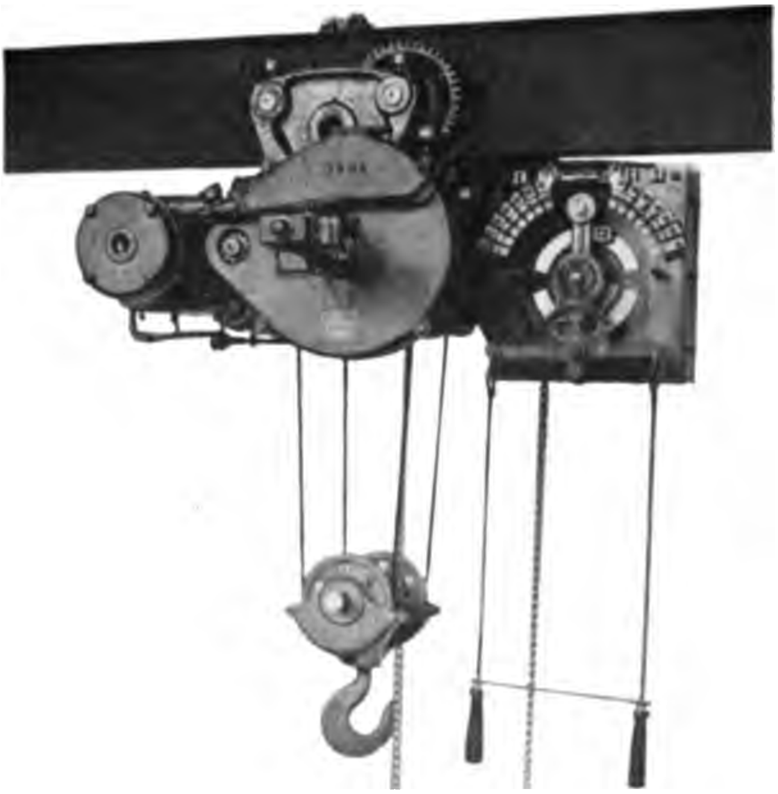
BUILDING FOR PITTSBURGH PLATE GLASS COMPANY, CRYSTAL CITY, MISSOURI

Grinder and Polisher Building, 408 feet by 532 feet, laying yard building 100 feet by 578 feet, Lehr building, 68 feet by 475 feet and 114 feet by 171 feet; stripper building 90 feet by 408 feet, warehouse, 153 feet by 308 feet and 90 feet by 173 feet. Total weight of steel work, 6,280,000 pounds.

Courtesy of American Bridge Company

by the angles is 3.86 − 0.55 = 3.31 square inches, two ¾-inch rivet-holes being taken out. This shows the angles to be amply safe in tension, and they will therefore be used for the section of the knee-braces.

25. **Runway Girders.** The runway girders extend from column to column on each side of the bay in which the girder runs. An



[illegible]

inspection of the figures of this article will give a clear idea of their position and their details. Along these girders run the wheels which support the end of the crane. The crane may be a small hoist, as indicated in Figs. 102, 103, and 104, in which case the crane girder consists of a simple I-beam supported by two wheels at the ends, and these are placed close together. In other cases the crane consists of

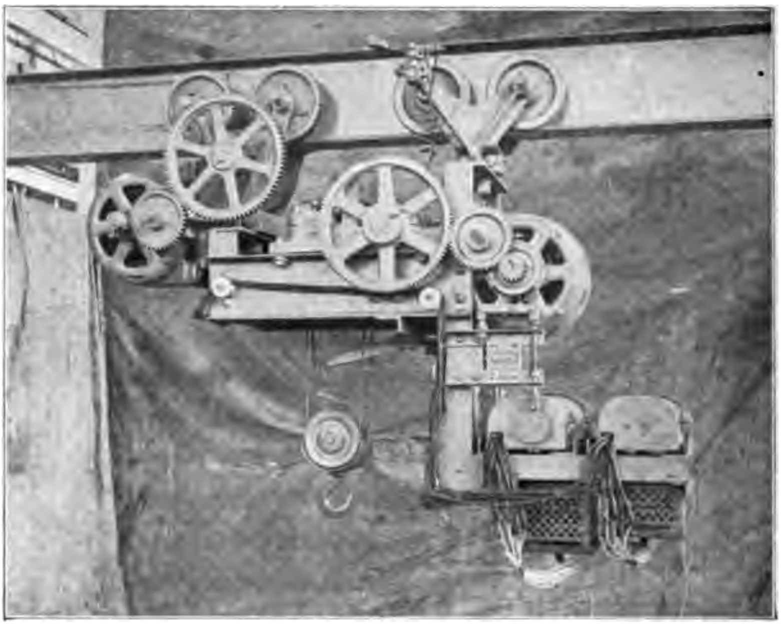


Fig. 103. Electric I-Beam Hoist.
Courtesy of Maris Brothers, Philadelphia, Pennsylvania.

two girders placed side by side, upon which runs the carriage carrying the hoist. This type of crane is supported upon four to eight wheels (see Figs. 105, 106, 107, 108, 109, and 110).

The maximum bending moment and shear in a runway girder

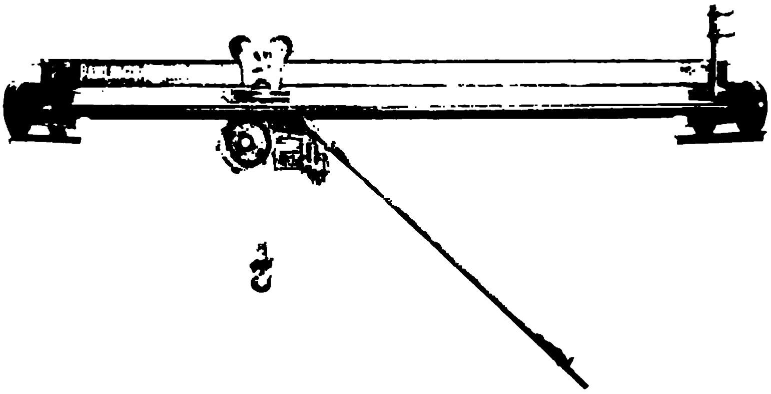


Fig. 104. Small Electric I-Beam Hoist: Capacity 500 Lbs.
Courtesy of Pawling & Harnischfeger, Milwaukee, Wis.

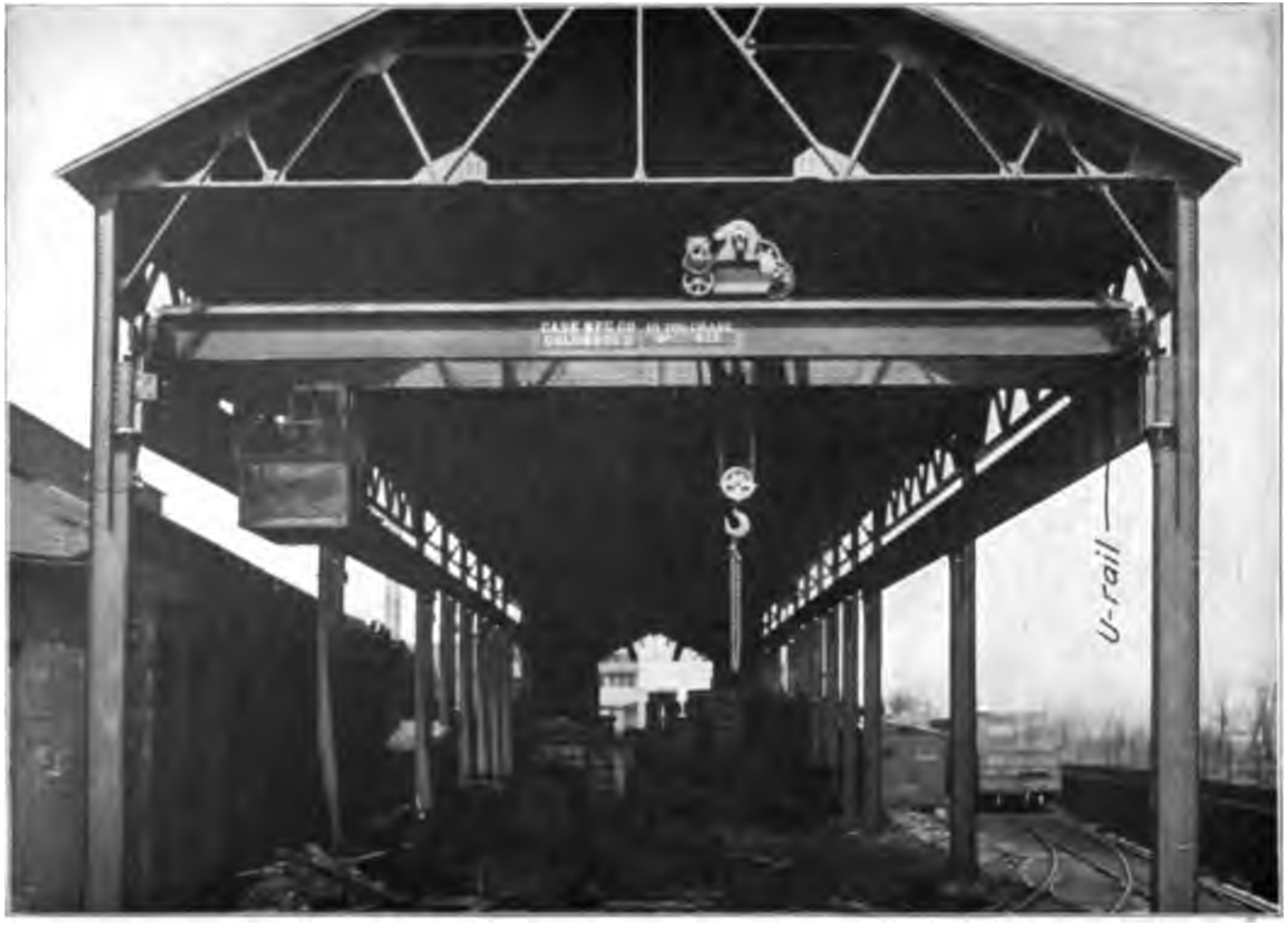


Fig 105 Ten Ton Three-Motor Electric Traveling Crane.
Courtesy of Case Manufacturing Company, Columbus, Ohio.

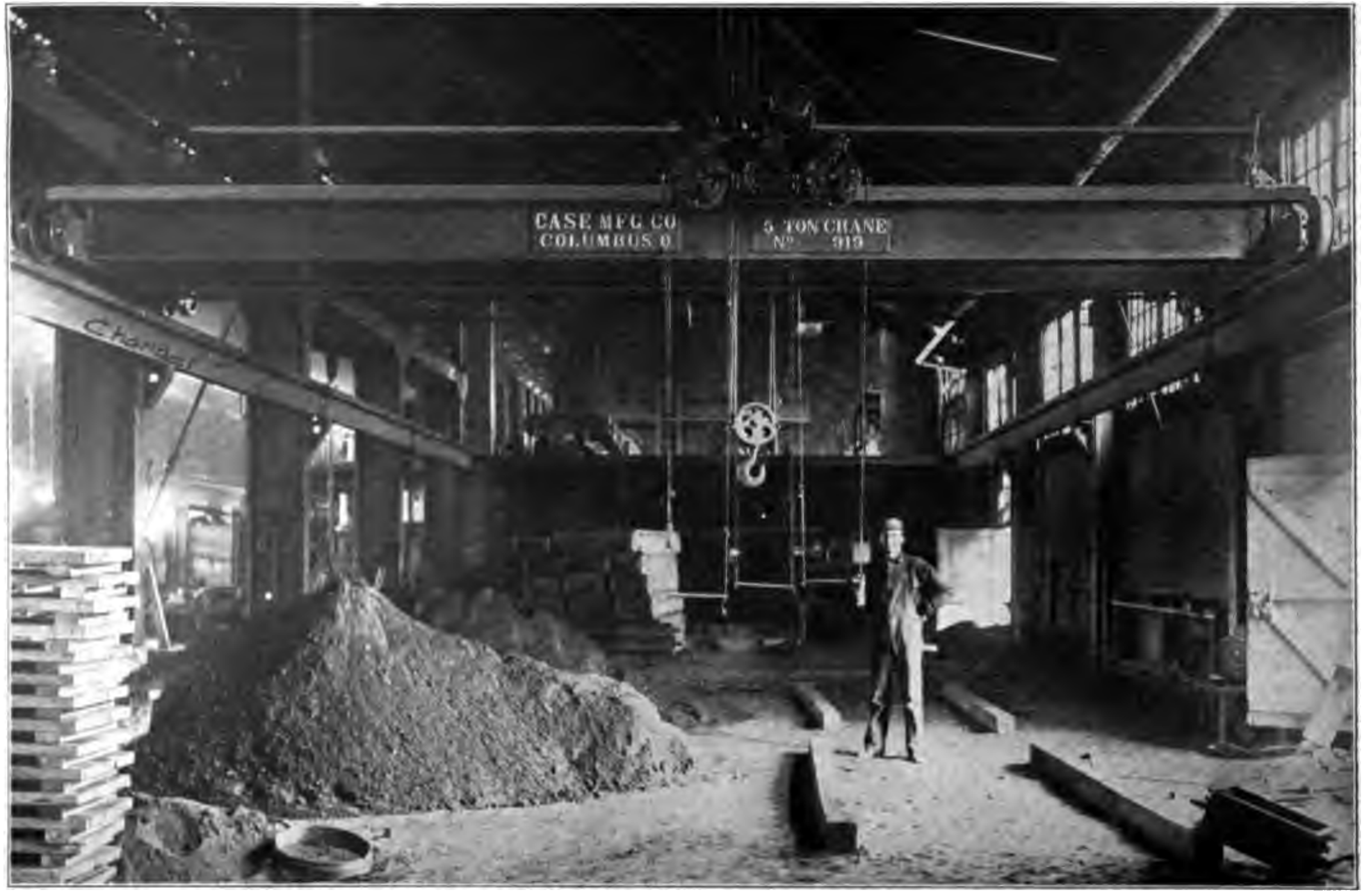


Fig [illegible]. Five-Ton Three Motor Electric Traveling Crane, Operated from Floor.
Courtesy of Case Manufacturing Company, Columbus, Ohio.

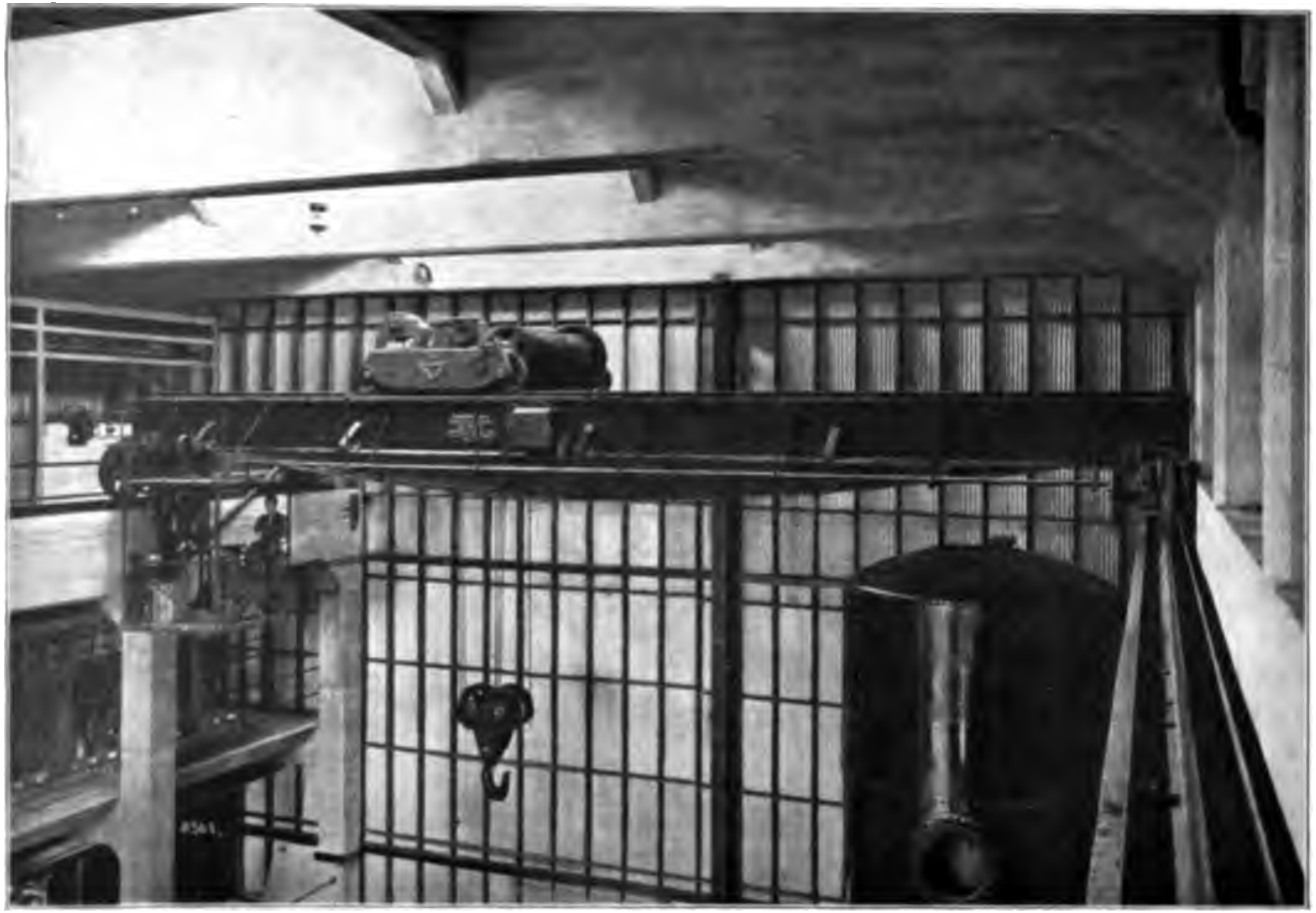


Fig 107. Fifty Ton Three-Motor Electric Travelling Crane.
Courtesy of Northern Engineering Works, Detroit, Mich.

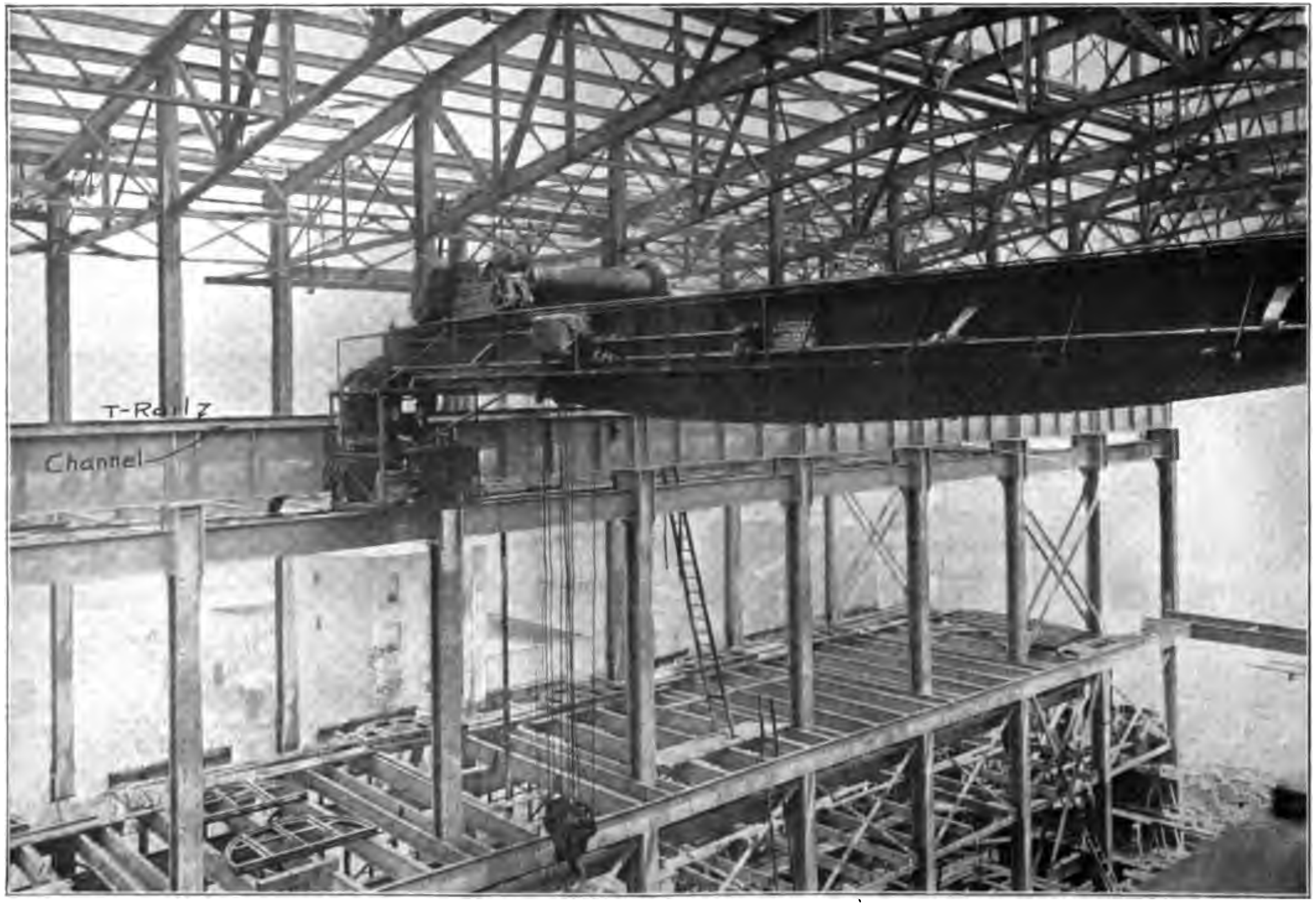


Fig. 108. Fifty Ton Four Motor Electric Traveling Crane.
Courtesy of Northern Engineering Works, Detroit, Mich.

will depend upon the capacity and span of the crane, and also upon the distance apart of the wheels at its ends. Where the bending moment is not too great, the runway girders may be composed of channels or I-beams (see Figs. 103, 104, and 106). In case the moment is too great to make the use of these possible, the runway girders are composed of plate-girders (see Figs. 105, 107 to 110)

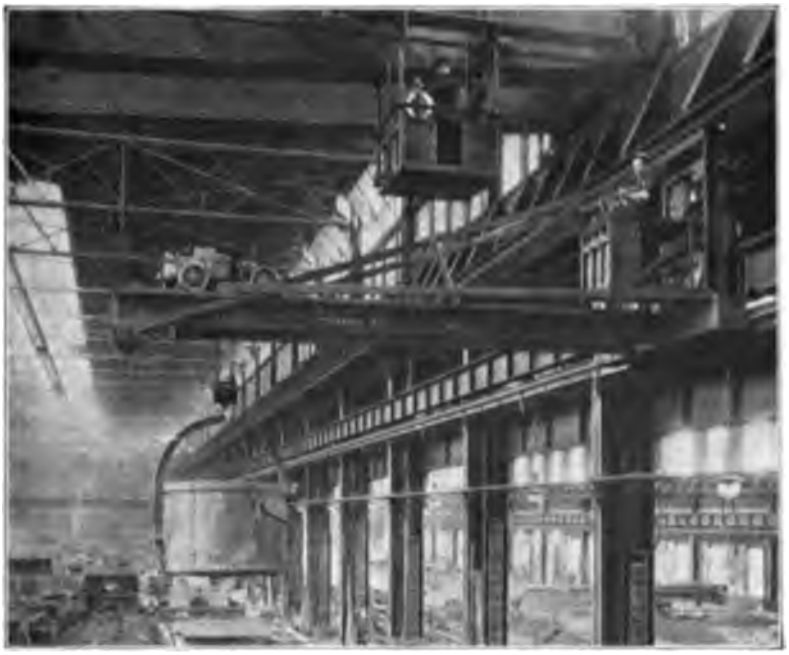


Fig. 108. Wall Jib Crane, Electrically Operated. Capacity, 10,000 Lbs.
Courtesy of Pawling & Harnischfeger, Milwaukee, Wis.

Plate-girders consist of a flat plate called a *web plate*, which has riveted to it at its upper and lower edges two angles, or two angles and one plate, called the *cover-plate*. The angles are called *flange angles*; and the two angles together, and the cover-plate when used, are called the *flanges*. At certain distances along its length, equal to or less than its depth, vertical angles are riveted on opposite sides of the web plate. These are called *stiffeners*, their function being to stiffen the web under the action of the shear. See Fig. 111 for a general view of a plate-girder, together with the names of the various parts.

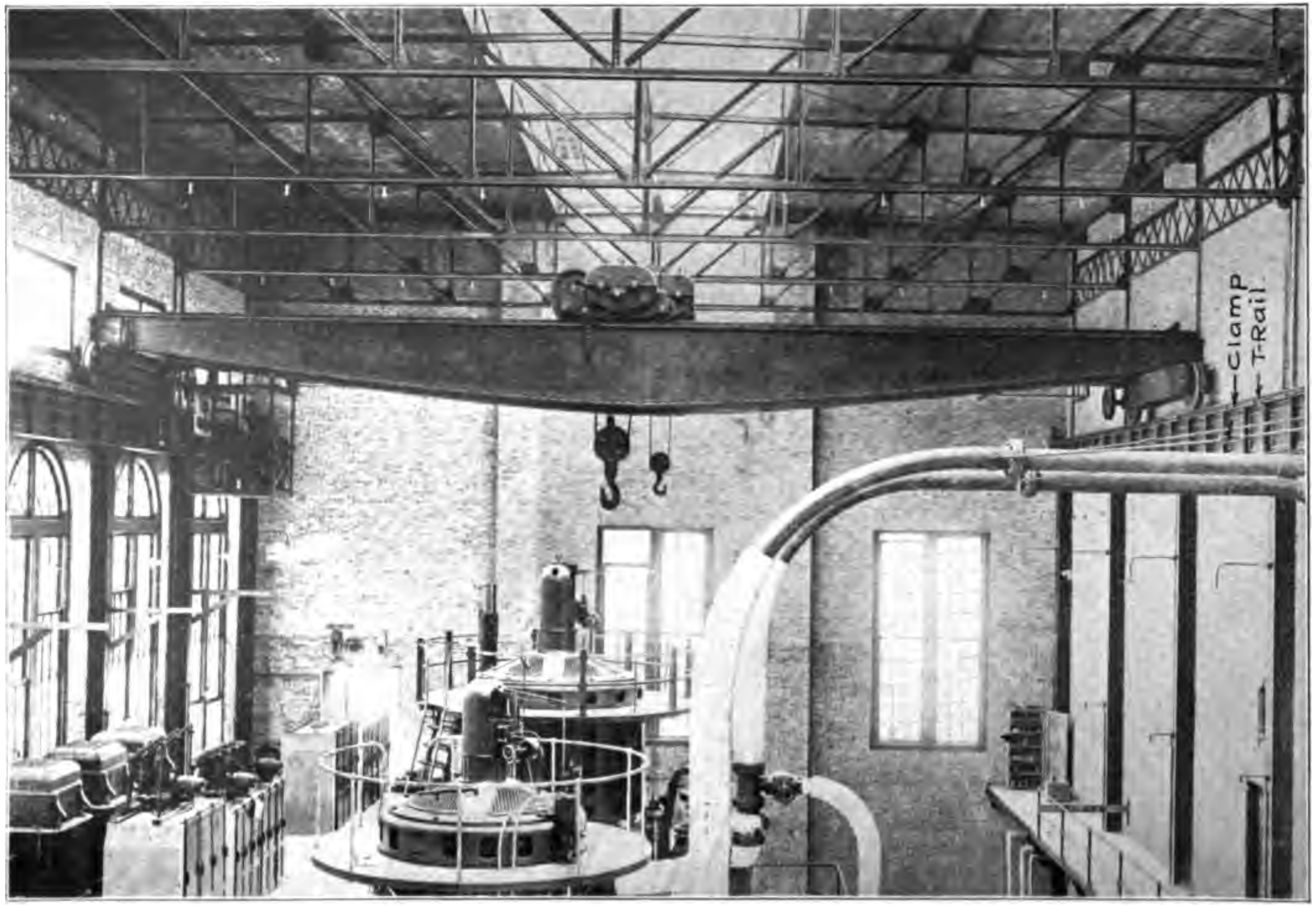


Fig 110 Thirty Ton Four Motor Electric Traveling Crane in Power Station.
Courtesy of Northern Engineering Works, Detroit, Mich.

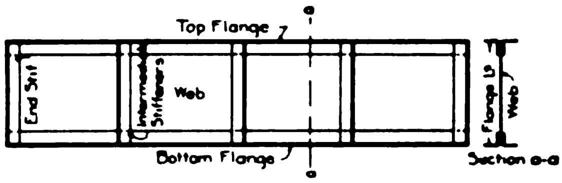


Fig. 111. Plate-Girder Notation.

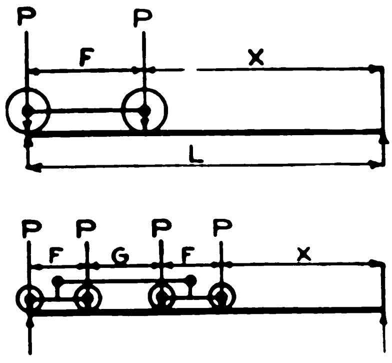


Fig. 112. Position of Crane Truck for Maximum End Shear of Runway Girder

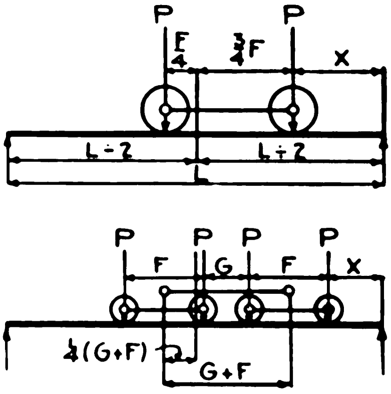


Fig. 113. Position of Crane Truck for Maximum Moment in Runway Girder

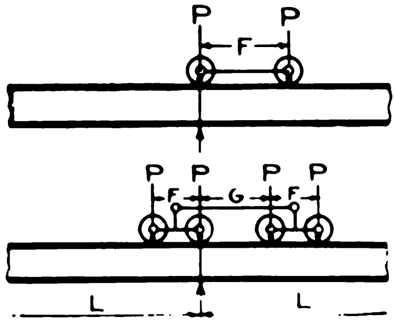


Fig. 114. Position of Crane Truck for Maximum Reaction on Column

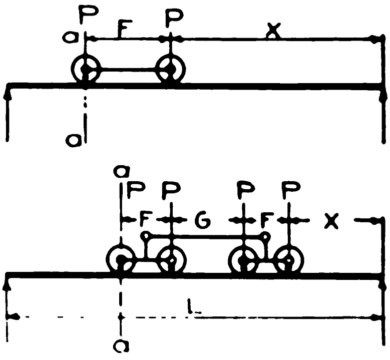
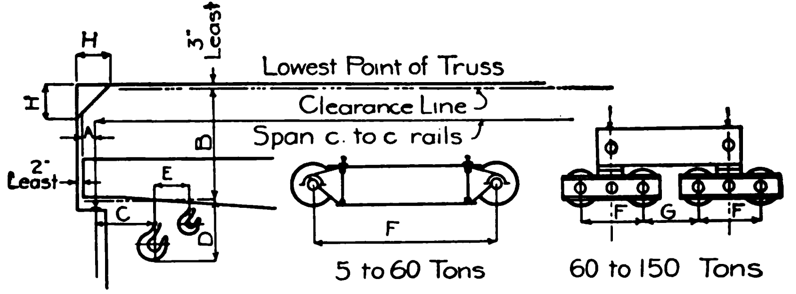


Fig. 115. Position of Crane Truck for Maximum Shear at any Section

The maximum shear in the runway girder will occur when the crane wheels are in the position shown in Fig. 112; and the maximum moment will occur under the wheel nearest the middle of the span, when the wheels are in the position shown in Fig. 113. The maximum reaction of the runway girders on the column will occur when the wheels are in the position shown in Fig. 114. In order to

TABLE XIV

Maximum Crane Reaction



	Capacity (tons)	Span (feet)	A	B	C	D	E	F	G	Wheel Load (in lbs.)	Tram Rail Am. Std.	H
			in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.			ft. in.
SINGLE DRUM. TYPE C	5	40	7	4- 5½	1- 8 2- 0	7		7- 6½		10 000	3½-1⅞- 40	1- 5
		60	8	4- 7½	1-11 2- 1	5		9- 0		12 500		
		80	8	4- 8½	1-11 2- 2	4		9- 3½		15 500		
	10	40	8	5- 0½	1- 0 2- 4	9		8- 5½		16 500	3½-1⅞- 40	1- 0
		60	8	4-11¼	1-11½ 2- 4	10		9- 7¼		19 500		
		80	8	5- 0½	1-11½ 2- 4½	9		9- 9½		23 000		
	15	40	8	5- 4¼	2- 0 2- 9	1-3½	2- 0	9- 6		24 500	3⅞-2⅛- 50	1- 3
		60	8	5- 4¼	2- 0 2- 9	1-3½	2- 0	10- 7½		28 000		
		80	8	5- 5½	2- 0 2- 9	1-2¼	2- 0	10- 9½		31 500		
	20	40	8	5- 5½	2- 0 2- 9	1-6¼	2- 0	9- 6		30 000	3⅞-2⅛- 50	1- 3
		60	8	5- 5½	2- 0 2- 9	1-6¼	2- 0	10- 7½		34 000		
		80	9	5- 6½	2- 0½ 2- 9	1-5¼	2- 0	11- 1½		38 000		
DOUBLE DRUM. TYPE A	25	40	9	6- 0¼	2- 6	3-9¼	2- 7½	11- 0½		38 600	4¼-2⅜- 60	1- 3
		60	9	6- 0¼	2- 6	3-9¼	2- 7½	11- 3½		42 400		
		80	10	6- 2¼	2- 6	3-7¼	2- 7½	11- 8½		47 000		
	30	40	10	6- 3¾	2- 7	3-8½	2-10	11- 4½		45 600	4¼-2⅜- 60	1- 3
		60	10	6- 3¾	2- 7	3-8½	2-10	11- 6½		49 600		
		80	10	6- 3¾	2- 7	3-8½	2-10	12- 0		54 600		
	40	40	11	7- 4½	3- 1½	4-2½	3- 8½	12- 8		59 600	5 -2½- 80	1- 6
		60	11	7- 4½	3- 1½	4-2½	3- 8½	13- 0		64 000		
		80	11	7- 4½	3- 1½	4-2½	3- 8½	13- 5		70 000		
	50	40	13	7-11½	3- 6	4-0½	3- 9	13-10½		77 000	5¾-2¾-100	1- 6
		60	13	7-11½	3- 6	4-0½	3- 9	13-10½		84 600		
		80	13	7-11½	3- 6	4-0½	3- 9	14- 0		92 000		
	60	40	14	8- 7¼	4- 1	4-3¾	4- 4½	15- 2		88 000	5¾-2¾-100	1-11
		60	14	8- 7¼	4- 1	4-3¾	4- 4½	15- 2		94 000		
		80	14	8- 7¼	4- 1	4-3¾	4- 4½	15- 4		103 000		
	60	40	12½	10- 3¼	4- 0	2-7¾	4- 1½	3- 6	5-4	44 000	5¾-2¾-100	
		60	12½	10- 3¼	4- 0	2-7¾	4- 4½	3- 6	5-4	47 000	6 -4 -150	
		80	12½	10- 3¼	4- 0	2-7¾	1- 4½	3- 6	5-4	51 500		
	75	40	12½	11- 3	4- 6	3-8½	4- 0½	5- 0	6-0	55 000	5¾-2¾-100	2- 2
		60	12½	11- 3	4- 6	3-8½	4- 0½	5- 0	6-0	60 000	6 -4 -150	
		80	12½	11- 3	4- 6	3-8½	4- 0½	5- 0	6-0	64 000		
	100	40	16½	13- 2¼	4- 1	2-0	SPECIAL	5- 0	6-0	83 000	6 -4 -150	4- 7
		60	16½	13- 2¼	4- 1	2-0	"	5- 0	6-0	86 000		
		80	16½	13- 2¼	4- 1	2-0	"	5- 0	6-0	89 000		
	150	40	17	15- 6⅛	6- 0	3-2	"	6- 0	6-0	130 000	6 -4 -150	4- 7
		60	17	15- 6⅛	6- 0	3-2	"	6- 0	6-0	134 000		
		80	17	15- 6⅛	6- 0	3-2	"	6- 0	6-0	139 000		

TABLE XV

Typical Electric Cranes

CAPACITY (Tons)	SPAN (Ft.)	WHEEL BASE F	WHEEL LOAD P	$A+2$ in.	B	WEIGHT OF RUNWAY RAIL	
						For Plate-Girders	I-Beams
5	40	8 ft. 6 in.	12 000	10 in.	7 ft	40 lbs. per yd.	40 lbs.
	60	9 " 0 "	13 000	10 "	7 "	40 "	40 "
10	40	9 " 0 "	19 000	10 "	7 "	45 "	40 "
	60	9 " 6 "	21 000	10 "	7 "	45 "	40 "
15	40	9 " 6 "	26 000	10 "	7 "	50 "	50 "
	60	10 " 0 "	29 000	10 "	7 "	50 "	50 "
20	40	10 " 0 "	33 000	12 "	8 "	55 "	50 "
	60	10 " 6 "	36 000	12 "	8 "	55 "	50 "
25	40	10 " 0 "	40 000	12 "	8 "	60 "	50 "
	60	10 " 6 "	44 000	12 "	8 "	60 "	50 "
30	40	10 " 6 "	48 000	12 "	8 "	70 "	60 "
	60	11 " 0 "	52 000	12 "	8 "	70 "	60 "
40	40	11 " 0 "	64 000	14 "	9 "	80 "	60 "
	60	12 " 0 "	70 000	14 "	9 "	80 "	60 "
50	40	11 " 0 "	72 000	14 "	9 "	100 "	60 "
	60	12 " 0 "	80 000	14 "	9 "	100 "	60 "

obtain the maximum shear at any section, as a a, the load should be placed as shown in Fig. 115; and the maximum shear will then be the left reaction, which is $R = 2P\left(x + \frac{F}{2}\right) \div l$, for two wheels; and $R = 4P\left(x + F + \frac{G}{2}\right) \div l$, for four wheels.

The values of P for traveling cranes of various capacities and spans may be obtained upon writing to the various crane manufacturing companies, whose addresses will be found in the advertising sections of the engineering periodicals. The distances between wheels may be obtained from their catalogues, which may be had upon application. The values of P, and the distances between wheels for cranes of various spans and capacities, are given in Table XIV, which is made from information furnished through the courtesy of Pawling & Harnischfeger, Milwaukee, Wisconsin.

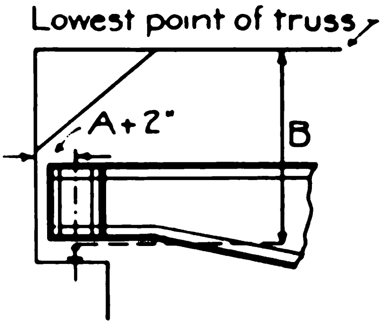


Fig. 116. Showing Notation used in Table XV

The values in Table XV are taken from the "Transactions" of

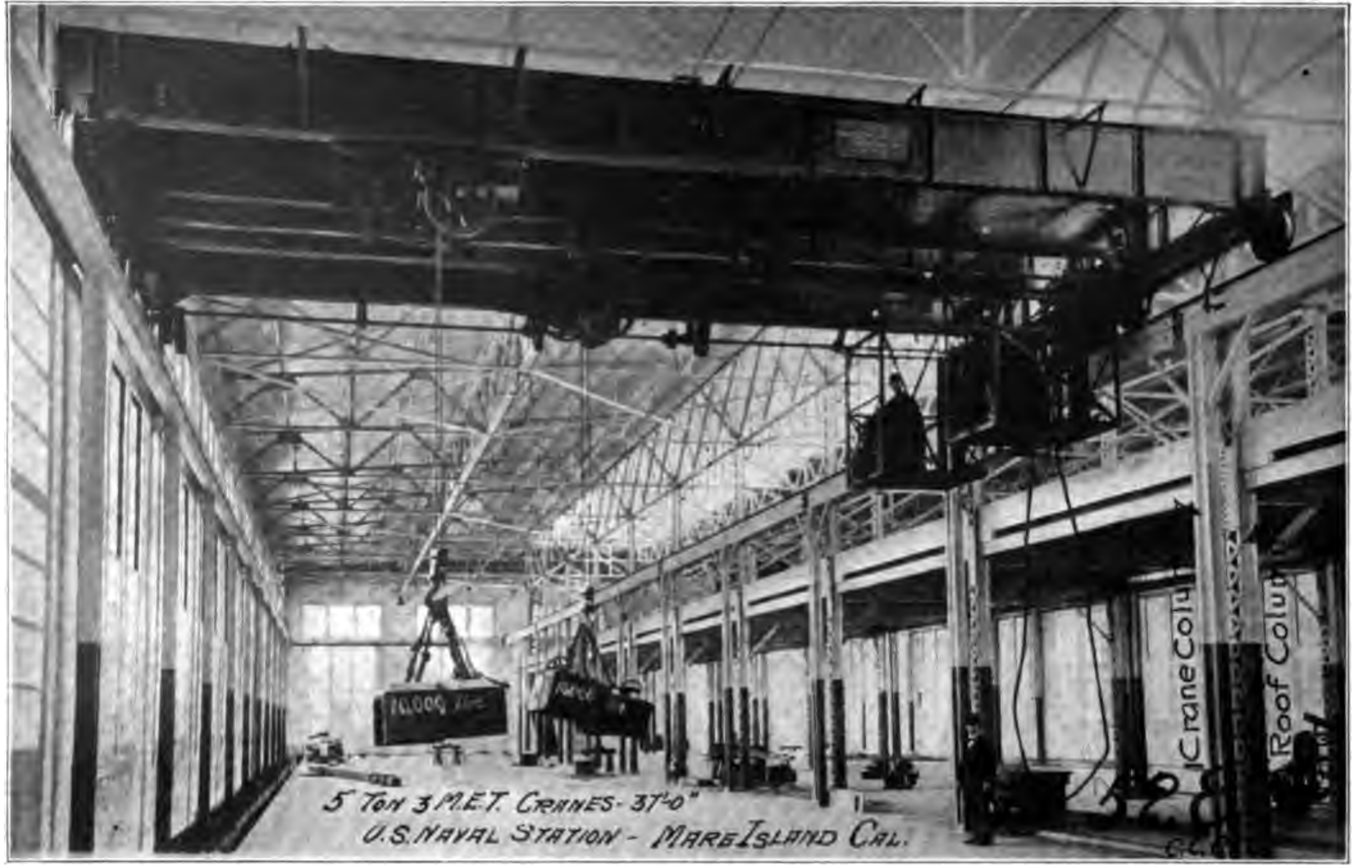

5 TON 3 M.E.T. CRANES - 37'-0"
U.S. NAVAL STATION - MARE ISLAND CAL.
10,000 lbs
Crane Colu
Roof Colu

Fig. 117. Five-Ton Electric Traveling Cranes Installed at U. S. Naval Station, Mare Island, California.
Courtesy of Cleveland Crane & Car Company, Cleveland, Ohio.

the American Society of Civil Engineers, Vol. 54, p. 400, 1905. They are for typical traveling electric cranes, and are proposed by Mr. C. C. Schneider, one of the most distinguished of structural engineers.

The side clearance B from the center of the rail, and the vertical clearance of the beam from the top of the rail, are given in this table (see Fig. 116). These values for the cranes of different manufacturers may be obtained from their catalogues; and they must be known, in order that the crane shall not interfere with the columns or the roof trusses.

If the runway girder is composed of an I-beam, a channel is usually riveted to its top; and on this the rail on which the crane wheels move is fastened down at intervals (see Fig. 107) of about $2\frac{1}{2}$ or 3 feet. Figs. 106 and 117 show details of this kind of girder. Note that the rails are U-shaped (see Fig. 105). This rail is used extensively, although in many cases the common T-rail is used and is fastened down by means of clamps around the edge of the flange of the girder (see Fig. 110).

In case plate-girders are necessary for runway girders, they must be designed. The depth of these girders should be $\frac{1}{10}$ to $\frac{1}{6}$ of the distance between trusses or columns—that is, $\frac{1}{10}$ of their span; The depths must be in the even inch. For example, if the trusses were 16 feet apart, the depth of the girder would be $16 \div 10 = 1.6$ feet, which is equal to 19.2 inches. The depth of the girder must then be made 20 inches, since, if it were made 19 inches, it would be difficult to obtain a web plate 19 inches wide, for the mills do not as a rule have plates of odd-inch widths in stock.

The thickness of the web plate is given by the formula:

$$t = \frac{V_0}{S_s\, d};$$

but in no case shall it be less than $\frac{5}{16}$ inch. In this formula, V_0 is the maximum end reaction of the runway girder. It is equal to R as given by the formula on p. 87, when x is equal to $l - F$, and d is the depth of the girder, which is equal to the depth of the web plate, and S_s is the unit allowable shearing stress.

The flanges are composed of two angles, placed with the long legs horizontal in case unequal-legged angles are used. The required **net area** of *one flange* is given by the formula:

$$A = \frac{M_m}{S_t\,(d-2)},$$

in which M_m is the moment obtained when the wheels are in the position shown in Fig. 113, S_t is the unit allowable tensile stress, and d is the width of the web plate. If the area A has been computed, two angles must be found from the tables in the Carnegie Handbook, such that when one ⅞-inch or ¾-inch rivet-hole, as the case may be, is taken out, each angle will give a net area equal to or slightly in excess of the area A.

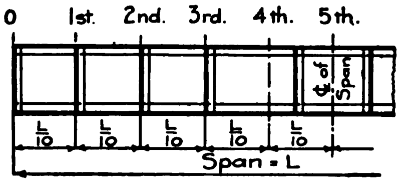


Fig. 118. Position of Tenth-Points.

These flange angles must be riveted to the web by rivets placed a certain distance apart. For convenience of manufacture, the girder is divided into ten equal parts, and the rivet spacing between any two of these divisions—or *tenth-points*, as they are called—is kept the same. These tenth-points are numbered (see Fig. 118). The rivet spacing in the first division is the same as that computed for the end of the girder, which is the zero tenth-point; the rivet spacing in the second division is the same as that computed for the 1st tenth-point; and so on. The rivet spacing at any point is given by the formula:

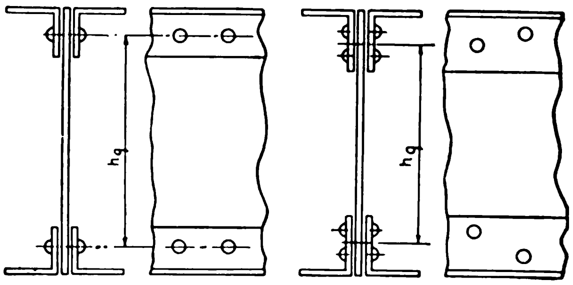


Fig. 119. Determination of Distance between Gauge Lines.

$$S = \frac{v}{\sqrt{\left(\frac{V_x}{h_g}\right)^2 + \left(\frac{P}{30}\right)^2}},$$

in which,

V_x = Maximum shear at the point;
v = Maximum allowable stress on one rivet; this will be the bearing value of the rivet in the web plate (see Table X, p. 47);
P = Maximum reaction of one crane wheel (see Table XIV or XV);
h_g = Distance between gauge lines of the angles.

In case there are two gauge lines on the angle, then the distance h_g is the distance between centers of these gauge lines (see Fig. 119).

Table IX, p. 46, gives the gauge lines for different lengths of angle legs. If S gives a value less than $2\frac{5}{8}$ inches, the leg of the angle against the web must be 5 inches or more, on account of practical limitations of manufacture.

26. *Examples.* In order to illustrate the preceding methods, two problems will be worked out.

1. Design a runway girder for a 5-ton crane of 40-foot span, the wheel loads and wheel base being as given in Table XV, p. 87, and the distance between trusses 20 feet.

In order to produce the maximum moment, the wheel must be placed as shown in Fig. 120. The left reaction is $12\,000\,(2.125 + 10.00 + 3.625) \div 20 = 9\,450$. The moment under wheel 1 is $9\,450 \times 7.875 \times 12 = 894\,000$ pound-inches, which requires a section modulus of $894\,000 \div 15\,000 = 59.60$. Looking in the Carnegie Handbook, pp. 97 and 98, it is seen that a 15-inch 42-pound I-beam with a section modulus of 58.9 will be sufficient, since the section modulus is less than $2\frac{1}{2}$ per cent under that required.

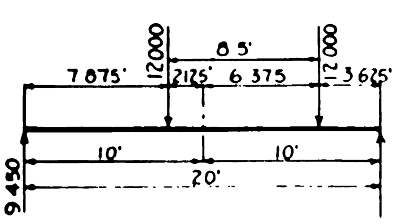


Fig. 120. Position for Maximum Moment for Problem 1 on Page 91.

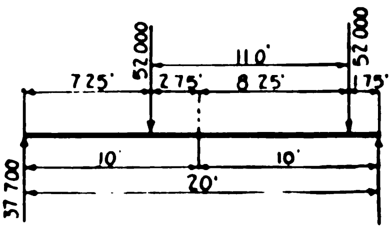


Fig. 121. Position for Maximum Moment for Problem 2 on Page 91.

2. Design a runway girder for a 30-ton crane of 60-foot span, the wheel loads and wheel base being as given in Table XV, and the distance between trusses 20 feet.

The wheels are placed in position as shown in Fig. 121. The left reaction is $\frac{52\,000\,(12.75 + 1.75)}{20} = 37\,700$ pounds; and the maximum moment, which occurs under wheel 1, is $37\,700 \times 7.25 \times 12 = 3\,285\,000$ pound-inches. The maximum shear occurs when the wheels are in position as shown in Fig. 112, p. 85, and is 75 400 pounds. The required thickness of the web is $\frac{75\,400}{10\,000 \times 24} = 0.314$ inch, the depth being $20 \div 10 = 2$ feet $= 24$ inches. The web will be made 24 inches wide and $\frac{3}{8}$ inch thick.

The required net flange area is $\frac{3\,285\,000}{15\,000 \times (24 - 2)} = 9.97$ square inches for two angles, or 4.99 square inches for one angle. An angle 6 by 6 by ½-inch gives a gross area of 5.75 square inches and a net area of 5.75 – 0.50 = 5.25 square inches, one ⅞-inch rivet-hole being taken out of the section. Since this area coincides quite closely with the required area and is larger, it will be used. A 6 by 3½ by ⅝-inch angle would have been better in regard to area, but the rivet spacing is less than 2⅝ inches at the end, and this required a double gauge line and therefore a leg 5 inches or over.

The maximum shears at the tenth-points are now computed, and are tabulated as follows:

V_0 = 75 400 pounds.
V_1 = 65 000 "
V_2 = 54 600 "
V_3 = 44 200 "
V_4 = 33 800 "
V_5 = 26 000 "

The value of the shear to be used in any particular case is given above. In this case, $P = 52\,000$ pounds; $v = 6\,570$ pounds, ⅞-inch rivets being used; and h_g is $24\frac{1}{4} - 2\left(2\frac{1}{4} + \frac{2\frac{1}{2}}{2}\right) = 17.5$ inches. The rivet spacing for the first division or first two feet of the span is:

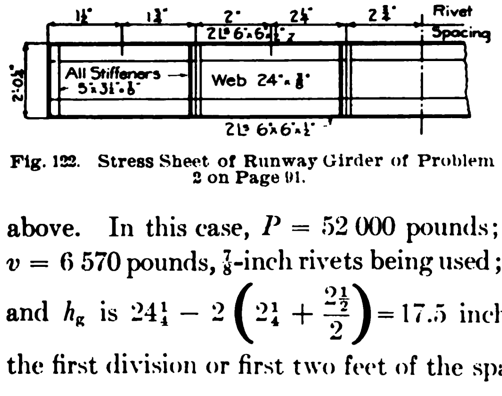


Fig. 122. Stress Sheet of Runway Girder of Problem 2 on Page 91.

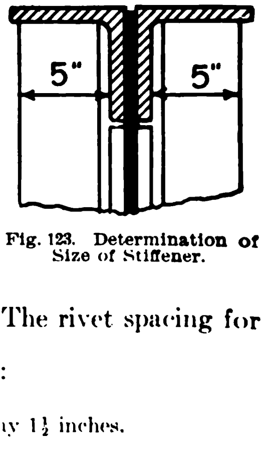
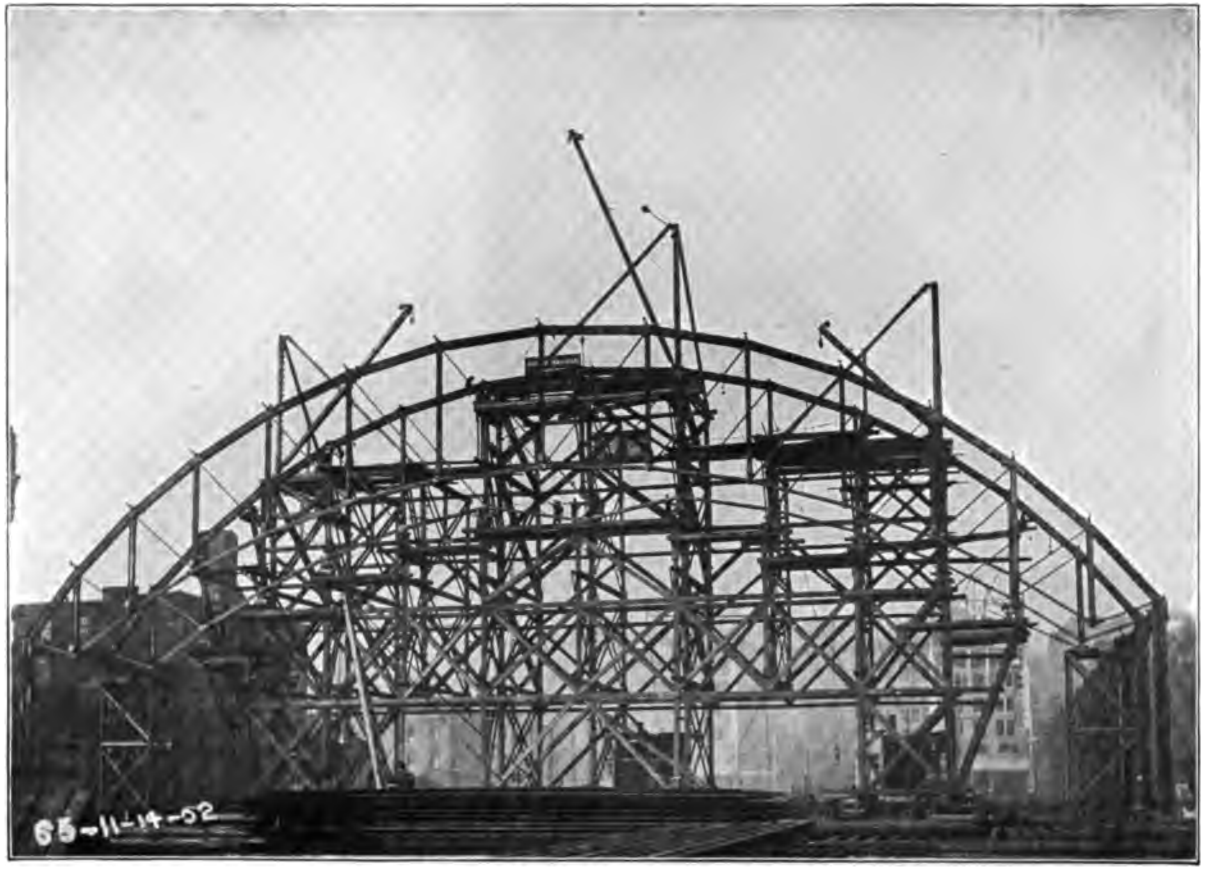


Fig. 123. Determination of Size of Stiffener.

$$S = \frac{6\,570}{\sqrt{\left(\frac{75\,400}{17.5}\right)^2 + \left(\frac{52\,000}{30}\right)^2}} = 1.414, \text{ say } 1\tfrac{1}{2} \text{ inches.}$$

The rivet spacing for the other divisions may be computed by the student. It is given in Fig. 122. The web of the girder should be stiffened as shown in the figure, by angles placed as there indicated The thickness of the angles should not be less than $\frac{5}{16}$ inch, nor greater than ½ inch. The size of the angles should be such that the



LA SALLE STATION, L. S. & M. S. AND C., R. I. & P. RAILROADS, CHICAGO

Frost & Granger, Architects; E. C. & R. M. Shankland, Engineers

Steel trusses over train shed; span of truss, 215 feet. Note the traveling crane, with three derricks on it, used in setting these trusses.

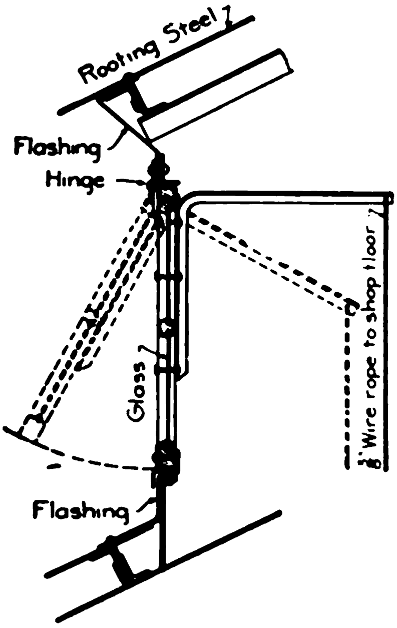


Fig. 124 Section of Glass Louvres in Monitor.

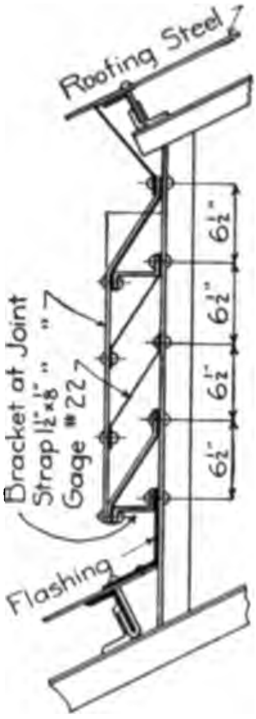


Fig. 125 Section of Metal Louvres in Monitor.

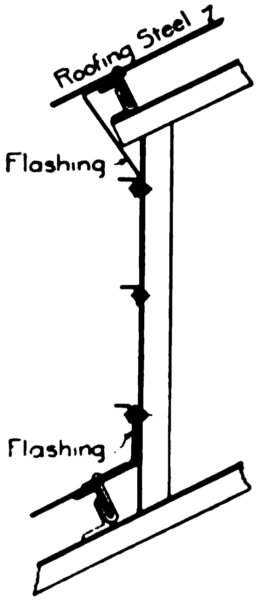


Fig. 126 Section of Open Monitor.

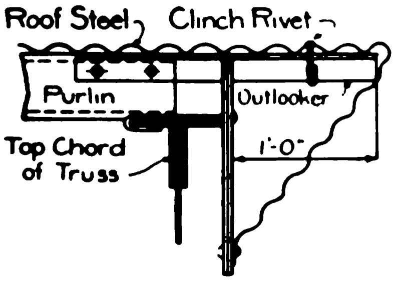


Fig. 127 Gable Details for Corrugated Steel.

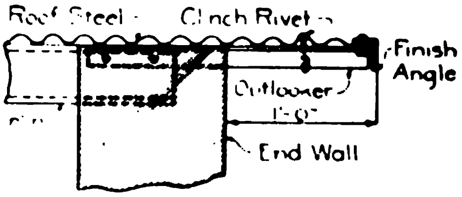


Fig. 128 Gable Details for Corrugated Steel

outstanding leg does not reach beyond the leg of the flange angle (see Fig. 123). This makes their size as shown in Fig. 122. The crane rail may be connected directly to these and the flange angles; or a channel may be placed over the flange angles and riveted to them in a manner similar to that employed in the case of I-beams, the crane rail being fastened to that. If this latter detail is employed, the area of the channel is reckoned as forming part of the upper flange; and the net area of the angle must then be equal to the required net area, less the net area of the channel.

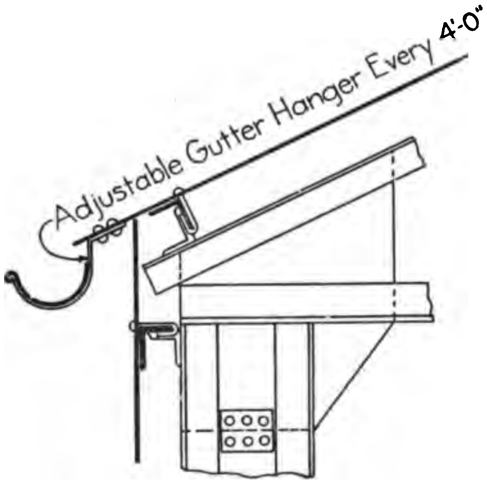


Fig. 129. Cornice Details for Steel Roof. See also Figs. 130 and 131.

27. **Ventilators.** Mill buildings may be ventilated by means of small circular ventilators such as shown in Fig. 69, p. 41, placed at certain intervals along the ridge or peak of the roof, or by means of monitors as shown in Fig. 71. The sides of these monitors may be fitted with swinging glass windows, with wooden or metal louvres, or, in case a large amount of ventilation is required, may be simply left open. Figs. 124, 125, and 126 give details of monitors, and show how they are connected to the trusses.

28. **Gable Details.** The gable is the end of the roof at that end of the building which is parallel to the roof trusses. Since this extends beyond the plane of the side of the building, some method must be employed in connecting the outer edge with the wall of the building, in order to keep out the rain and wind. Figs. 127 and 128 give several details which are efficient and at the same time economical.

29. **Cornice Details.** The cornice is that edge of the roof which is perpendicular to the planes of the roof trusses. In addition to being necessarily so constructed as to keep out the wind and the elements, it must have in many cases some form of gutter connected to it, which takes the water off the roof. This gutter should be con-

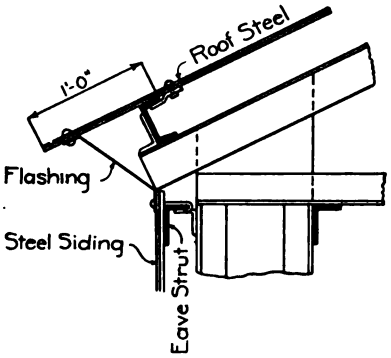


Fig. 130. Cornice Details for Steel Roof. See also Figs. 129 and 131.

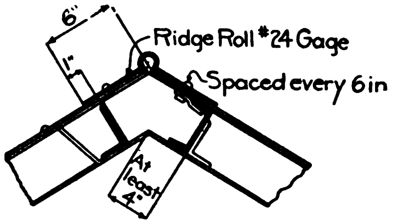


Fig. 133. Ridge Roll.

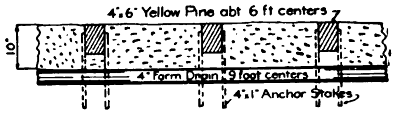

4"x6" Yellow Pine abt 6 ft centers
10"
4" Form Drain 9 foot centers
4"x1" Anchor Stakes

Fig. 134. Detail of Cinder Floor.

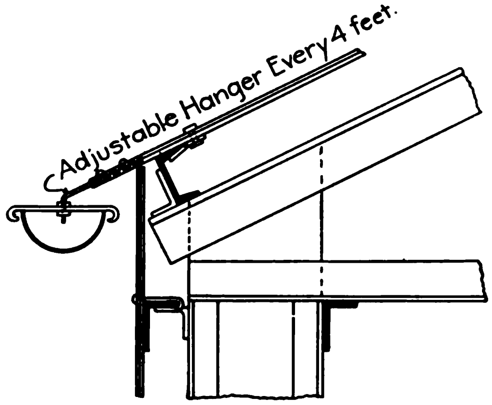


Fig. 131. Cornice Details for Steel Roof. See also Figs. 129 and 130.

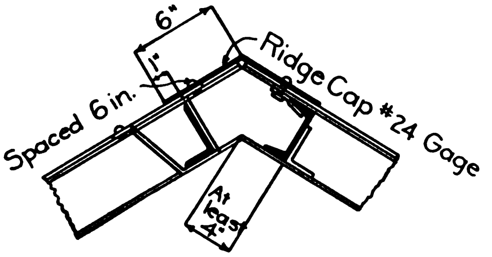


Fig. 132. Ridge Cap.

nected at intervals of every three bays—or, in case this exceeds 50 feet, every two bays—with a pipe or conductor to carry the water to the ground. Gutters, as a general thing, are semicircular or nearly so; and for ordinary spans they should not be less than 6 inches wide. Conductors should not be less than 5 inches in diameter. It is not to be supposed that the water entirely fills either the conductors or the gutters. The sizes are made so as to allow for any obstruction such as dirt or ice. Gutters should preferably have a pitch of one inch in every 10 feet. Figs. 129 to 131 give details of cornices with various forms of gutters attached.

The ridge, or peak of the roof, is usually covered with a plain sheet of metal, in which case it is called the *ridge cap*; or with a metallic roll with flared sides, in which case it is called a *ridge roll.* Figs. 132 and 133 show cross-sections of a ridge cap and a ridge roll.

30. **Floors.** The floor of the shop depends very largely upon the purpose for which the building is intended. It may consist of earth, cinders, boards, concrete, or sheet steel. In cases where men are required to work standing, cinders or boards give the best results. Earth floors will wear into holes in places where the men stand, and concrete or steel makes them foot-weary on account of its inelasticity. Where heavy machinery is installed, and men are seldom present except for a short time at certain periods, concrete makes an ideal floor. Figs. 134, 135, and 136 show details of various kinds of floors. Fig. 137 gives a detail of the floor in the Steam Engineering Laboratory of the University of Illinois. This consists of channels imbedded in concrete. These channels, which are placed in pairs a small distance apart, run both lengthwise and crosswise of the shop. The advantage of this form of construction is that machinery can be placed anywhere on the shop floor and quickly bolted into place by means of T-bolts, a detail of which is shown in the figure.

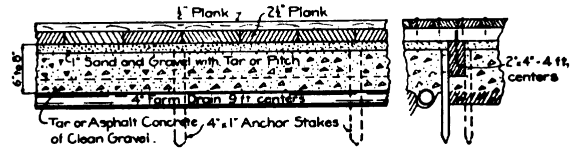

1" Plank
2½" Plank
6" to 8"
1" Sand and Gravel with Tar or Pitch
4" Farm Drain 9 ft. centers
Tar or Asphalt Concrete of Clean Gravel.
4"x1" Anchor Stakes
2"x4"-4 ft. centers

Fig. 135. Detail of Asphalt or Coal-Tar Concrete Floor.

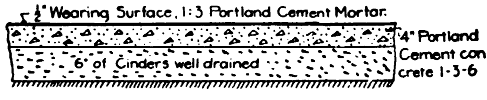

½" Wearing Surface, 1:3 Portland Cement Mortar.
6" of Cinders well drained
4" Portland Cement concrete 1-3-6

Fig. 136. Detail of Concrete Floor.

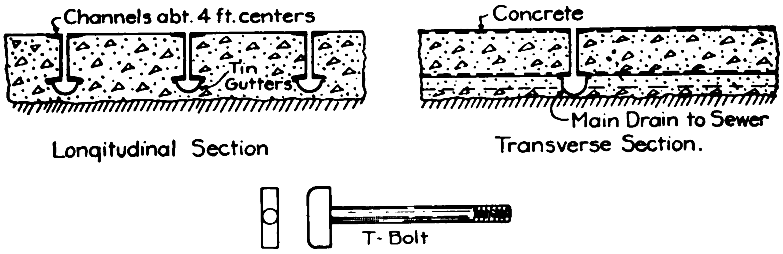
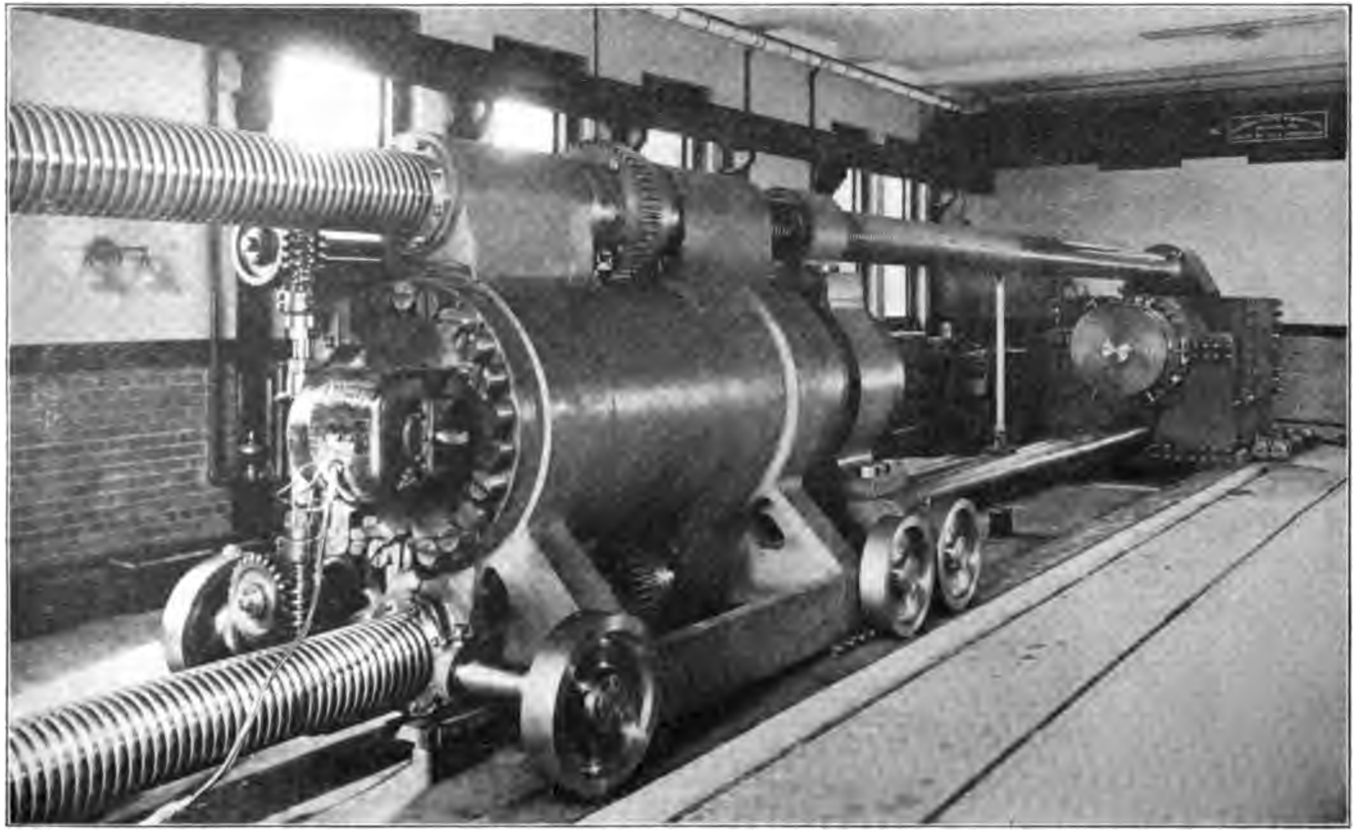


Fig. 137. Detail of Floor of Steam Laboratory of the University of Illinois.



EMERY TESTING MACHINE, BUREAU OF STANDARDS, WASHINGTON, D. C.

This Immense Machine Is Used for Both Tension and Compression Tests. **Capacity, 2,300,000 Pounds.**

STRENGTH OF MATERIALS.

PART I

SIMPLE STRESS.

1. Stress. When forces are applied to a body they tend in a greater or less degree to break it. Preventing or tending to prevent the rupture, there arise, generally, forces between every two adjacent parts of the body. Thus, when a load is suspended by means of an iron rod, the rod is subjected to a downward pull at its lower end and to an upward pull at its upper end, and these two forces tend to pull it apart. At any cross-section of the rod the iron on either side "holds fast" to that on the other, and these forces which the parts of the rod exert upon each other prevent the tearing of the rod. For example, in Fig. 1, let *a* represent the rod and its suspended load, 1,000 pounds; then the pull on the lower end equals 1,000 pounds. If we neglect the weight of the rod, the pull on the upper end is also 1,000 pounds, as shown in Fig. 1 (*b*); and the upper part A exerts on the lower part B an upward pull Q equal to 1,000 pounds, while the lower part exerts on the upper a force P also equal to 1,000 pounds. These two forces, P and Q, prevent rupture of the rod at the "section" C; at any other section there are two forces like P and Q preventing rupture at that section.

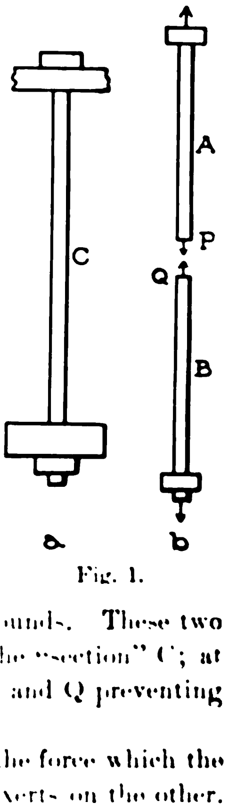


Fig. 1.

By *stress at a section* of a body is meant the force which the part of the body on either side of the section exerts on the other. Thus, the stress at the section C (Fig. 1) is P (or Q), and it equals 1,000 pounds.

2. Stresses are usually expressed in America in pounds, sometimes in tons. Thus the stress P in the preceding article is

1,000 pounds, or ½ ton. Notice that this value has nothing to do with the size of the cross-section on which the stress acts.

3. Kinds of Stress. (a) When the forces acting on a body (as a rope or rod) are such that they tend to tear it, the stress at any cross-section is called a *tension* or a *tensile stress.* The stresses P and Q, of Fig. 1, are tensile stresses. Stretched ropes, loaded "tie rods" of roofs and bridges, etc., are under tensile stress.

(b.) When the forces acting on a body (as a short post, brick, etc.) are such that they tend to crush it, the stress at any section at right angles to the direction of the crushing forces is called a *pressure* or a *compressive stress.* Fig. 2 (*a*) represents a loaded post, and Fig. 2 (*b*) the upper and lower parts. The upper part presses down on B, and the lower part presses up on A, as shown. P or Q is the compressive stress in the post at section C. Loaded posts, or struts, piers, etc., are under compressive stress.

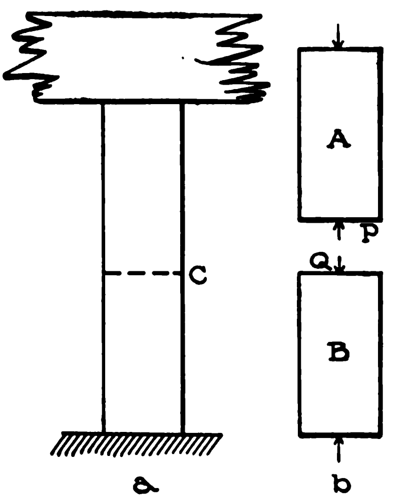


Fig. 2.

(c.) When the forces acting on a body (as a rivet in a bridge joint) are such that they tend to cut or "shear" it across, the stress at a section along which there is a tendency to cut is called a *shear* or a *shearing stress.* This kind of stress takes its name from the act of cutting with a pair of shears. In a material which is being cut in this way, the stresses that are being "overcome" are shearing stresses. Fig. 3 (*a*) represents a riveted joint, and Fig. 3 (*b*) two parts of the rivet. The forces applied to the joint are such that A tends to slide to the left, and B to the right; then B exerts on A a force P toward the right, and A on B a force Q toward the left as shown. P or Q is the shearing stress in the rivet.

Tensions, Compressions and Shears are called *simple stresses.* Forces may act upon a body so as to produce a combination of simple stresses on some section; such a combination is called a *complex* *stress.* The stresses in beams are usually complex. There are other terms used to describe stress; they will be defined farther on.

4. **Unit-Stress.** It is often necessary to specify not merely the amount of the entire stress which acts on an area, but also the amount which acts on each unit of area (square inch for example). By unit-stress is meant stress per unit area.

To find the value of a unit-stress: *Divide the whole stress by the whole area of the section on which it acts, or over which it is distributed.* Thus, let

P denote the value of the whole stress,
A the area on which it acts, and
S the value of the unit-stress; then

$$S = \frac{P}{A}, \text{ also } P = AS. \qquad (1)$$

Strictly these formulas apply only when the stress P is uniform, that is, when it is uniformly distributed over the area, each square inch for example sustaining the same amount of stress. When the stress is not uniform, that is, when the stresses on different square inches are not equal, then P : A equals the *average value* of the unit-stress.

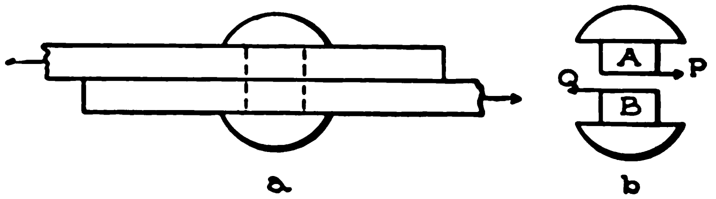


Fig. 3.

5. Unit-stresses are usually expressed (in America) in pounds per square inch, sometimes in tons per square inch. If P and A in equation 1 are expressed in pounds and square inches respectively, then S will be in pounds per square inch; and if P and A are expressed in tons and square inches, S will be in tons per square inch.

Examples. 1. Suppose that the rod sustaining the load in Fig. 1 is 2 square inches in cross-section, and that the load weighs 1,000 pounds. What is the value of the unit-stress?

Here $P = 1{,}000$ pounds, $A = 2$ square inches; hence.

$$S = \frac{1{,}000}{2} = 500 \text{ pounds per square inch.}$$

2. Suppose that the rod is one-half square inch in cross-section. What is the value of the unit-stress?

$A = \frac{1}{2}$ square inch, and, as before, $P = 1{,}000$ pounds; hence

$$S = 1{,}000 \div \frac{1}{2} = 2{,}000 \text{ pounds per square inch.}$$

Notice that one must always divide the whole stress by the area to get the unit-stress, whether the area is greater or less than one.

6. Deformation. Whenever forces are applied to a body it changes in size, and usually in shape also. This change of size and shape is called deformation. Deformations are usually measured in inches; thus, if a rod is stretched 2 inches, the "elongation" = 2 inches.

7. Unit-Deformation. It is sometimes necessary to specify not merely the value of a total deformation but its amount per unit length of the deformed body. Deformation per unit length of the deformed body is called unit-deformation.

To find the value of a unit-deformation: *Divide the whole deformation by the length over which it is distributed.* Thus, if

D denotes the value of a deformation,
l the length,
s the unit-deformation, then

$$s = \frac{D}{l}, \text{ also } D = ls. \qquad (2)$$

Both D and l should always be expressed in the same unit.

Example. Suppose that a 4-foot rod is elongated $\frac{1}{2}$ inch. What is the value of the unit-deformation?

Here $D = \frac{1}{2}$ inch, and $l = 4$ feet $= 48$ inches;
hence $s = \frac{1}{2} \div 48 = \frac{1}{96}$ inch per inch.

That is, each inch is elongated $\frac{1}{96}$ inch.

Unit-elongations are sometimes expressed in per cent. To express an elongation in per cent: *Divide the elongation in inches by the original length in inches, and multiply by 100.*

8. Elasticity. Most solid bodies when deformed will regain more or less completely their natural size and shape when the de-forming forces cease to act. This property of regaining size and shape is called elasticity.

We may classify bodies into kinds depending on the degree of elasticity which they have, thus:

1. *Perfectly elastic* bodies; these will regain their original form and size no matter how large the applied forces are if less than breaking values. Strictly there are no such materials, but rubber, practically, is perfectly elastic.

2. *Imperfectly elastic* bodies; these will fully regain their original form and size if the applied forces are not too large, and practically even if the loads are large but less than the breaking value. Most of the constructive materials belong to this class.

3. *Inelastic* or *plastic* bodies; these will not regain in the least their original form when the applied forces cease to act. Clay and putty are good examples of this class.

9. Hooke's Law, and Elastic Limit. If a gradually increasing force is applied to a perfectly elastic material, the deformation increases proportionally to the force; that is, if P and P′ denote two values of the force (or stress), and D and D′ the values of the deformation produced by the force,

$$\text{then } P:P'::D:D'.$$

This relation is also true for imperfectly elastic materials, provided that the loads P and P′ do not exceed a certain limit depending on the material. Beyond this limit, the deformation increases much faster than the load; that is, if within the limit an addition of 1,000 pounds to the load produces a stretch of 0.01 inch, beyond the limit an equal addition produces a stretch larger and usually much larger than 0.01 inch.

Beyond this limit of proportionality a part of the deformation is permanent; that is, if the load is removed the body only partially recovers its form and size. The permanent part of a deformation is called *set*.

The fact that for most materials the deformation is proportional to the load within certain limits, is known as Hooke's Law. The unit-stress within which Hooke's law holds, or above which the deformation is not proportional to the load or stress, is called *elastic limit*.

10. Ultimate Strength. By ultimate tensile, compressive, or shearing strength of a material is meant the greatest tensile, compressive, or shearing unit-stress which it can withstand.

As before mentioned, when a material is subjected to an increasing load the deformation increases faster than the load beyond the elastic limit, and much faster near the stage of rupture. Not only do tension bars and compression blocks elongate and shorten respectively, but their cross-sectional areas change also; tension bars thin down and compression blocks "swell out" more or less. The value of the ultimate strength for any material is ascertained by subjecting a specimen to a gradually increasing tensile, compressive, or shearing stress, as the case may be, until rupture occurs, and measuring the greatest load. *The breaking load divided by the area of the original cross-section sustaining the stress, is the value of the ultimate strength.*

Example. Suppose that in a tension test of a wrought-iron rod $\frac{1}{2}$ inch in diameter the greatest load was 12,540 pounds. What is the value of the ultimate strength of that grade of wrought iron?

The original area of the cross-section of the rod was $0.7854\ (\text{diameter})^2=0.7854\times\frac{1}{4}=0.1964$ square inches; hence the ultimate strength equals

$$12{,}540\div0.1964=63{,}850 \text{ pounds per square inch. (nearly).}$$

11. Stress-Deformation Diagram. A "test" to determine the elastic limit, ultimate strength, and other information in regard to a material is conducted by applying a gradually increasing load until the specimen is broken, and noting the deformation corresponding to many values of the load. The first and second columns of the following table are a record of a tension test on a steel rod one inch in diameter. The numbers in the first column are the values of the pull, or the loads, at which the elongation of the specimen was measured. The elongations are given in the second column. The numbers in the third and fourth columns are the values of the unit-stress and unit-elongation corresponding to the values of the load opposite to them. The numbers in the third column were obtained from those in the first by dividing the latter by the area of the cross-section of the rod, 0.7854 square inches. Thus,

$$3{,}930\div0.7854=5{,}000$$
$$7{,}850\div0.7854=10{,}000, \text{ etc.}$$

Total Pull in pounds, P	Deformation in inches, D	Unit-Stress in pounds per square inch, S	Unit-Deformation, s
3930	0.00136	5000	0.00017
7850	.00280	10000	.00035
11780	.00404	15000	.00050
15710	.00538	20000	.00067
19635	.00672	25000	.00084
23560	.00805	30000	.00101
27490	.00942	35000	.00118
31415	.01080	40000	.00135
35345	.01221	45000	.00153
39270	.0144	50000	.00180
43200	.0800	55000	.0100
47125	.1622	60000	.0202
51050	.201	65000	.0251
54980	.281	70000	.0351
58910	.384	75000	.048
62832	.560	80000	.070
65200	1.600	83000	.200

The numbers in the fourth column were obtained by dividing those in the second by the length of the specimen (or rather the length of that part whose elongation was measured), 8 inches. Thus,

$$0.00136 : 8 = 0.00017,$$
$$.00280 : 8 = .00035, \text{ etc.}$$

Looking at the first two columns it will be seen that the elongations are practically proportional to the loads up to the ninth load, the increase of stretch for each increase in load being about 0.00135 inch; but beyond the ninth load the increases of stretch are much greater. Hence the elastic limit was reached at about the ninth load, and its value is about 45,000 pounds per square inch. The greatest load was 65,200 pounds, and the corresponding unit-stress, 83 000 pounds per square inch, is the ultimate strength.

Nearly all the information revealed by such a test can be well represented in a diagram called a *stress-deformation diagram*. It is made as follows: Lay off the values of the unit-deformation (fourth column) along a horizontal line, according to some convenient scale, from some fixed point in the line. At the points on the horizontal line representing the various unit-elongations, lay off perpendicular distances equal to the corresponding unit stresses. Then connect by a smooth curve the upper ends of all those distances, last distances laid off. Thus, for instance, the highest unit-elongation (0.20) laid off from *o* (Fig. 4) fixes the point *a*, and a perpendicular distance to represent the highest unit-stress (83,000) fixes the point *b*. All the points so laid off give the curve *ocb*. The part *oc*, within the elastic limit, is straight and nearly vertical while the remainder is curved and more or less horizontal, especially toward the point of rupture *b*. Fig. 5 is a typical stress-deformation diagram for timber, cast iron, wrought iron, soft and hard steel, in tension and compression.

12. Working Stress and Strength, and Factor of Safety. The greatest unit-stress in any part of a structure when it is sustaining its loads is called the *working stress* of that part. If it is under tension, compression and shearing stresses, then the corresponding highest unit-stresses in it are called its working stress in tension, in compression, and in shear respectively; that is, we speak of as many working stresses as it has kinds of stress.

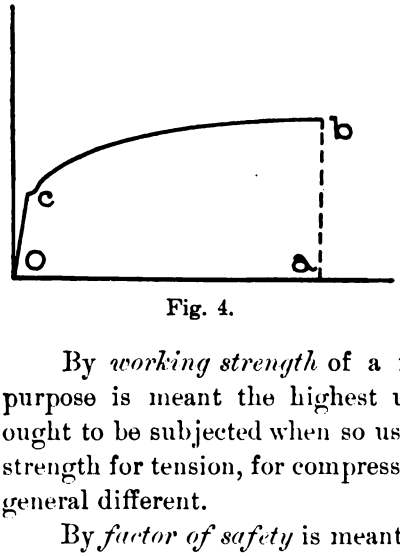


Fig. 4.

By *working strength* of a material to be used for a certain purpose is meant the highest unit-stress to which the material ought to be subjected when so used. Each material has a working strength for tension, for compression, and for shear, and they are in general different.

By *factor of safety* is meant the ratio of the ultimate strength of a material to its working stress or strength. Thus, if

S_u denotes ultimate strength,
S_w denotes working stress or strength, and
f denotes factor of safety, then

$$f = \frac{S_u}{S_w}; \text{ also } S_w = \frac{S_u}{f}. \qquad (3)$$

When a structure which has to stand certain loads is about to be designed, it is necessary to select working strengths or factors of safety for the materials to be used. Often the selection is a matter of great importance, and can be wisely performed only by an experienced engineer, for this is a matter where hard-and-fast rules should not govern but rather the judgment of the expert. But there are certain principles to be used as guides in making a selection, chief among which are:

1. The working strength should be considerably below the elastic limit. (Then the deformations will be small and not permanent.)

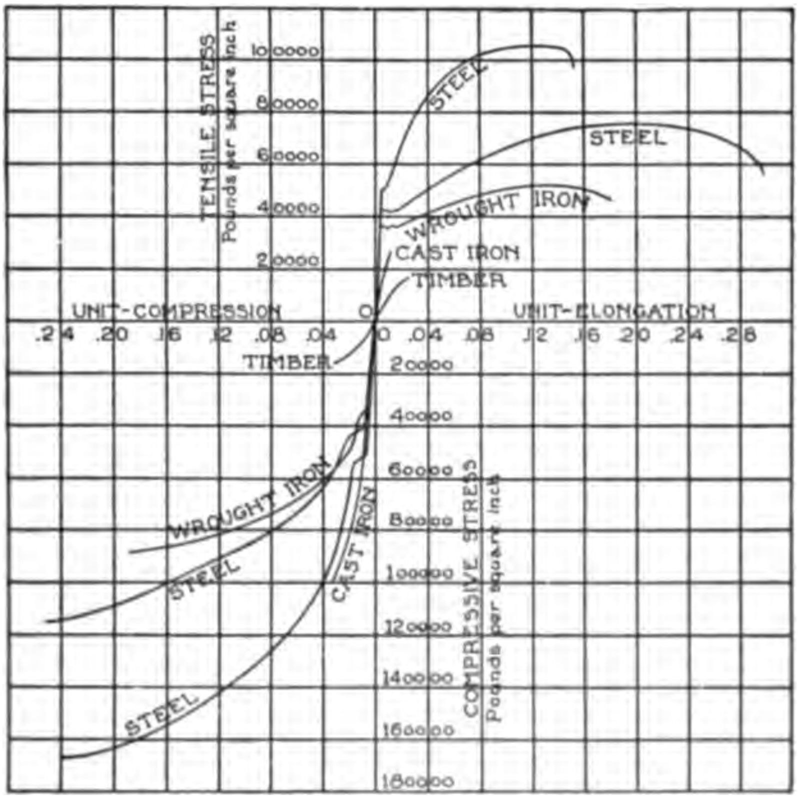


Fig. 5. (After Johnson.)

2. The working strength should be smaller for parts of a structure sustaining varying loads than for those whose loads are steady. (Actual experiments have disclosed the fact that the strength of a specimen depends on the kind of load put upon it, and that in a general way it is less the less steady the load is.)

3. The working strength must be taken low for non uniform material, where poor workmanship may be expected, when the

loads are uncertain, etc. Principles 1 and 2 have been reduced to figures or formulas for many particular cases, but the third must remain a subject for display of judgment, and even good guessing in many cases.

The following is a table of factors of safety* which will be used in the problems:

Factors of Safety.

Materials.	For steady stress. (Buildings.)	For varying stress. (Bridges.)	For shocks. (Machines.)
Timber	8	10	15
Brick and stone	15	25	30
Cast iron	6	15	20
Wrought iron	4	6	10
Steel	5	7	15

They must be regarded as average values and are not to be adopted in every case in practice.

Examples. 1. A wrought-iron rod 1 inch in diameter sustains a load of 30,000 pounds. What is its working stress? If its ultimate strength is 50,000 pounds per square inch, what is its factor of safety?

The area of the cross-section of the rod equals 0.7854× (diameter)^2=0.7854×1^2=0.7854 square inches. Since the whole stress on the cross-section is 30,000 pounds, equation 1 gives for the unit working stress

$$S=\frac{30,000}{0.7854}=38,197 \text{ pounds per square inch.}$$

Equation 3 gives for factor of safety

$$f=\frac{50,000}{38,197}=1.3$$

2. How large a steel bar or rod is needed to sustain a steady pull of 100,000 pounds if the ultimate strength of the material is 65,000 pounds?

The load being steady, we use a factor of safety of 5 (see table above); hence the working strength to be used (see equation 3) is

$$S=\frac{65,000}{5}=13,000 \text{ pounds per square inch.}$$

The proper area of the cross-section of the rod can now be computed from equation 1 thus:

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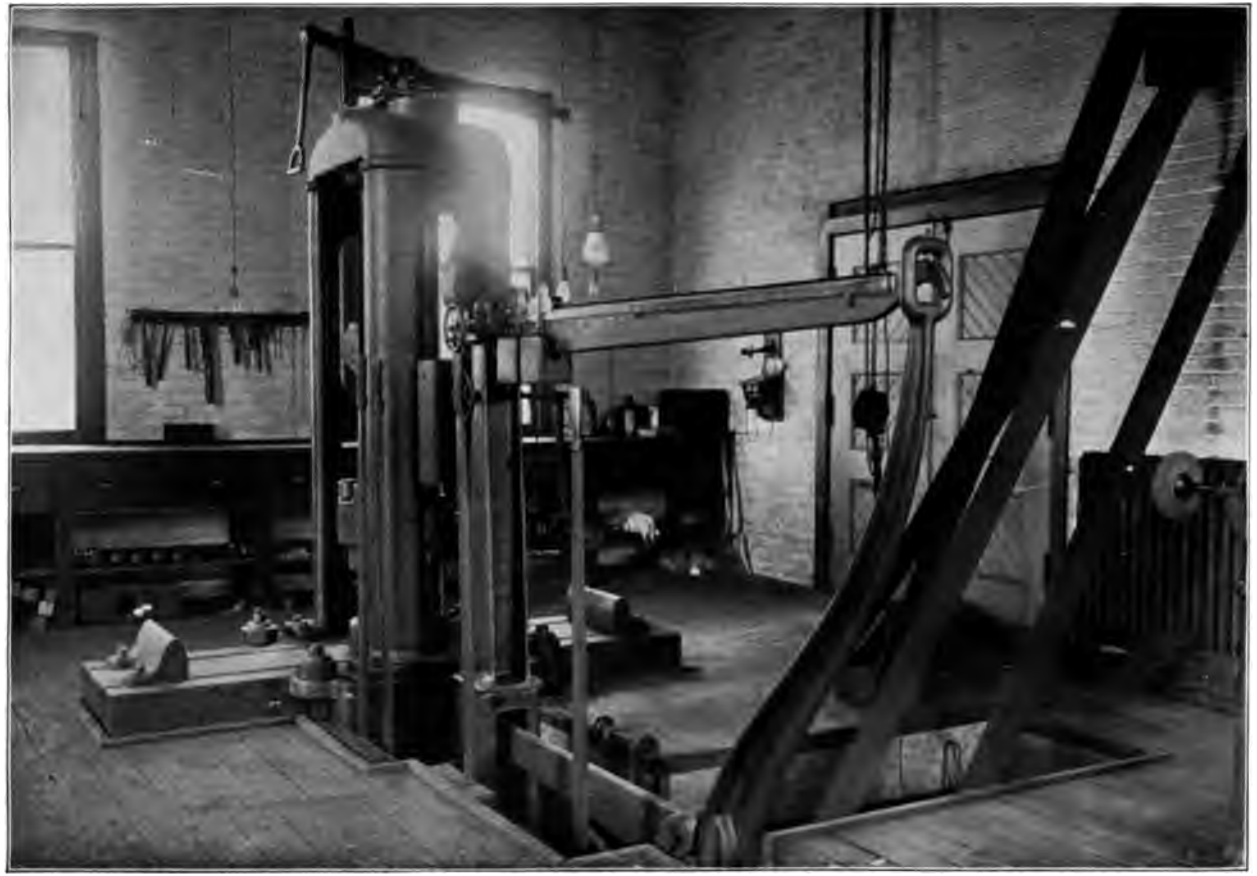
*Taken from Merriman's "Mechanics of Materials."



A 3000-POUND RIEHLE TESTING MACHINE
Machine located in the Chicago Physical Testing Laboratory
Robert W. Hunt and Company, Engineers, Chicago, New York, Pittsburgh, and London
Courtesy of Riehle Brothers Testing Machine Company, Philadelphia, Pennsylvania

$$A = \frac{P}{S} = \frac{100,000}{13,000} = 7.692 \text{ square inches.}$$

A bar 2×4 inches in cross-section would be a little stronger than necessary. To find the diameter (d) of a round rod of sufficient strength, we write $0.7854\ d^2 = 7.692$, and solve the equation for d; thus:

$$d^2 = \frac{7.692}{0.7854} = 9.794, \text{ or } d = 3.129 \text{ inches.}$$

3. How large a steady load can a short timber post safely sustain if it is 10×10 inches in cross-section and its ultimate compressive strength is 10,000 pounds per square inch?

According to the table (page 12) the proper factor of safety is 8, and hence the working strength according to equation 3 is

$$S = \frac{10,000}{8} = 1,250 \text{ pounds per square inch.}$$

The area of the cross-section is 100 square inches; hence the safe load (see equation 1) is

$$P = 100 \times 1,250 = 125,000 \text{ pounds.}$$

4. When a hole is punched through a plate the shearing strength of the material has to be overcome. If the ultimate shearing strength is 50,000 pounds per square inch, the thickness of the plate ½ inch, and the diameter of the hole ¾ inch, what is the value of the force to be overcome?

The area shorn is that of the cylindrical surface of the hole or the metal punched out; that is

$$3.1416 \times \text{diameter} \times \text{thickness} = 3.1416 \times \tfrac{3}{4} \times \tfrac{1}{2} = 1.178 \text{ sq. in.}$$

Hence, by equation 1, the total shearing strength or resistance to punching is

$$P = 1.178 \times 50,000 = 58,900 \text{ pounds.}$$

STRENGTH OF MATERIALS UNDER SIMPLE STRESS.

13. Materials in Tension. Practically the only materials used extensively under tension are timber, wrought iron and steel, and to some extent cast iron.

14. Timber. A successful tension test of wood is difficult, as the specimen usually crushes at the ends when held in the testing machine, splits, or fails otherwise than as desired. Hence the tensile strengths of woods are not well known, but the following may be taken as approximate average values of the ultimate strengths of the woods named, when "dry out of doors."

Hemlock,	7,000	pounds per square inch.
White pine,	8,000	" "
Yellow pine, long leaf,	12,000	" "
" " , short leaf,	10,000	" "
Douglas spruce,	10,000	" "
White oak,	12,000	" "
Red oak,	9,000	" "

15. Wrought Iron. The process of the manufacture of wrought iron gives it a "grain," and its tensile strengths along and across the grain are unequal, the latter being about three-fourths of the former. The ultimate tensile strength of wrought iron along the grain varies from 45,000 to 55,000 pounds per square inch. Strength along the grain is meant when not otherwise stated.

The strength depends on the size of the piece, it being greater for small than for large rods or bars, and also for thin than for thick plates. The elastic limit varies from 25,000 to 40,000 pounds per square inch, depending on the size of the bar or plate even more than the ultimate strength. Wrought iron is very ductile, a specimen tested in tension to destruction elongating from 5 to 25 per cent of its length.

16. Steel. Steel has more or less of a grain but is practically of the same strength in all directions. To suit different purposes, steel is made of various grades, chief among which may be mentioned rivet steel, sheet steel (for boilers), medium steel (for bridges and buildings), rail steel, tool and spring steel. In general, these grades of steel are hard and strong in the order named, the ultimate tensile strength ranging from about 50,000 to 160,000 pounds per square inch.

There are several grades of structural steel, which may be described as follows:*

1. Rivet steel:

   Ultimate tensile strength, 48,000 to 58,000 pounds per square inch.
   Elastic limit, not less than one-half the ultimate strength.
   Elongation, 26 per cent.
   Bends 180 degrees flat on itself without fracture.

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*Taken from "Manufacturer's Standard Specifications."

2. **Soft steel:**

   Ultimate tensile strength, 52,000 to 62,000 pounds per square inch.
   Elastic limit, not less than one half the ultimate strength.
   Elongation, 25 per cent.
   Bends 180 degrees flat on itself.

3. **Medium steel:**

   Ultimate tensile strength, 60,000 to 70,000 pounds per square inch.
   Elastic limit, not less than one half the ultimate strength.
   Elongation, 22 per cent.
   Bends 180 degrees to a diameter equal to the thickness of the specimen without fracture.

17. Cast Iron. As in the case of steel, there are many grades of cast iron. The grades are not the same for all localities or districts, but they are based on the appearance of the fractures, which vary from coarse dark grey to fine silvery white.

The ultimate tensile strength does not vary uniformly with the grades but depends for the most part on the percentage of "combined carbon" present in the iron. This strength varies from 15,000 to 35,000 pounds per square inch, 20,000 being a fair average.

Cast iron has no well-defined elastic limit (see curve for cast iron, Fig. 5). Its ultimate elongation is about one per cent.

EXAMPLES FOR PRACTICE.

1. A steel wire is one-eighth inch in diameter, and the ultimate tensile strength of the material is 150,000 pounds per square inch. How large is its breaking load? Ans. 1,845 pounds.

2. A wrought-iron rod (ultimate tensile strength 50,000 pounds per square inch) is 2 inches in diameter. How large a steady pull can it safely bear? Ans. 39,270 pounds.

18. Materials in Compression. Unlike the tensile, the compressive strength of a specimen or structural part depends on its dimension in the direction in which the load is applied, for, in compression, a long bar or rod is weaker than a short one. At present we refer only to the strength of short pieces such as do not bend under the load, the longer ones (columns) being discussed farther on.

Different materials break or fail under compression, in two very different ways:

1. Ductile materials (structural steel, wrought iron, etc.),

and wood compressed across the grain, do not fail by breaking into two distinct parts as in tension, but the former bulge out and flatten under great loads, while wood splits and mashes down. There is no particular point or instant of failure under increasing loads, and such materials have no definite ultimate strength in compression.

2. Brittle materials (brick, stone, hard steel, cast iron, etc.), and wood compressed along the grain, do not mash gradually, but fail suddenly and have a definite ultimate strength in compression. Although the surfaces of fracture are always much inclined to the direction in which the load is applied (about 45 degrees), the ultimate strength is computed by dividing the total breaking load by the cross-sectional area of the specimen.

The principal materials used under compression in structural work are timber, wrought iron, steel, cast iron, brick and stone.

19. Timber. As before noted, timber has no definite ultimate compressive strength across the grain. The U. S. Forestry Division has adopted certain amounts of compressive *deformation* as marking stages of failure. Three per cent compression is regarded as "a working limit allowable," and fifteen per cent as "an extreme limit, or as failure." The following (except the first) are values for compressive strength from the Forestry Division Reports, all in pounds per square inch:

	Ultimate strength along the grain.	3% Compression across the grain
Hemlock	6,000	
White pine	5,400	700
Long-leaf yellow pine	8,000	1,260
Short-leaf yellow pine	6,500	1,050
Douglas spruce	5,700	800
White oak	8,500	2,200
Red oak	7,200	2,300

20. Wrought Iron. The elastic limit of wrought iron, as before noted, depends very much upon the size of the bars or plate, it being greater for small bars and thin plates. Its value for compression is practically the same as for tension, 25,000 to 40,000 pounds per square inch.

21. Steel. The hard steels have the highest compressive strength; there is a recorded value of nearly 400,000 pounds per square inch, but 150,000 is probably a fair average.

The elastic limit in compression is practically the same as in tension, which is about 60 per cent of the ultimate tensile strength, or, for structural steel, about 25,000 to 42,000 pounds per square inch.

22. Cast Iron. This is a very strong material in compression, in which way, principally, it is used structurally. Its ultimate strength depends much on the proportion of "combined carbon" and silicon present, and varies from 50,000 to 200,000 pounds per square inch, 90,000 being a fair average. As in tension, there is no well-defined elastic limit in compression (see curve for cast iron, Fig. 5).

23. Brick. The ultimate strengths are as various as the kinds and makes of brick. For soft brick, the ultimate strength is as low as 500 pounds per square inch, and for pressed brick it varies from 4,000 to 20,000 pounds per square inch, 8,000 to 10,000 being a fair average. The ultimate strength of good paving brick is still higher, its average value being from 12,000 to 15,000 pounds per square inch.

24. Stone. Sandstone, limestone and granite are the principal building stones. Their ultimate strengths in pounds per square inch are about as follows:

Sandstone,* 5,000 to 16,000, average 8,000.
Limestone,* 8,000 " 16,000, " 10,000.
Granite, 14,000 " 24,000, " 16,000.

*Compression at right angles to the "bed" of the stone.

EXAMPLES FOR PRACTICE.

1. A limestone 12×12 inches on its bed is used as a pier cap, and bears a load of 120,000 pounds. What is its factor of safety? Ans. 12.

2. How large a post (short) is needed to sustain a steady load of 100,000 pounds if the ultimate compressive strength of the wood is 10,000 pounds per square inch? Ans. 8 × 10 inches.

25. Materials in Shear. The principal materials used under shearing stress are timber, wrought iron, steel and cast iron. Partly on account of the difficulty of determining shearing strengths, these are not well known.

26. Timber. The ultimate shearing strengths of the more important woods *along the grain* are about as follows:

Hemlock,	**300**	**pounds per square inch.**
White pine,	**400**	**" "**
Long-leaf yellow pine,	**850**	**" "**
Short-leaf " "	**775**	**" "**
Douglas spruce,	**500**	**" "**
White oak,	**1,000**	**" "**
Red oak,	**1,100**	**" "**

Wood rarely fails by shearing across the grain. Its ultimate shearing strength in that direction is probably four or five times the values above given.

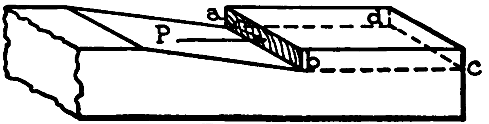

Fig. 6 *a*.

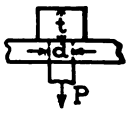

Fig. 6 *b*.

27. Metals. The ultimate shearing strength of wrought iron, steel, and cast iron is about 80 per cent of their respective ultimate tensile strengths.

EXAMPLES FOR PRACTICE.

1. How large a pressure P (Fig. 6 *a*) exerted on the shaded area can the timber stand before it will shear off on the surface *abcd*, if $ab = 6$ inches and $bc = 10$ inches, and the ultimate shearing strength of the timber is 400 pounds per square inch?

Ans. 24,000 pounds.

2. When a bolt is under tension, there is a tendency to tear the bolt and to "strip" or shear off the head. The shorn area would be the surface of the cylindrical hole left in the head. Compute the tensile and shearing unit-stresses when P (Fig. 6 *b*) equals 30,000 pounds, $d = 2$ inches, and $t = 3$ inches.

Ans. { Tensile unit-stress, 9,550 pounds per square inch.
Shearing unit-stress, 1,591 pounds per square inch.

REACTIONS OF SUPPORTS.

28. Moment of a Force. By moment of a force with respect to a point is meant its tendency to produce rotation about that point. Evidently the tendency depends on the magnitude of the force and on the perpendicular distance of the line of action of the force from the point: the greater the force and the perpendicular distance, the greater the tendency; hence *the moment* *of a force with respect to a point equals the product of the force and the perpendicular distance from the force to the point.*

The point with respect to which the moment of one or more forces is taken is called an *origin* or *center of moments*, and the perpendicular distance from an origin of moments to the line of action of a force is called the *arm* of the force with respect to that origin. Thus, if F_1 and F_2 (Fig. 7) are forces, their arms with respect to O′ are a_1' and a_2' respectively, and their moments are $F_1a'_1$ and $F_2a'_2$. With respect to O″ their arms are a_1'' and a_2'' respectively, and their moments are F_1a_1'' and F_2a_2''.

If the force is expressed in pounds and its arm in feet, the moment is in foot-pounds; if the force is in pounds and the arm in inches, the moment is in inch-pounds.

29. A *sign* is given to the moment of a force for convenience; the rule used herein is as follows: *The moment of a force about a point is positive or negative according as it tends to turn the body about that point in the clockwise or counter-clockwise direction**.

Thus the moment (Fig. 7)

of F_1 about O′ is negative, about O″ positive;
" F_2 " O′ " " , about O″ negative.

30. Principle of Moments. In general, a single force of proper magnitude and line of action can balance any number of forces. That single force is called the *equilibrant* of the forces, and the single force that would balance the equilibrant is called the *resultant* of the forces. Or, otherwise stated, the resultant of any number of forces is a force which produces the same effect. It can be proved that *The algebraic sum of the moments of any number of forces with respect to a point, equals the moment of their resultant about that point.*

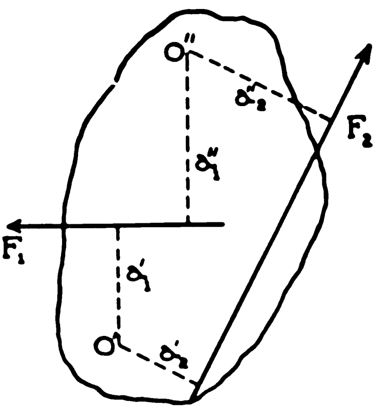

Fig. 7.

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*By clockwise direction is meant that in which the hands of a clock rotate; and by counter clockwise, the opposite direction.

This is a useful principle and is called "principle of moments."

31. All the forces acting upon a body which is at rest are said to be *balanced* or *in equilibrium*. No force is required to balance such forces and hence their equilibrant and resultant are zero.

Since their resultant is zero, *the algebraic sum of the moments of any number of forces which are balanced or in equilibrium equals zero.*

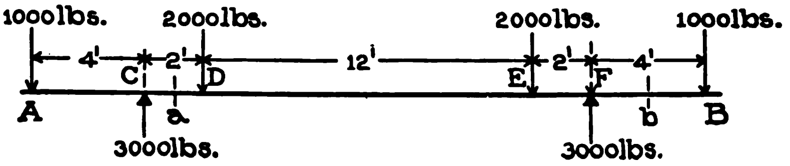


Fig. 8.

This is known as the principle of moments for forces in equilibrium; for brevity we shall call it also "the principle of moments."

The principle is easily verified in a simple case. Thus, let AB (Fig. 8) be a beam resting on supports at C and F. It is evident from the symmetry of the loading that each reaction equals one-half of the whole load, that is, ½ of 6,000=3,000 pounds. (We neglect the weight of the beam for simplicity.)

With respect to C, for example, the moments of the forces are, taking them in order from the left:

—1,000 × 4 = — 4,000 foot-pounds
3,000 × 0 = 0 "
2,000 × 2 = 4,000 "
2,000 × 14 = 28,000 "
—3,000 × 16 = —48,000 "
1,000 × 20 = 20,000 "

The algebraic sum of these moments is seen to equal zero.

Again, with respect to B the moments are:

— 1,000 × 24 = — 24,000 foot-pounds
3,000 × 20 = 60,000 "
—2,000 × 18 = —36,000 "
—2,000 × 6 = — 12,000 "
3,000 × 4 = 12,000 "
1,000 × 0 = 0 "

The sum of these moments also equals zero. In fact, no matter where the center of moments is taken, it will be found in this and any other balanced system of forces that the algebraic sum of their moments equals zero. The chief use that we shall make of this principle is in finding the supporting forces of loaded beams.

32. Kinds of Beams. A *cantilever beam* is one resting on one support or fixed at one end, as in a wall, the other end being free.

A *simple beam* is one resting on two supports.

A *restrained beam* is one fixed at both ends; a beam fixed at one end and resting on a support at the other is said to be restrained at the fixed end and simply supported at the other.

A *continuous beam* is one resting on more than two supports.

33. Determination of Reactions on Beams. The forces which the supports exert on a beam, that is, the "supporting forces," are called *reactions*. We shall deal chiefly with simple beams. The reaction on a cantilever beam supported at one point evidently equals the total load on the beam.

When the loads on a horizontal beam are all vertical (and this is the usual case), the supporting forces are also vertical and *the sum of the reactions equals the sum of the loads*. This principle is sometimes useful in determining reactions, but in the case of simple beams the principle of moments is sufficient. The general method of determining reactions is as follows:

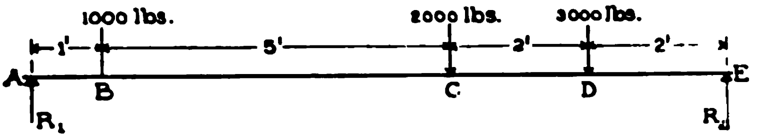


Fig. 9.

1. Write out two equations of moments for all the forces (loads and reactions) acting on the beam with origins of moments at the supports.

2. Solve the equations for the reactions.

3. As a check, try if the sum of the reactions equals the sum of the loads.

Examples. 1. Fig. 9 represents a beam supported at its ends and sustaining three loads. We wish to find the reactions due to these loads.

Let the reactions be denoted by R_1 and R_2 as shown; then the moment equations are:
For origin at A,

$$1{,}000\times 1+2{,}000\times 6+3{,}000\times 8-R_2\times 10=0.$$

For origin at E,

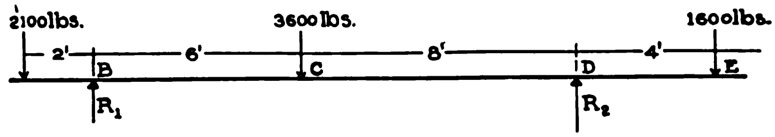


Fig. 10.

$$R_1\times 10-1{,}000\times 9-2{,}000\times 4-3{,}000\times 2=0.$$

The first equation reduces to

$$10\ R_2=1{,}000+12{,}000+24{,}000=37{,}000;\ \text{or}$$

$R_2= 3{,}700$ pounds.

The second equation reduces to

$$10\ R_1=9{,}000+8{,}000+6{,}000=23{,}000;\ \text{or}$$

$R_1= 2{,}300$ pounds.

The sum of the loads is 6,000 pounds and the sum of the reactions is the same; hence the computation is correct.

2. Fig. 10 represents a beam supported at B and D (that is, it has overhanging ends) and sustaining three loads as shown. We wish to determine the reactions due to the loads.

Let R_1 and R_2 denote the reactions as shown; then the moment equations are:
For origin at B,

$$-2{,}100\times 2+0+3{,}600\times 6-R_2\times 14+1{,}600\times 18=0.$$

For origin at D,

$$-2{,}100\times 16+R_1\times 14-3{,}600\times 8+0+1{,}600\times 4=0.$$

The first equation reduces to

$$14\ R_2=-4{,}200+21{,}600+28{,}800=46{,}200;\ \text{or}$$

$R_2= 3{,}300$ pounds.

The second equation reduces to

$$14\ R_1=33{,}600+28{,}800-6{,}400=56{,}000;\ \text{or}$$

$R_1= 4{,}000$ pounds.

The sum of the loads equals 7,300 pounds and the sum of the reactions is the same; hence the computation checks.

3. What are the total reactions in example 1 if the beam weighs 400 pounds?

(1.) Since we already know the reactions due to the loads (2,300 and 3,700 pounds at the left and right ends respectively (see illustration 1 above), we need only to compute the reactions due to the weight of the beam and add. Evidently the reactions due to the weight equal 200 pounds each; hence the

left reaction = 2,300 + 200 = 2,500 pounds, and the
right " = 3,700 + 200 = 3,900 " .

(2.) Or, we might compute the reactions due to the loads and weight of the beam together and directly. In figuring the moment due to the weight of the beam, we imagine the weight as concentrated at the middle of the beam; then its moments with respect to the left and right supports are (400 × 5) and —(400 × 5) respectively. The moment equations for origins at A and E are like those of illustration 1 except that they contain one more term, the moment due to the weight; thus they are respectively:

$$1{,}000 \times 1 + 2{,}000 \times 6 + 3{,}000 \times 8 - R_2 \times 10 + 400 \times 5 = 0,$$

$$R_1 \times 10 - 1{,}000 \times 9 - 2{,}000 \times 4 - 3{,}000 \times 2 - 400 \times 5 = 0.$$

The first one reduces to

$$10\,R_2 = 39{,}000, \text{ or } R = 3{,}900 \text{ pounds};$$

and the second to

$$10\,R_1 = 25{,}000, \text{ or } R_1 = 2{,}500 \text{ pounds}.$$

4. What are the total reactions in example 2 if the beam weighs 42 pounds per foot?

As in example 3, we might compute the reactions due to the weight and then add them to the corresponding reactions due to the loads (already found in example 2), but we shall determine the total reactions due to load and weight directly.

The beam being 20 feet long, its weight is 42 × 20, or 840 pounds. Since the middle of the beam is 8 feet from the left and 6 feet from the right support, the moments of the weight with to the left and right supports are respectively:

$$840 \times 8 = 6{,}720, \text{ and } 840 \times 6 = 5{,}040 \text{ foot-pounds}.$$

The moment equations for all the forces applied to the beam for origins at B and D are like those in example 2, with an additional term, the moment of the weight; they are respectively:

$$-2{,}100 \times 2 + 0 + 3{,}600 \times 6 - R_2 \times 14 + 1{,}600 \times 18 + 6{,}720 = 0,$$

$$-2{,}100 \times 16 + R_1 \times 14 - 3{,}600 \times 8 + 0 + 1{,}600 \times 4 - 5{,}040 = 0.$$

The first equation reduces to

$$14\ R_2=52{,}920, \text{ or } R_2=3{,}780 \text{ pounds},$$

and the second to

$$14\ R_1=61{,}040, \text{ or } R_1=4{,}360 \text{ pounds}.$$

The sum of the loads and weight of beam is 8,140 pounds; and since the sum of the reactions is the same, the computation checks.

EXAMPLES FOR PRACTICE.

1. AB (Fig. 11) represents a simple beam supported at its ends. Compute the reactions, neglecting the weight of the beam.

Ans. { Right reaction = 1,443.75 pounds.
Left reaction = 1,556.25 pounds.

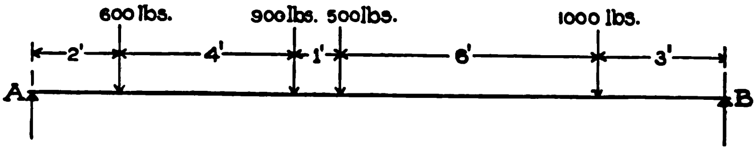


Fig. 11.

2. Solve example 1 taking into account the weight of the beam, which suppose to be 400 pounds.

Ans. { Right reaction = 1,643.75 pounds.
Left reaction = 1,756.25 pounds.

3. Fig. 12 represents a simple beam weighing 800 pounds supported at A and B, and sustaining three loads as shown. What are the reactions ?

Ans. { Right reaction = 2,014.28 pounds.
Left reaction = 4,785.72 pounds.

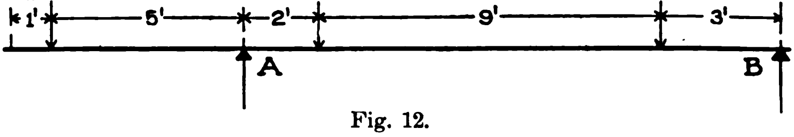


Fig. 12.

4. Suppose that in example 3 the beam also sustains a uniformly distributed load (as a floor) over its entire length, of 500 pounds per foot. Compute the reactions due to all the loads and the weight of the beam.

Ans. { Right reaction = 4,871.43 pounds.
Left reaction = 11,928.57 pounds.

EXTERNAL SHEAR AND BENDING MOMENT.

On almost every cross-section of a loaded beam there are three kinds of stress, namely tension, compression and shear. The first two are often called *fibre stresses* because they act along the real fibres of a wooden beam or the imaginary ones of which we may suppose iron and steel beams composed. Before taking up the subject of these stresses in beams it is desirable to study certain quantities relating to the loads, and on which the stresses in a beam depend. These quantities are called *external shear* and *bending moment*, and will now be discussed.

34. External Shear. By external shear at (or for) any section of a loaded beam is meant the algebraic sum of all the loads (including weight of beam) and reactions on *either side* of the section. This sum is called external shear because, as is shown later, it equals the shearing stress (internal) at the section. For brevity, we shall often say simply "shear" when external shear is meant.

35. Rule of Signs. In computing external shears, it is customary to give the plus sign to the reactions and the minus sign to the loads. But in order to get the same sign for the external shear whether computed from the right or left, we *change the sign* of the sum when computed from the loads and reactions *to the right*. Thus for section *a* of the beam in Fig. 8 the algebraic sum is, when computed from the left,

$$-1{,}000 + 3{,}000 = +2{,}000 \text{ pounds;}$$

and when computed from the right,

$$1{,}000 - 3{,}000 + 2{,}000 - 2{,}000 = -2{,}000 \text{ pounds.}$$

The external shear at section *a* is $+2{,}000$ pounds.

Again, for section *b* the algebraic sum is, when computed from the left,

$$-1{,}000 + 3{,}000 - 2{,}000 - 2{,}000 + 3{,}000 = +1{,}000 \text{ pounds;}$$

and when computed from the right, $-1{,}000$ pounds.

The external shear at the section is $+1{,}000$ pounds.

It is usually convenient to compute the shear at a section from the forces to the right or left according as there are fewer forces (loads and reactions) on the right or left sides of the section.

36. Units for Shears. It is customary to express external shears in pounds, but any other unit for expressing force and weight (as the ton) may be used.

37. Notation. We shall use V to stand for external shear at any section, and the shear at a particular section will be denoted by that letter subscripted; thus V_1, V_2, etc., stand for the shears at sections one, two, etc., feet from the left end of a beam.

The shear has different values just to the left and right of a support or concentrated load. We shall denote such values by V' and V''; thus V_5' and V_5'' denote the values of the shear at sections a little less and a little more than 5 feet from the left end respectively.

Examples. 1. Compute the shears for sections one foot apart in the beam represented in Fig. 9, neglecting the weight of the beam. (The right and left reactions are 3,700 and 2,300 pounds respectively; see example 1, Art. 33.)

All the following values of the shear are computed from the left. The shear just to the right of the left support is denoted by V_0'', and $V_0'' = 2{,}300$ pounds. The shear just to the left of B is denoted by V_1', and since the only force to the left of the section is the left reaction, $V_1' = 2{,}300$ pounds. The shear just to the right of B is denoted by V_1'', and since the only forces to the left of this section are the left reaction and the 1,000-pound load, $V_1'' = 2{,}300 - 1{,}000 = 1{,}300$ pounds. To the left of all sections between B and C, there are but two forces, the left reaction and the 1,000-pound load; hence the shear at any of those sections equals $2{,}300 - 1{,}000 = 1{,}300$ pounds, or

$$V_2 = V_3 = V_4 = V_5 = V_6' = 1{,}300 \text{ pounds.}$$

The shear just to the right of C is denoted by V_6''; and since the forces to the left of that section are the left reaction and the 1,000- and 2,000-pound loads,

$$V_6'' = 2{,}300 - 1{,}000 - 2{,}000 = -700 \text{ pounds.}$$

Without further explanation, the student should understand that

$$V_7 = +2{,}300 - 1{,}000 - 2{,}000 = -700 \text{ pounds,}$$
$$V_8' = -700,$$
$$V_8'' = +2{,}300 - 1{,}000 - 2{,}000 - 3{,}000 = -3{,}700,$$
$$V_9 = V_{10}' = -3{,}700,$$
$$V_{10}'' = +2.300 - 1{,}000 - 2{,}000 - 3{,}000 + 3{,}700 = 0$$

2. A simple beam 10 feet long, and supported at each end, weighs 400 pounds, and bears a uniformly distributed load of 1,600 pounds. Compute the shears for sections two feet apart.

Evidently each reaction equals one-half the sum of the load and weight of the beam, that is, $\frac{1}{2}(1,600 + 400) = 1,000$ pounds. To the left of a section 2 feet from the left end, the forces acting on the beam consist of the left reaction, the load on that part of the beam, and the weight of that part; then since the load and weight of the beam *per foot* equal 200 pounds,

$$V_2 = 1,000 - 200 \times 2 = 600 \text{ pounds.}$$

To the left of a section four feet from the left end, the forces are the left reaction, the load on that part of the beam, and the weight; hence

$$V_4 = 1,000 - 200 \times 4 = 200 \text{ pounds.}$$

Without further explanation, the student should see that

$$V_6 = 1,000 - 200 \times 6 = -200 \text{ pounds,}$$
$$V_8 = 1,000 - 200 \times 8 = -600 \text{ pounds,}$$
$$V_{10}' = 1,000 - 200 \times 10 = -1,000 \text{ pounds,}$$
$$V_{10}'' = 1,000 - 200 \times 10 + 1,000 = 0.$$

3. Compute the values of the shear in example 1, taking into account the weight of the beam (400 pounds). (The right and left reactions are then 3,900 and 2,500 pounds respectively; see example 3, Art. 33.)

We proceed just as in example 1, except that in each computation we include the weight of the beam to the left of the section (or to the right when computing from forces to the right). The weight of the beam being 40 pounds per foot, then (computing from the left)

$$V_0'' = +2,500 \text{ pounds,}$$
$$V_1' = +2,500 - 40 = +2,460,$$
$$V_1'' = +2,500 - 40 - 1,000 = +1,460,$$
$$V_2 = +2,500 - 1,000 - 40 \times 2 = +1,420,$$
$$V_3 = +2,500 - 1,000 - 40 \times 3 = +1,380,$$
$$V_4 = +2,500 - 1,000 - 40 \times 4 = +1,340,$$
$$V_5 = +2,500 - 1,000 - 40 \times 5 = +1,300,$$
$$V_6' = +2,500 - 1,000 - 40 \times 6 = +1,260,$$
$$V_6'' = +2,500 - 1,000 - 40 \times 6 - 2,000 = -740,$$
$$V_7 = +2,500 - 1,000 - 2,000 - 40 \times 7 = -780,$$

$$
\begin{aligned}
V_8' &= +2{,}500-1{,}000-2{,}000-40\times 8 = -820,\\
V_8'' &= +2{,}500-1{,}000-2{,}000-40\times 8-3{,}000 = -3{,}820,\\
V_9 &= +2{,}500-1{,}000-2{,}000-3{,}000-40\times 9 = -3{,}860,\\
V'_{10} &= +2{,}500-1{,}000-2{,}000-3{,}000-40\times 10 = -3{,}900,\\
V''_{10} &= +2{,}500-1{,}000-2{,}000-3{,}000-40\times 10 + 3{,}900 = 0.
\end{aligned}
$$

Computing from the right, we find, as before, that

$$
\begin{aligned}
V_7 &= -(3{,}900-3{,}000-40\times 3) = -780 \text{ pounds},\\
V_8' &= -(3{,}900-3{,}000-40\times 2) = -820,\\
V_8'' &= -(3{,}900-40\times 2) = -3{,}820,
\end{aligned}
$$

etc., etc.

EXAMPLES FOR PRACTICE.

1. Compute the values of the shear for sections of the beam represented in Fig. 10, neglecting the weight of the beam. (The right and left reactions are 3,300 and 4,000 pounds respectively; see example 2, Art. 33.)

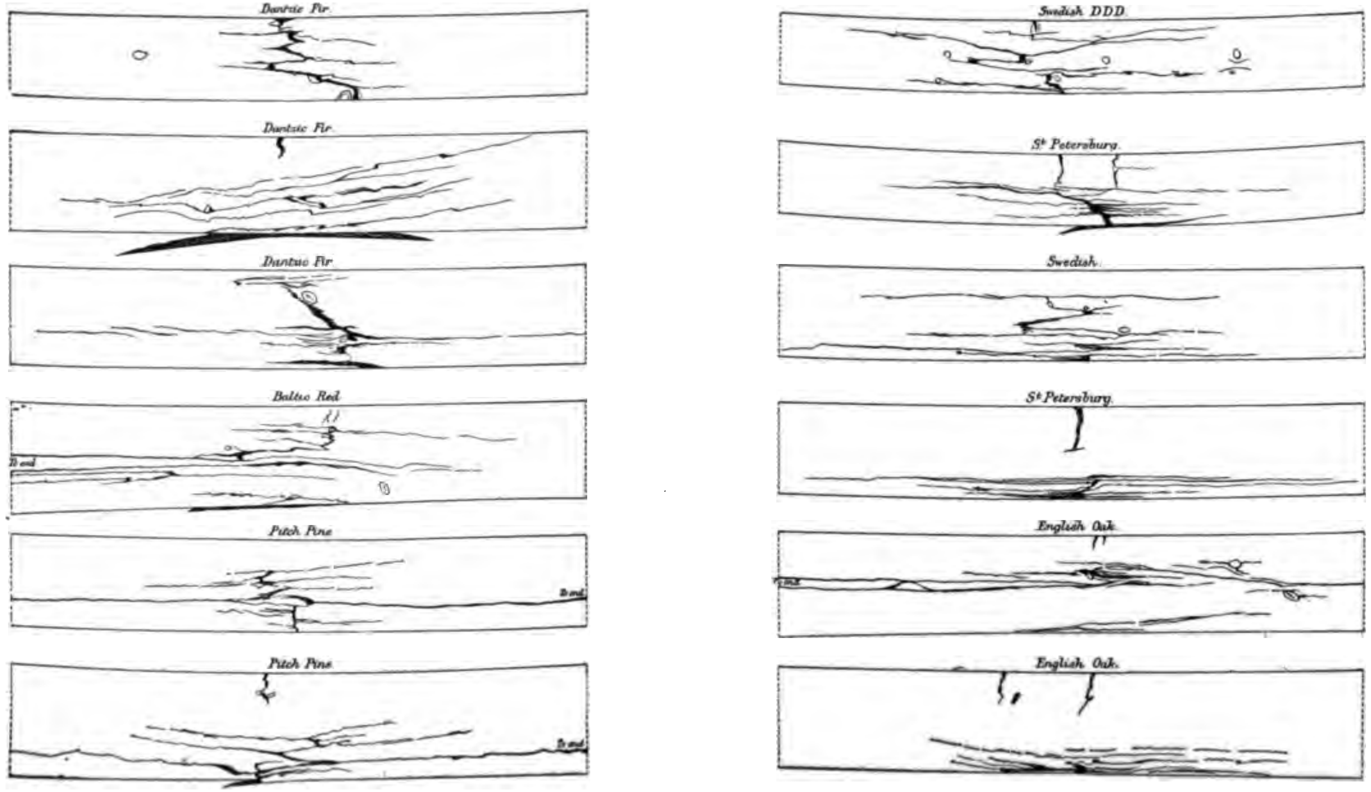
$$
\text{Ans.}\begin{cases}
V_1 = V_2' = -2{,}100 \text{ pounds},\\
V_2'' = V_3 = V_4 = V_5 = V_6 = V_7 = V'_8 = +1{,}900,\\
V_8'' = V_9 = V_{10} = V_{11} = V_{12} = V_{13} = V_{14} = V_{15} = V'_{16} = -1{,}700,\\
V''_{16} = V_{17} = V_{18} = V_{19} = V'_{20} = +1{,}600.
\end{cases}
$$

2. Solve the preceding example, taking into account the weight of the beam, 42 pounds per foot. (The right and left reactions are 3,780 and 4,360 pounds respectively; see example 4, Art. 33.)

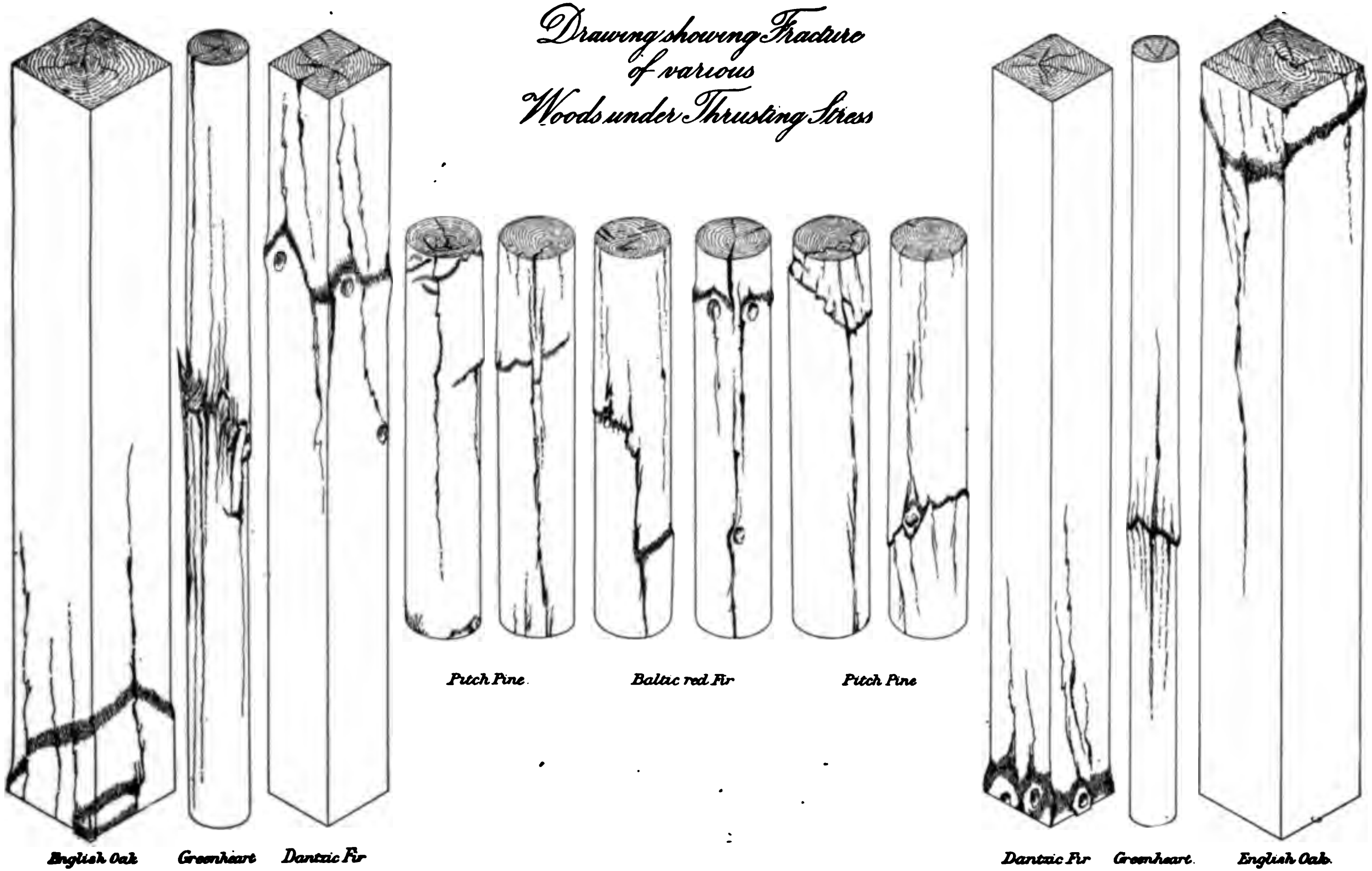
$$
\text{Ans.}\begin{cases}
\begin{array}{lll}
V_0'' = -2{,}100 \text{ lbs.} & V_7 = +1{,}966 \text{ lbs.} & V_{14} = -1{,}928 \text{ lbs.}\\
V_1 = -2{,}142 & V_8' = +1{,}924 & V_{15} = -1{,}970\\
V_2' = -2{,}184 & V_8'' = -1{,}676 & V_{16}' = -2{,}012\\
V_2'' = +2{,}176 & V_9 = -1{,}718 & V_{16}'' = +1{,}768\\
V_3 = +2{,}134 & V_{10} = -1{,}760 & V_{17} = +1{,}726\\
V_4 = +2{,}092 & V_{11} = -1{,}802 & V_{18} = +1{,}684\\
V_5 = +2{,}050 & V_{12} = -1{,}844 & V_{19} = +1{,}642\\
V_6 = +2{,}008 & V_{13} = -1{,}886 & V_{20}' = +1{,}600
\end{array}
\end{cases}
$$

3. Compute the values of the shear at sections one foot apart in the beam of Fig. 11, neglecting the weight. (The right and left reactions are 1,444 and 1,556 pounds respectively; see example 1, Art. 33.)

Drawing showing Fracture of various Woods under Bending Stress.



DRAWING SHOWING FRACTURE OF VARIOUS WOODS UNDER BENDING STRESS
Reproduced from "Strength and Properties of Materials," by W. G. Kirkaldy.



DRAWING SHOWING FRACTURE OF VARIOUS WOODS UNDER THRUSTING STRESS

Reproduced from "Strength and Properties of Materials," by W. G. Kirkaldy.

$$\text{Ans.}\left\{\begin{aligned}
&V_0'' = V_1 = V_2' = +1{,}556 \text{ pounds},\\
&V_2'' = V_3 = V_4 = V_5 = V_6' = +956,\\
&V_6'' = V_7' = +56,\\
&V_7'' = V_8 = V_9 = V_{10} = V_{11} = V_{12} = V_{13}' = -444,\\
&V_{13}'' = V_{14} = V_{15} = V_{16}' = -1{,}444.
\end{aligned}\right.$$

4. Compute the vertical shear at sections one foot apart in the beam of Fig. 12, taking into account the weight of the beam 800 pounds, and a distributed load of 500 pounds per foot. (The right and left reactions are 4,870 and 11,930 pounds respectively: see examples 3 and 4, Art. 33.)

$$\text{Ans.}\left\{\begin{aligned}
&V_0 = 0 && V_7 = +6{,}150 \text{ lbs.} && V_{15} = +830 \text{ lbs.}\\
&V_1' = -540 \text{ lbs.} && V_8' = +5{,}610 && V_{16} = +290\\
&V_1'' = -2{,}540 && V_8'' = +4{,}610 && V_{17}' = -250\\
&V_2 = -3{,}080 && V_9 = +4{,}070 && V_{17}'' = -3{,}250\\
&V_3 = -3{,}620 && V_{10} = +3{,}530 && V_{18} = -3{,}790\\
&V_4 = -4{,}160 && V_{11} = +2{,}990 && V_{19} = -4{,}330\\
&V_5 = -4{,}700 && V_{12} = +2{,}450 && V_{20}' = -4{,}870\\
&V_6' = -5{,}240 && V_{13} = +1{,}910 && V_{20}'' = 0\\
&V_6'' = +6{,}690 && V_{14} = +1{,}370 &&
\end{aligned}\right.$$

38. Shear Diagrams. The way in which the external shear varies from section to section in a beam can be well represented by means of a diagram called a *shear diagram*. To construct such a diagram for any loaded beam,

1. Lay off a line equal (by some scale) to the length of the beam, and mark the positions of the supports and the loads. (This is called a "base-line.")

2. Draw a line such that the distance of any point of it from the base equals (by some scale) the shear at the corresponding section of the beam, and so that the line is above the base where the shear is positive, and below it where negative. (This is called a *shear line*, and the distance from a point of it to the base is called the "ordinate" from the base to the shear line at that point.)

We shall explain these diagrams further by means of illustrative examples.

Examples. 1. It is required to construct the shear diagram for the beam represented in Fig. 13, *a* (a copy of Fig. 9).

Lay off A'E' (Fig. 13, *b*) to represent the beam, and mark the positions of the loads B', C' and D'. In example 1, Art. 37, we computed the values of the shear at sections one foot apart; hence we lay off ordinates at points on A'E' one foot apart, to represent those shears.

Use a scale of 4,000 pounds to one inch. Since the shear for any section in AB is 2,300 pounds, we draw a line *ab* parallel to the base 0.575 inch (2,300÷4,000) therefrom; this is the shear line for the portion AB. Since the shear for any section in BC equals 1,300 pounds, we draw a line *b'c* parallel to the base and 0.325 inch (1,300÷4,000) therefrom; this is the shear line for the portion BC. Since the shear for any section in CD is −700 pounds, we draw a line *c'd* below the base and 0.175 inch (700÷4,000) therefrom; this is the shear line for the portion CD. Since the shear for any section in DE equals −3,700 lbs., we draw a line *d'e* below the base and 0.925 inch (3,700÷4,000) therefrom; this is the shear line for the portion DE. Fig. 13, *b*, is the required shear diagram.

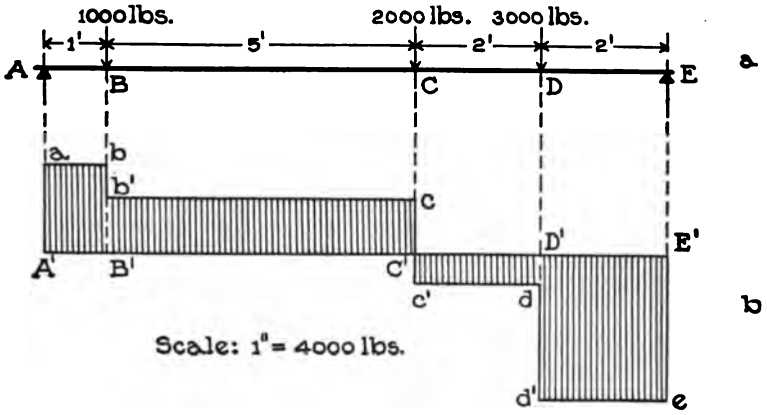


Fig. 13.

2. It is required to construct the shear diagram for the beam of Fig. 14, *a* (a copy of Fig. 9), taking into account the weight of the beam, 400 pounds.

The values of the shear for sections one foot apart were computed in example 3, Art. 37, so we have only to erect ordinates at the various points on a base line A'E' (Fig. 14, *b*), equal to those values. We shall use the same scale as in the preceding illustration, 4,000 pounds to an inch. Then the lengths of the ordinates corresponding to the values of the shear (see example 3, Art. 37) are respectively:

$$2{,}500 \div 4{,}000 = 0.625 \text{ inch}$$
$$2{,}460 \div 4{,}000 = 0.615 \quad \text{"}$$
$$1{,}460 \div 4{,}000 = 0.365 \quad \text{"}$$
$$\text{etc.} \qquad \text{etc.}$$

Laying these ordinates off from the base (upwards or downwards according as they correspond to positive or negative shears), we get *ab*, *b'c*, *c'd*, and *d'e* as the shear lines.

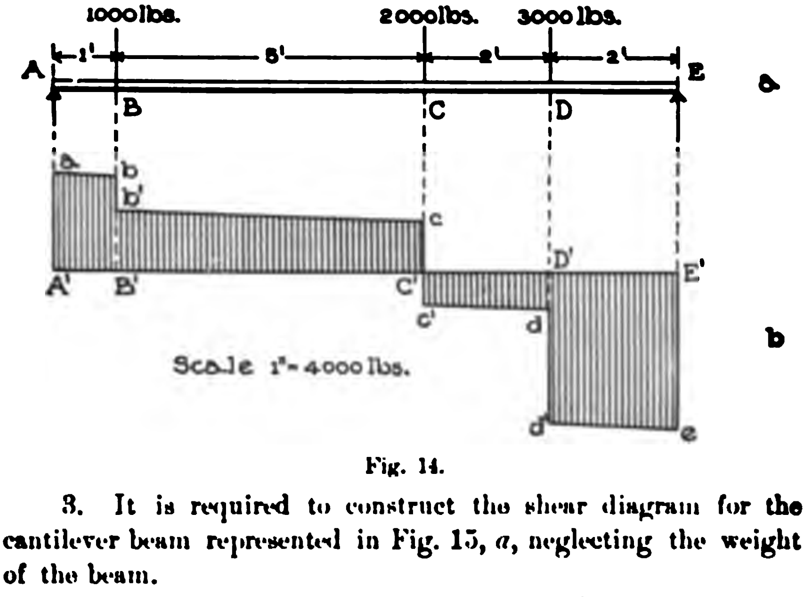


Fig. 14.

3. It is required to construct the shear diagram for the cantilever beam represented in Fig. 15, *a*, neglecting the weight of the beam.

The value of the shear for any section in AB is − 500 pounds; for any section in BC, − 1,500 pounds; and for any section in CD, − 3,500 pounds. Hence the shear lines are *ab*, *b'c*, *c'd*. The scale being 5,000 pounds to an inch,

$$A'a = 500 \div 5{,}000 = 0.1 \text{ inch,}$$
$$B'b' = 1{,}500 \div 5{,}000 = 0.3 \quad \text{"}$$
$$C'c' = 3{,}500 \div 5{,}000 = 0.7 \quad \text{"}$$

The shear lines are all below the base because all the values of the shear are negative.

4. Suppose that the cantilever of the preceding illustration sustains also a uniform load of 200 pounds per foot (see Fig. 16, *a*). Construct a shear diagram.

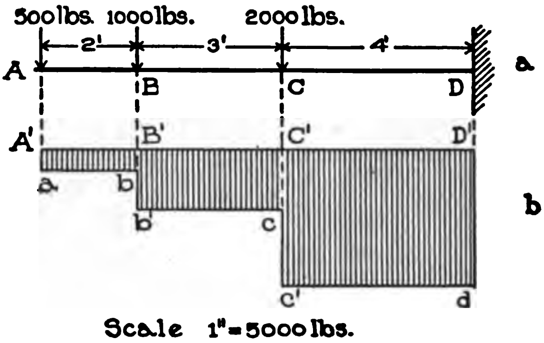


Fig. 15.

First, we compute the values of the shear at several sections. Thus

$$
\begin{aligned}
V_0'' &= -500 \text{ pounds},\\
V_1 &= -500 - 200 = -700,\\
V_2' &= -500 - 200 \times 2 = -900,\\
V_2'' &= -500 - 200 \times 2 - 1{,}000 = -1{,}900,\\
V_3 &= -500 - 1{,}000 - 200 \times 3 = -2{,}100,\\
V_4 &= -500 - 1{,}000 - 200 \times 4 = -2{,}300,\\
V_5' &= -500 - 1{,}000 - 200 \times 5 = -2{,}500,\\
V_5'' &= -500 - 1{,}000 - 200 \times 5 - 2{,}000 = -4{,}500,\\
V_6 &= -500 - 1{,}000 - 2{,}000 - 200 \times 6 = -4{,}700,\\
V_7 &= -500 - 1{,}000 - 2{,}000 - 200 \times 7 = -4{,}900,\\
V_8 &= -500 - 1{,}000 - 2{,}000 - 200 \times 8 = -5{,}100,\\
V_9 &= -500 - 1{,}000 - 2{,}000 - 200 \times 9 = -5{,}300.
\end{aligned}
$$

The values, being negative, should be plotted downward. To a scale of 5,000 pounds to the inch they give the shear lines ab, $b'c$, $c'd$ (Fig. 16, *b*).

EXAMPLES FOR PRACTICE.

1. Construct a shear diagram for the beam represented in Fig. 10, neglecting the weight of the beam (see example 1, Art. 37).

2. Construct the shear diagram for the beam represented in Fig. 11, neglecting the weight of the beam (see example 3, Art. 37).

3. Construct the shear diagram for the beam of Fig. 12 when it sustains, in addition to the loads represented, its own weight, 800 pounds, and a uniform load of 500 pounds per foot (see example 4, Art. 37).

4. Figs. *a*, cases 1 and 2, Table B, represent two cantilever beams, the first bearing a concentrated load P at the free end, and the second a uniform load W. Figs. *b* are the corresponding shear diagrams. Take P and W equal to 1,000 pounds, and satisfy yourself that the diagrams are correct.

and satisfy yourself that the diagrams are correct.

5. Figs. *a*, cases 3 and 4, same table, represent simple beams supported at their ends, the first bearing a concentrated load P at the middle, and the second a uniform load W. Figs. *b* are the corresponding shear diagrams. Take P and W equal to 1,000 pounds, and satisfy yourself that they are correct.

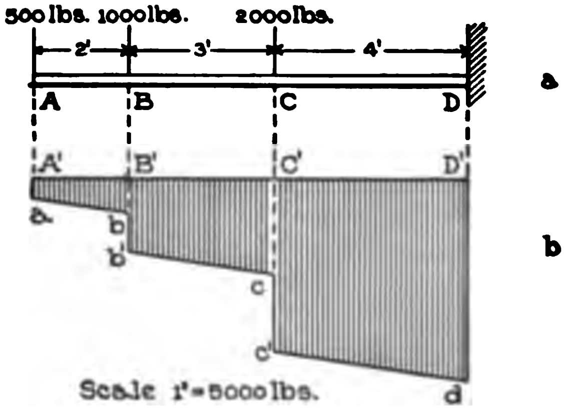


Fig. 16.

39. Maximum Shear. It is sometimes desirable to know the greatest or maximum value of the shear in a given case. This value can always be found with certainty by constructing the shear diagram, from which the maximum value of the shear is evident at a glance. In any case it can most readily be computed if one knows the section for which the shear is a maximum. The student should examine all the shear diagrams in the preceding articles and those that he has drawn, and see that

1. *In cantilevers fixed in a wall, the maximum shear occurs at the wall.*

2. *In simple beams, the maximum shear occurs at a section next to one of the supports.*

By the use of these propositions one can determine the value of the maximum shear without constructing the whole shear diagram. Thus, it is easily seen (referring to the diagrams, page 53) that for a

Cantilever, end load P,	maximum shear	=P
“ , uniform load W,	“ “	=W
Simple beam, middle load P,	“ “	$=\frac{1}{2}$P
“ “ , uniform “ W,	“ “	$=\frac{1}{2}$W

40. Bending Moment. By bending moment at (or for) a section of a loaded beam, is meant the algebraic sum of the moments of all the loads (including weight of beam) and reactions to the left or right of the section with respect to any point in the section.

41. Rule of Signs. We follow the rule of signs previously stated (Art. 29) that the moment of a force which tends to produce clockwise rotation is plus, and that of a force which tends to produce counter-clockwise rotation is minus; but in order to get the same sign for the bending moment whether computed from the right or left, we *change the sign* of the sum of the moments when computed from the loads and reactions *on the right.* Thus for section *a*, Fig. 8, the algebraic sums of the moments of the forces are:

when computed from the left,

$$-1{,}000\times5+3{,}000\times1=-2{,}000 \text{ foot-pounds;}$$

and when computed from the right,

$$1{,}000\times19-3{,}000\times15+2{,}000\times13+2{,}000\times1=+2{,}000 \text{ foot-pounds.}$$

The bending moment at section *a* is –2,000 foot-pounds.

Again, for section *b*, the algebraic sums of the moments of the forces are:

when computed from the left,

$$-1{,}000\times22+3{,}000\times18-2{,}000\times16-2{,}000\times4+3{,}000\times2= -2{,}000 \text{ foot-pounds;}$$

and when computed from the right,

$$1{,}000\times2=+2{,}000 \text{ foot-pounds.}$$

The bending moment at the section is –2,000 foot-pounds.

It is usually convenient to compute the bending moment for a section from the forces to the right or left according as there are fewer forces (loads and reactions) on the right or left side of the section.

42. Units. It is customary to express bending moments in inch-pounds, but often the foot-pound unit is more convenient. *To reduce foot-pounds to inch-pounds, multiply by twelve.*

43. Notation. We shall use M to denote bending moment at any section, and the bending moment at a particular section will be denoted by that letter subscripted; thus M_1, M_2, etc., denote values of the bending moment for sections one, two, etc., feet from the left end of the beam.

Examples. 1. Compute the bending moments for sections one foot apart in the beam represented in Fig. 9, neglecting the weight of the beam. (The right and left reactions are 3,700 and 2,300 pounds respectively. See example 1, Art. 33.)

Since there are no forces acting on the beam to the left of the left support, $M_0 = 0$. To the left of the section one foot from the left end there is but one force, the left reaction, and its arm is one foot; hence $M_1 = 2{,}300 \times 1 = 2{,}300$ foot-pounds. To the left of a section two feet from the left end there are two forces, 2,300 and 1,000 pounds, and their arms are 2 feet and 1 foot respectively; hence $M_2 = 2{,}300 \times 2 - 1{,}000 \times 1 = 3{,}600$ foot-pounds. At the left of all sections between B and C there are only two forces, 2,300 and 1,000 pounds; hence

$$
\begin{aligned}
M_3 &= 2{,}300 \times 3 - 1{,}000 \times 2 = 4{,}900 \text{ foot-pounds,}\\
M_4 &= 2{,}300 \times 4 - 1{,}000 \times 3 = 6{,}200 \quad \text{"}\\
M_5 &= 2{,}300 \times 5 - 1{,}000 \times 4 = 7{,}500 \quad \text{"}\\
M_6 &= 2{,}300 \times 6 - 1{,}000 \times 5 = 8{,}800 \quad \text{"}
\end{aligned}
$$

To the right of a section seven feet from the left end there are two forces, the 3,000 pound load and the right reaction (3,700 pounds), and their arms with respect to an origin in that section are respectively one foot and three feet; hence

$$M_7 = 3{,}700 \times 3 - 3{,}000 \times 1 = 8{,}100 \text{ foot pounds.}$$

To the right of any section between E and D there is only one force, the right reaction; hence

$$M_8 = -(-3{,}700 \times 2) = 7{,}400 \text{ foot-pounds,}$$
$$M_9 = -(-3{,}700 \times 1) = 3{,}700 \quad \text{``}$$

Clearly $M_{10}=0$.

2. A simple beam 10 feet long and supported at its ends weighs 400 pounds, and bears a uniformly distributed load of 1,600 pounds. Compute the bending moments for sections two feet apart.

Each reaction equals one-half the whole load, that is, $\frac{1}{2}$ of $(1{,}600+400)=1{,}000$ pounds, and the load per foot including weight of the beam is 200 pounds. The forces acting on the beam to the left of the first section, two feet from the left end, are the left reaction (1,000 pounds) and the load (including weight) on the part of the beam to the left of the section (400 pounds). The arm of the reaction is 2 feet and that of the 400-pound force is 1 foot (the distance from the middle of the 400-pound load to the section). Hence

$$M_2 = +1{,}000 \times 2 - 400 \times 1 = +1{,}600 \text{ foot-pounds.}$$

The forces to the left of the next section, 4 feet from the left end, are the left reaction and all the load (including weight of beam) to the left (800 pounds). The arm of the reaction is 4 feet, and that of the 800-pound force is 2 feet; hence

$$M_4 = +1{,}000 \times 4 - 800 \times 2 = +2{,}400 \text{ foot-pounds.}$$

Without further explanation the student should see that

$$M_6 = +1{,}000 \times 6 - 1{,}200 \times 3 = +2{,}400 \text{ foot-pounds,}$$
$$M_8 = +1{,}000 \times 8 - 1{,}600 \times 4 = +1{,}600 \quad \text{``}$$

Evidently $M_0 = M_{10} = 0$.

3. Compute the values of the bending moment in example 1, taking into account the weight of the beam, 400 pounds. (The right and left reactions are respectively 3,900 and 2,500 pounds; see example 3, Art. 33.)

We proceed as in example 1, except that the moment of the weight of the beam to the left of each section (or to the right when computing from forces to the right) must be included in the respective moment equations. Thus, computing from the left,

$M_0 = 0$

$M_1 = +2{,}500\times 1-40\times \tfrac{1}{2} = +2{,}480$ foot-pounds,

$M_2 = +2{,}500\times 2-1{,}000\times 1-80\times 1 = +3{,}920,$

$M_3 = +2{,}500\times 3-1{,}000\times 2-120\times 1\tfrac{1}{2} = +5{,}320,$

$M_4 = +2{,}500\times 4-1{,}000\times 3-160\times 2 = +6{,}680,$

$M_5 = +2{,}500\times 5-1{,}000\times 4-200\times 2\tfrac{1}{2} = +8{,}000,$

$M_6 = +2{,}500\times 6-1{,}000\times 5-240\times 3 = +9{,}280.$

Computing from the right,

$M_7 = -(-3{,}900\times 3+3{,}000\times 1+120\times 1\tfrac{1}{2}) = +8{,}520,$

$M_8 = -(-3{,}900\times 2+80\times 1) = +7{,}720,$

$M_9 = -(-3{,}900\times 1+40\times \tfrac{1}{2}) = +3{,}880,$

$M_{10} = 0.$

EXAMPLES FOR PRACTICE.

1. Compute the values of the bending moment for sections one foot apart, beginning one foot from the left end of the beam represented in Fig. 10, neglecting the weight of the beam. (The right and left reactions are 3,300 and 4,000 pounds respectively; see example 2, Art. 33.)

	$M_1 = +2{,}100$	$M_6 = +3{,}400$	$M_{11} = +2{,}100$	$M_{16} = 6{,}400$
Ans.	$M_2 = +4{,}200$	$M_7 = +5{,}300$	$M_{12} = -400$	$M_{17} = 4{,}800$
(in foot-	$M_3 = 2{,}300$	$M_8 = +7{,}200$	$M_{13} = 1{,}300$	$M_{18} = 3{,}200$
pounds)	$M_4 = -400$	$M_9 = +5{,}500$	$M_{14} = 3{,}000$	$M_{19} = 1{,}600$
	$M_5 = +1{,}500$	$M_{10} = +3{,}800$	$M_{15} = +4{,}700$	$M_{20} = 0$

2. Solve the preceding example, taking into account the weight of the beam, 42 pounds per foot. (The right and left reactions are 3,780 and 4,360 pounds respectively; see example 4, Art. 33.)

	$M_1 = 2{,}121$	$M_6 = +4{,}084$	$M_{11} = +2{,}799$	$M_{16} = 6{,}736$
Ans.	$M_2 = 4{,}284$	$M_7 = +6{,}071$	$M_{12} = 976$	$M_{17} = 4{,}989$
(in foot-	$M_3 = +2{,}129$	$M_8 = +8{,}016$	$M_{13} = 889$	$M_{18} = 3{,}284$
pounds)	$M_4 = -16$	$M_9 = +6{,}319$	$M_{14} = 2{,}796$	$M_{19} = 1{,}621$
	$M_5 = +2{,}055$	$M_{10} = +4{,}580$	$M_{15} = 4{,}745$	$M_{20} = 0$

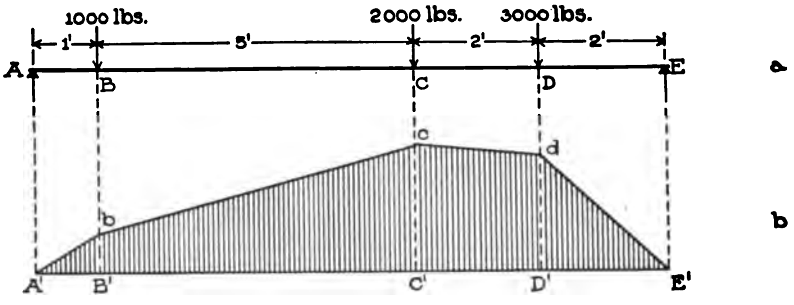
3. Compute the bending moments for sections one foot apart, of the beam represented in Fig. 11, neglecting the weight. (The right and left reactions are 1,444 and 1,556 pounds respectively; see example 1, Art. 33.)

$$\begin{array}{l}\text{Ans.}\\ \text{(in foot-}\\ \text{pounds)}\end{array}\left\{\begin{array}{llll} M_1=+1{,}556 & M_5=+5{,}980 & M_9=+6{,}104 & M_{13}=+4{,}328\\ M_2=+3{,}112 & M_6=+6{,}936 & M_{10}=+5{,}660 & M_{14}=+2{,}884\\ M_3=+4{,}068 & M_7=+6{,}992 & M_{11}=+5{,}216 & M_{15}=+1{,}440\\ M_4=+5{,}024 & M_8=+6{,}548 & M_{12}=+4{,}772 & M_{16}=\quad 0 \end{array}\right.$$

4 Compute the bending moments at sections one foot apart in the beam of Fig. 12, taking into account the weight of the beam, 800 pounds, and a uniform load of 500 pounds per foot. (The right and left reactions are 4,870 and 11,930 pounds respectively; see Exs. 3 and 4, Art. 33.)

$$\begin{array}{l}\text{Ans.}\\ \text{(in foot-}\\ \text{pounds)}\end{array}\left\{\begin{array}{llll} M_1=-\ \ 270 & M_6=-19{,}720 & M_{11}=+\ 3{,}980 & M_{16}=12{,}180\\ M_2=-\ 3{,}080 & M_7=-13{,}300 & M_{12}=+\ 6{,}700 & M_{17}=12{,}200\\ M_3=-\ 6{,}430 & M_8=-\ 7{,}420 & M_{13}=+\ 8{,}880 & M_{18}=\ 8{,}680\\ M_4=-10{,}320 & M_9=-\ 3{,}080 & M_{14}=+10{,}520 & M_{19}=\ 4{,}620\\ M_5=-14{,}750 & M_{10}=+\ \ 720 & M_{15}=+11{,}620 & M_{20}=\quad 0 \end{array}\right.$$

44. Moment Diagrams. The way in which the bending moment varies from section to section in a loaded beam can be well represented by means of a diagram called a *moment diagram*. To construct such a diagram for any loaded beam,



Scale: 1 = 10000 ft.-lbs

Fig. 17.

1. Lay off a base-line just as for a shear diagram (see Art. 38).

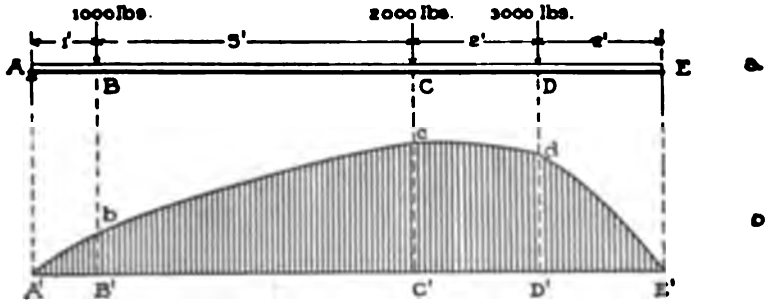
2. Draw a line such that the distance from any point of it to the base-line equals (by some scale) the value of the bending moment at the corresponding section of the beam, and so that the line is above the base where the bending moment is positive and below it where it is negative. (This line is called a "moment line.")

Examples. 1. It is required to construct a moment diagram for the beam of Fig. 17, *a* (a copy of Fig. 9), loaded as there shown.

Lay off A'E' (Fig. 17, *b*) as a base. In example 1, Art. 43, we computed the values of the bending moment for sections one foot apart, so we erect ordinates at points of A'E' one foot apart, to represent the bending moments.

We shall use a scale of 10,000 foot-pounds to the inch; then the ordinates (see example 1, Art. 43, for values of M) will be:

One foot from left end, 2,300 : 10,000 = 0.23 inch,
Two feet " " " 3,600 : 10,000 = 0.36 "
Three " " " " 4,900 : 10,000 = 0.49 "
Four " " " " 6,200 : 10,000 = 0.62 "
etc., etc.



Scale: 1" = 10000 ft-lbs.

Fig. 18.

Laying these ordinates off, and joining their ends in succession, we get the line *A'bcdE'*, which is the bending moment line. Fig. 17, *b*, is the moment diagram.

2. It is required to construct the moment diagram for the beam, Fig. 18, *a* (a copy of Fig. 9), taking into account the weight of the beam, 400 pounds.

The values of the bending moment for sections one foot apart were computed in example 3, Art. 43. So we have only to lay off ordinates equal to those values, one foot apart, on the base A'E' (Fig. 18, *b*).

To a scale of 10,000 foot-pounds to the inch the ordinates (see example 3, Art. 43, for values of M) are:

At left end, 0
One foot from left end, $2{,}480 \div 10{,}000 = 0.248$ inch
Two feet " " " $3{,}920 \div 10{,}000 = 0.392$ "
Three " " " " $5{,}320 \div 10{,}000 = 0.532$ "
Four " " " " $6{,}680 \div 10{,}000 = 0.668$ "

Laying these ordinates off at the proper points, we get A'*bcd*E as the moment line.

3. It is required to construct the moment diagram for the cantilever beam represented in Fig. 19, *a*, neglecting the weight of the beam. The bending moment at B equals

$$-500 \times 2 = -1{,}000 \text{ foot-pounds;}$$

at C,

$$-500 \times 5 - 1{,}000 \times 3 = -5{,}500;$$

and at D,

$$-500 \times 9 - 1{,}000 \times 7 - 2{,}000 \times 4 = -19{,}500.$$

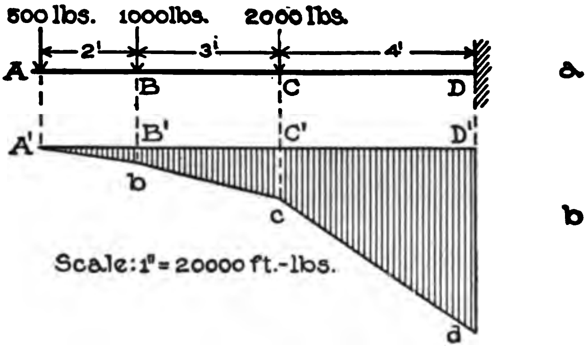


Fig. 19.

Using a scale of 20,000 foot-pounds to one inch, the ordinates in the bending moment diagram are:

At B, $1{,}000 \div 20{,}000 = 0.05$ inch,
" C, $5{,}500 \div 20{,}000 = 0.275$ "
" D, $19{,}500 \div 20{,}000 = 0.975$ "

Hence we lay these ordinates off, and downward because the bending moments are negative, thus fixing the points *b*, *c* and *d*. The bending moment at A is zero; hence the moment line connects A *b*, *c* and *d*. Further, the portions A*b*, *bc* and *cd* are straight, as can be shown by computing values of the bending moment for sections in AB, BC and CD, and laying off the corresponding ordinates in the moment diagram.

4. Suppose that the cantilever of the preceding illustration sustains also a uniform load of **100** pounds per foot (see Fig. 20, *a*). Construct a moment diagram.

First, we compute the values of the bending moment at several sections; thus,

$M_1 = -500 \times 1 - 100 \times \frac{1}{2} = -550$ foot-pounds,
$M_2 = -500 \times 2 - 200 \times 1 = -1,200,$
$M_3 = -500 \times 3 - 1,000 \times 1 - 300 \times 1\frac{1}{2} = -2,950,$
$M_4 = -500 \times 4 - 1,000 \times 2 - 400 \times 2 = -4,800,$
$M_5 = -500 \times 5 - 1,000 \times 3 - 500 \times 2\frac{1}{2} = -6,750,$
$M_6 = -500 \times 6 - 1,000 \times 4 - 2,000 \times 1 - 600 \times 3 = -10,800,$
$M_7 = -500 \times 7 - 1,000 \times 5 - 2,000 \times 2 - 700 \times 3\frac{1}{2} = -14,950,$
$M_8 = -500 \times 8 - 1,000 \times 6 - 2,000 \times 3 - 800 \times 4 = -19,200,$
$M_9 = -500 \times 9 - 1,000 \times 7 - 2,000 \times 4 - 900 \times 4\frac{1}{2} = -23,550.$

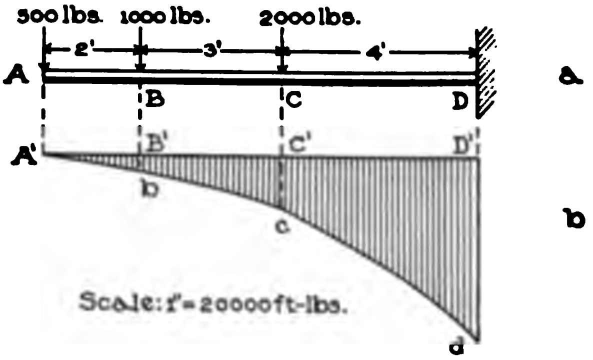


Fig. 20.

These values all being negative, the ordinates are all laid off downwards. To a scale of 20,000 foot-pounds to one inch, they fix the moment line *A'bcd*.

EXAMPLES FOR PRACTICE.

1. Construct a moment diagram for the beam represented in Fig. 10, neglecting the weight of the beam. (See example 1, Art. 43).

2. Construct a moment diagram for the beam represented in Fig. 11, neglecting the weight of the beam. (See example 3, Art. 43).

3. Construct the moment diagram for the beam of Fig. 12

when it sustains, in addition to the loads represented and its own weight (800 pounds), a uniform load of 500 pounds per foot. (See example 4, Art. 43.)

4. Figs. *a*, cases 1 and 2, page 53, represent two cantilever beams, the first bearing a load P at the free end, and the second a uniform load W. Figs. *c* are the corresponding moment diagrams. Take P and W equal to 1,000 pounds, and *l* equal to 10 feet, and satisfy yourself that the diagrams are correct.

5. Figs. *a*, cases 3 and 4, page 53, represent simple beams on end supports, the first bearing a middle load P, and the other a uniform load W. Figs. *c* are the corresponding moment diagrams. Take P and W equal to 1,000 pounds, and *l* equal to 10 feet, and satisfy yourself that the diagrams are correct.

45. Maximum Bending Moment. It is sometimes desirable to know the greatest or maximum value of the bending moment in a given case. This value can always be found with certainty by constructing the moment diagram, from which the maximum value of the bending moment is evident at a glance. But in any case, it can be most readily computed if one knows the section for which the bending moment is greatest. If the student will compare the corresponding shear and moment diagrams which have been constructed in foregoing articles (Figs. 13 and 17, 14 and 18, 15 and 19, 16 and 20), and those which he has drawn, he will see that—*The maximum bending moment in a beam occurs where the shear changes sign.*

By the help of the foregoing principle we can readily compute the maximum moment in a given case. We have only to construct the shear line, and observe from it where the shear changes sign; then compute the bending moment for that section. If a simple beam has one or more overhanging ends, then the shear changes sign more than once—twice if there is one overhanging end, and three times if two. In such cases we compute the bending moment for each section where the shear changes sign; the largest of the values of these bending moments is the maximum for the beam.

The section of maximum bending moment in a cantilever fixed at one end (as when built into a wall) is always at the wall.

Thus, without reference to the moment diagrams, it is readily seen that,

for a cantilever whose length is l,
with an end load P, the maximum moment is Pl,
" a uniform " W, " " " $\frac{1}{2}Wl$.

Also by the principle, it is seen that,

for a beam whose length is l, on end supports,
with a middle load P, the maximum moment is $\frac{1}{4}Pl$,
" uniform " W, " " " $\frac{1}{8}Wl$.

46. Table of Maximum Shears, Moments, etc. Table B on page 53 shows the shear and moment diagrams for eight simple cases of beams. The first two cases are built-in cantilevers; the next four, simple beams on end supports; and the last two, restrained beams built in walls at each end. In each case l denotes the length.

CENTER OF GRAVITY AND MOMENT OF INERTIA.

It will be shown later that the strength of a beam depends partly on the form of its cross-section. The following discussion relates principally to cross-sections of beams, and the results reached (like shear and bending moment) will be made use of later in the subject of strength of beams.

47. Center of Gravity of an Area. The student probably knows what is meant by, and how to find, the center of gravity of any flat disk, as a piece of tin. Probably his way is to balance the piece of tin on a pencil point, the point of the tin at which it so balances being the center of gravity. (Really it is midway between the surfaces of the tin and over the balancing point.) The center of gravity of the piece of tin, is also that point of it through which the resultant force of gravity on the tin (that is, the weight of the piece) acts.

By "center of gravity" of a plane area of any shape we mean that point of it which corresponds to the center of gravity of a piece of tin when the latter is cut out in the shape of the area. The center of gravity of a quite irregular area can be found most readily by balancing a piece of tin or stiff paper cut in the shape of the area. But when an area is simple in shape, or consists of parts which are simple, the center of gravity of the whole can be found readily by computation, and such a method will now be described.

48. Principle of Moments Applied to Areas. Let Fig. 21 represent a piece of tin which has been divided off into any number of parts in any way, the weight of the whole being W, and that of the parts W_1, W_2, W_3, etc. Let C_1, C_2, C_3, etc., be the centers of gravity of the parts, C that of the whole, and c_1, c_2, c_3, etc., and c the distances from those centers of gravity respectively to some line (L L) in the plane of the sheet of tin. When the tin is lying in a horizontal position, the moment of the weight of the entire piece about L L is Wc, and the moments of the parts are W_1c_1, W_2c_2, etc. Since the weight of the whole is the resultant of the weights of the parts, the moment of the weight of the whole equals the sum of the moments of the weights of the parts; that is,

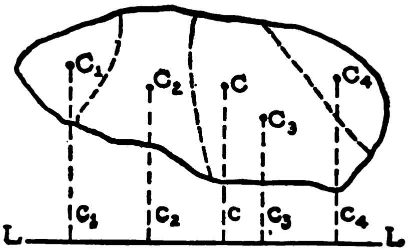


Fig. 21.

$$Wc = W_1c_1 + W_2c_2 + \text{etc} \ldots .$$

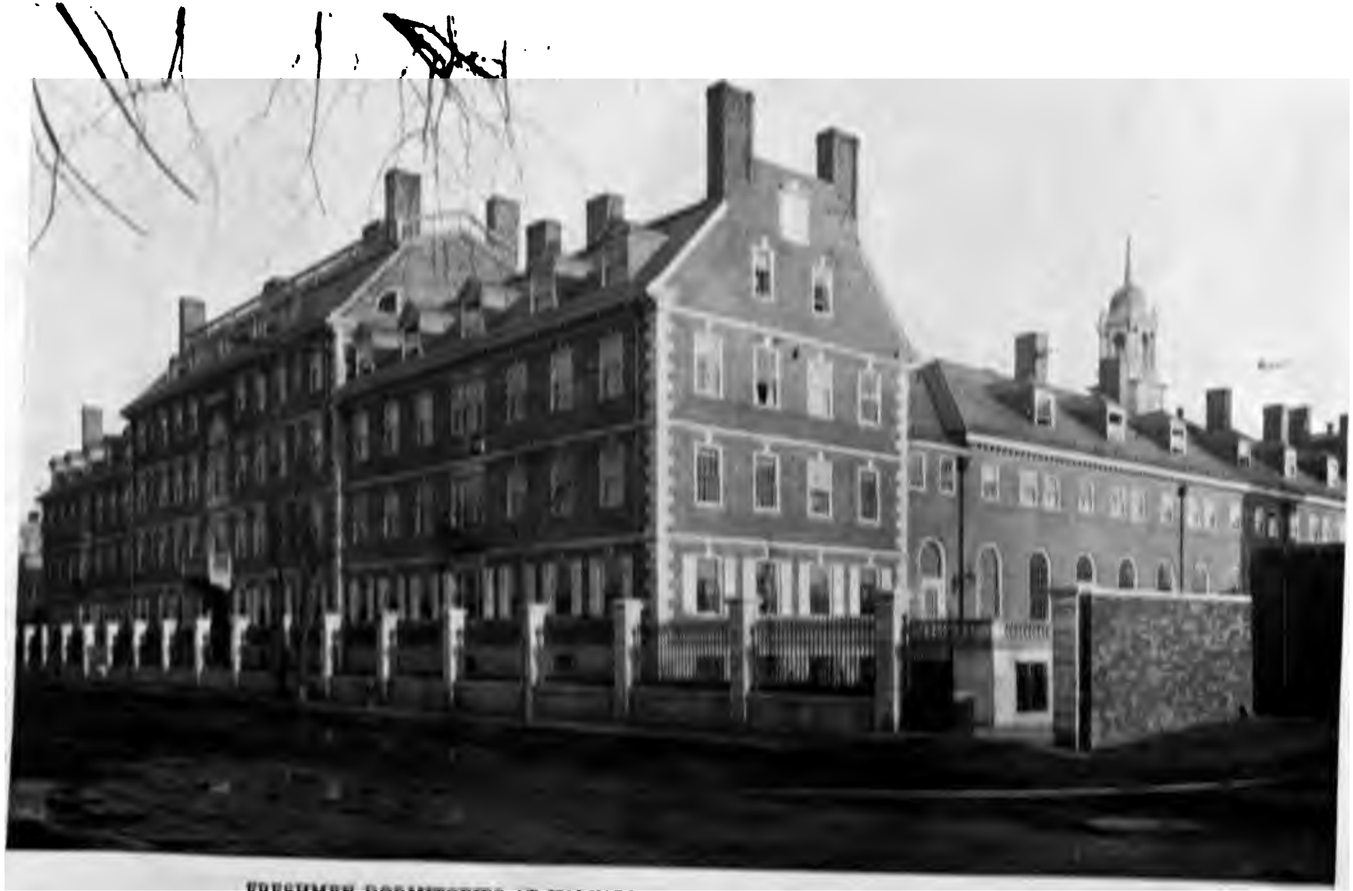
Now let A_1, A_2, etc. denote the areas of the parts of the pieces of tin, and A the area of the whole; then since the weights are proportional to the areas, we can replace the W's in the preceding equation by corresponding A's, thus:

$$Ac = A_1c_1 + A_2c_2 + \text{etc} \ldots . \qquad (4)$$

If we call the product of an area and the distance of its center of gravity from some line in its plane, the "moment" of the area with respect to that line, then the preceding equation may be stated in words thus:

The moment of an area with respect to any line equals the algebraic sum of the moments of the parts of the area.

If all the centers of gravity are on one side of the line with respect to which moments are taken, then all the moments should be given the plus sign; but if some centers of gravity are on one side and some on the other side of the line, then the moments of the areas whose centers of gravity are on one side should be given the



FRESHMEN DORMITORIES AT HARVARD UNIVERSITY, CAMBRIDGE, MASSACHUSETTS

Shepley, Rutan and Coolidge, Architects, Boston, Massachusetts

The Freshmen dormitories consist of Smith, Standish and Gore Halls, accommodating 500 students same sign, and the moments of the others the opposite sign. The foregoing is the principle of moments for areas, and it is the basis of all rules for finding the center of gravity of an area.

To find the center of gravity of an area which can be divided up into simple parts, we write the principle in forms of equations for two different lines as "axes of moments," and then solve the equations for the unknown distances of the center of gravity of the whole from the two lines. We explain further by means of specific examples.

Examples. 1. It is required to find the center of gravity of Fig. 22, *a*, the width being uniformly one inch.

The area can be divided into two rectangles. Let C_1 and C_2 be the centers of gravity of two such parts, and C the center of gravity of the whole. Also let a and b denote the distances of C from the two lines OL′ and OL″ respectively.

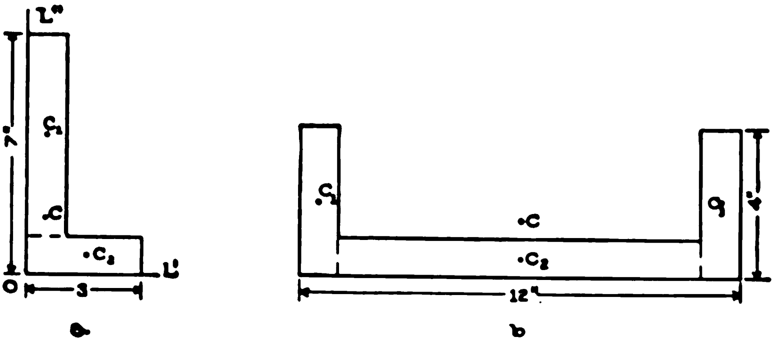


Fig. 22.

The areas of the parts are 6 and 3 square inches, and their arms with respect to OL′ are 4 inches and ½ inch respectively, and with respect to OL″ ½ inch and 1½ inches. Hence the equations of moments with respect to OL′ and OL″ (the whole area being 9 square inches) are:

$$9 \times a = 6 \times 4 + 3 \times \tfrac{1}{2} = 25.5,$$

$$9 \times b = 6 \times \tfrac{1}{2} + 3 \times 1\tfrac{1}{2} = 7.5.$$

Hence,

$$a = 25.5 + 9 = 2.83 \text{ inches},$$

$$b = 7.5 + 9 = 0.83 \quad \text{"} \quad .$$

2. It is required to locate the center of gravity of Fig. 22, *b*, the width being uniformly one inch.

The figure can be divided up into three rectangles. Let C_1, C_2 and C_3 be the centers of gravity of such parts, C the center of gravity of the whole; and let a denote the (unknown) distance of C from the base. The areas of the parts are 4, 10 and 4 square inches, and their "arms" with respect to the base are 2, ½ and 2 inches respectively; hence the equation of moments with respect to the base (the entire area being 18 square inches) is:

$$18 \times a = 4 \times 2 + 10 \times \tfrac{1}{2} + 4 \times 2 = 21.$$

Hence, $a = 21 \div 18 = 1.17$ inches.

From the symmetry of the area it is plain that the center of gravity is midway between the sides.

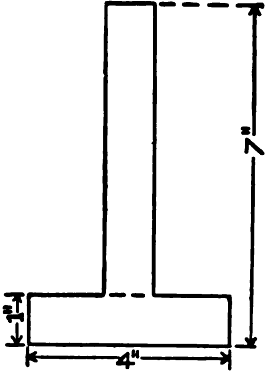


Fig. 23.

EXAMPLE FOR PRACTICE.

1. Locate the center of gravity of Fig. 23.

Ans. 2.3 inches above the base.

49. Center of Gravity of Built-up Sections. In Fig. 24 there are represented cross-sections of various kinds of rolled steel, called "shape steel," which is used extensively in steel construction. Manufacturers of this material publish "handbooks" giving full information in regard thereto, among other things, the position of the center of gravity of each cross section. With such a handbook available, it is therefore not necessary actually to compute the position of the center of gravity of any section, as we did in the preceding article; but sometimes several shapes are riveted together to

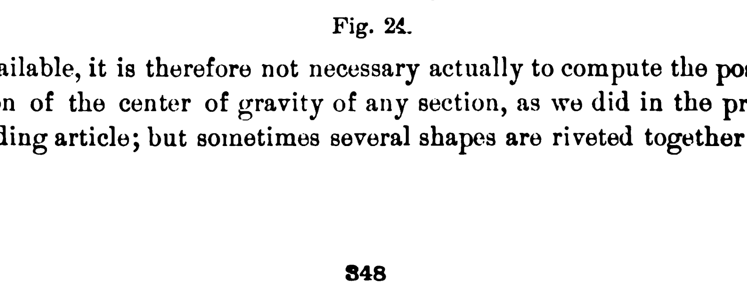


Fig. 24.

make a "built-up" section (see Fig. 25), and then it may be necessary to compute the position of the center of gravity of the section.

Example. It is desired to locate the center of gravity of the section of a built-up beam represented in Fig. 25. The beam consists of two channels and a plate, the area of the cross-section of a channel being 6.03 square inches.

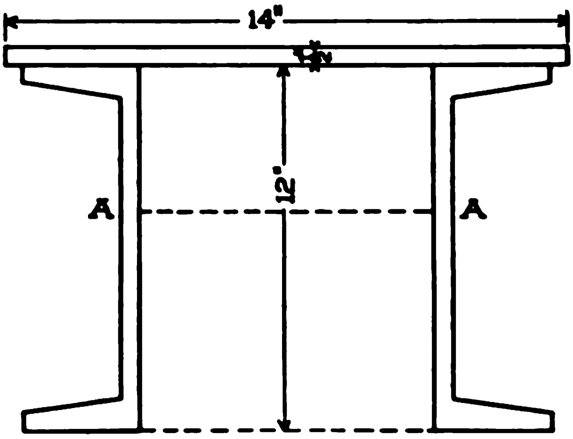


Fig. 25.

Evidently the center of gravity of each channel section is 6 inches, and that of the plate section is 12¼ inches, from the bottom. Let c denote the distance of the center of gravity of the whole section from the bottom; then since the area of the plate section is 7 square inches, and that of the whole section is 19.06,

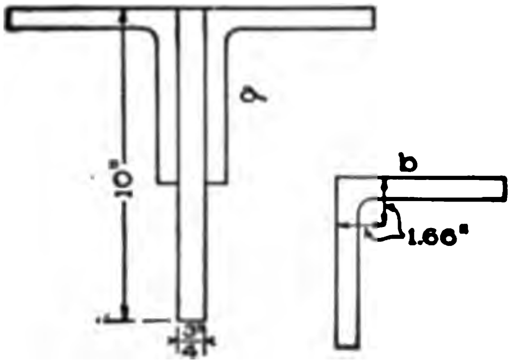


Fig. 26.

$$19.06 \times c = 6.03 \times 6 + 6.03 \times 6 + 7 \times 12\tfrac{1}{4} = 158.11.$$

Hence, $c = 158.11 \div 19.06 = 8.30$ inches. (about).

EXAMPLES FOR PRACTICE.

1. Locate the center of gravity of the built-up section of

Fig. 26, *a*, the area of each "angle" being 5.06 square inches, and the center of gravity of each being as shown in Fig. 26, *b*.

Ans. Distance from top, 3.08 inches.

2. Omit the left-hand angle in Fig. 26, *a*, and locate the center of gravity of the remainder.

Ans. { Distance from top, 3.65 inches,
" " left side, 1.19 inches.

50. Moment of Inertia. If a plane area be divided into an infinite number of infinitesimal parts, then the sum of the products obtained by multiplying the area of each part by the square of its distance from some line is called the *moment of inertia* of the area with respect to the line. The line to which the distances are measured is called the *inertia-axis;* it may be taken anywhere in the plane of the area. In the subject of beams (where we have sometimes to compute the moment of inertia of the cross-section of a beam), the inertia-axis is taken through the center of gravity of the section and horizontal.

An approximate value of the moment of inertia of an area can be obtained by dividing the area into small parts (not infinitesimal), and adding the products obtained by multiplying the area of each part by the square of the distance from its center to the inertia-axis.

Example. If the rectangle of Fig. 27, *a*, is divided into 8 parts as shown, the area of each is one square inch, and the distances from the axis to the centers of gravity of the parts are $\frac{1}{2}$ and $1\frac{1}{2}$ inches. For the four parts lying nearest the axis the product (area times distance squared) is:

$1\times(\frac{1}{2})^2=\frac{1}{4}$; and for the other parts it is
$1\times(1\frac{1}{2})^2=\frac{9}{4}$.

Hence the approximate value of the moment of inertia of the area with respect to the axis, is

$$4(\tfrac{1}{4})+4(\tfrac{9}{4})=10.$$

If the area is divided into 32 parts, as shown in Fig. 27, *b*, the area of each part is $\frac{1}{4}$ square inch. For the eight of the little squares farthest away from the axis, the distance from their centers of gravity to the axis is $1\frac{3}{4}$ inches; for the next eight it is $1\frac{1}{4}$; for the next eight $\frac{3}{4}$; and for the remainder $\frac{1}{4}$ inch. Hence an approximate value of the moment of inertia of the rectangle with respect to the axis is:

$$8\times\tfrac{1}{4}\times(1\tfrac{3}{4})^2+8\times\tfrac{1}{4}\times(1\tfrac{1}{4})^2+8\times\tfrac{1}{4}\times(\tfrac{3}{4})^2+8\times\tfrac{1}{4}\times(\tfrac{1}{4})^2=10\tfrac{1}{2}.$$

If we divide the rectangle into still smaller parts and form the products

$$(\text{small area})\times(\text{distance})^2,$$

and add the products just as we have done, we shall get a larger answer than 10½. The smaller the parts into which the rectangle is divided, the larger will be the answer, but it will never be larger than 10⅔. This 10⅔ is the sum corresponding to a division of the rectangle into an infinitely large number of parts (infinitely small) and it is the exact value of the moment of inertia of the rectangle with respect to the axis selected.

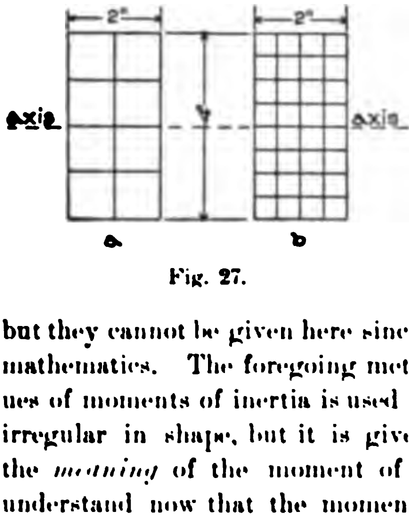


Fig. 27.

There are short methods of computing the exact values of the moments of inertia of simple figures (rectangles, circles, etc.), but they cannot be given here since they involve the use of difficult mathematics. The foregoing method to obtain approximate values of moments of inertia is used especially when the area is quite irregular in shape, but it is given here to explain to the student the *meaning* of the moment of inertia of an area. He should understand now that the moment of inertia of an area is simply a name for such sums as we have just computed. The name is not a fitting one, since the sum has nothing whatever to do with inertia. It was first used in this connection because the sum is very similar to certain other sums which had previously been called moments of inertia.

51. Unit of Moment of Inertia. The product (area × distance²) is really the product of four lengths, two in each factor; and since a moment of inertia is the sum of such products, a moment of inertia is also the product of four lengths. Now the product of two lengths is an area, the product of three is a volume, and the product of four is moment of inertia—unthinkable in the way in which we can think of an area or volume, and therefore the source of much difficulty to the student. The units of these quantities (area, volume, and moment of inertia) are respectively:

the square inch, square foot, etc.,
" cubic " , cubic " " ,
" biquadratic inch, biquadratic foot, etc.;

but the biquadratic inch is almost exclusively used in this connection; that is, the inch is used to compute values of moments of inertia, as in the preceding illustration. It is often written thus: Inches⁴.

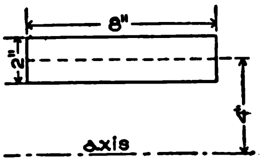


Fig. 28.

52. Moment of Inertia of a Rectangle. Let b denote the base of a rectangle, and a its altitude; then by higher mathematics it can be shown that the moment of inertia of the rectangle with respect to a line through its center of gravity and parallel to its base, is $\frac{1}{12} ba^3$.

Example. Compute the value of the moment of inertia of a rectangle 4×12 inches with respect to a line through its center of gravity and parallel to the long side.

Here $b=12$, and $a = 4$ inches ; hence the moment of inertia desired equals

$$\tfrac{1}{12}(12\times 4^3)=64 \text{ inches}^4.$$

EXAMPLE FOR PRACTICE.

1. Compute the moment of inertia of a rectangle 4×12 inches with respect to a line through its center of gravity and parallel to the short side. Ans. 576 inches⁴.

53. Reduction Formula. In the previously mentioned "handbooks" there can be found tables of moments of inertia of all the cross-sections of the kinds and sizes of rolled shapes made. The inertia-axes in those tables are always taken through the center of gravity of the section, and usually parallel to some edge of the section. Sometimes it is necessary to compute the moment of inertia of a "rolled section" with respect to some other axis, and if the two axes (that is, the one given in the tables, and the other) are parallel, then the desired moment of inertia can be easily computed from the one given in the tables by the following rule:

The moment of inertia of an area with respect to any axis equals the moment of inertia with respect to a parallel axis through the center of gravity, plus the product of the area and the square of the distance between the axes. Or, if I denotes the moment of inertia with respect to any axis; I_o the moment of inertia with respect to a parallel axis through the center of gravity; A the area; and *d* the distance between the axes, then

$$I = I_o + Ad^2 \ldots . \quad \textbf{(5)}$$

Example. It is required to compute the moment of inertia of a rectangle 2×8 inches with respect to a line parallel to the long side and 4 inches from the center of gravity.

Let I denote the moment of inertia sought, and I_o the moment of inertia of the rectangle with respect to a line parallel to the long side and through the center of gravity (see Fig. 28). Then

$I_o = \frac{1}{12}ba^3$ (see Art. 52); and, since $b = 8$ inches and $a = 2$ inches,

$I_o = \frac{1}{12}(8 \times 2^3) = 5\frac{1}{3}$ biquadratic inches.

The distance between the two inertia-axes is 4 inches, and the area of the rectangle is 16 square inches, hence equation 5 becomes

$$I = 5\tfrac{1}{3} + 16 \times 4^2 = 261\tfrac{1}{3} \text{ biquadratic inches.}$$

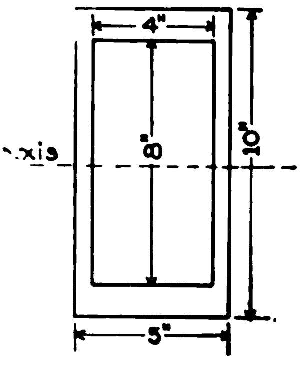


Fig. 28.

EXAMPLE FOR PRACTICE.

1. The moment of inertia of an "angle" $2\frac{1}{2} \times 2 \times \frac{1}{2}$ inches (lengths of sides and width respectively) with respect to a line through the center of gravity and parallel to the long side, is 0.64 inches^4. The area of the section is 2 square inches, and the distance from the center of gravity to the long side is 0.63 inches. (These values are taken from a "handbook".) It is required to compute the moment of inertia of the section with respect to a line parallel to the long side and 4 inches from the center of gravity. Ans. 32.64 inches^4.

54. Moment of Inertia of Built-up Sections. As before stated, beams are sometimes "built up" of rolled shapes (angles, channels, etc.). The moment of inertia of such a section with respect to a definite axis is computed by adding the moments of inertia of the parts, *all with respect to that same axis.* This is the method for computing the moment of any area which can be divided into simple parts.

The moment of inertia of an area which may be regarded as consisting of a larger area *minus* other areas, is computed by subtracting from the moment of inertia of the large area those of the "minus areas."

Examples. 1. Compute the moment of inertia of the built-up section represented in Fig. 30 (in part same as Fig. 25) with respect to a horizontal axis passing through the center of gravity, it being given that the moment of inertia of each channel section with respect to a horizontal axis through its center of gravity is 128.1 inches^4, and its area 6.03 square inches.

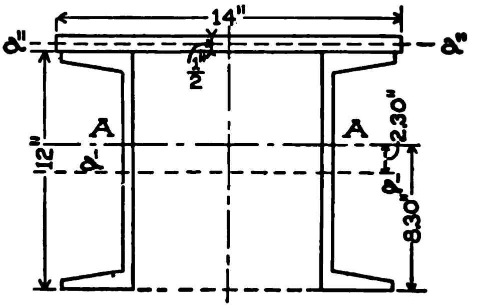


Fig. 30.

The center of gravity of the whole section was found in the example of Art. 49 to be 8.30 inches from the bottom of the section; hence the distances from the inertia-axis to the centers of gravity of the channel section and the plate are 2.30 and 3.95 inches respectively (see Fig. 30).

The moment of inertia of one channel section with respect to the axis AA (see equation 5, Art. 53) is:

$$128.1+6.03\times 2.30^2=\mathbf{160.00}\text{ inches}^4.$$

The moment of inertia of the plate section (rectangle) with respect to the line $a''a''$ (see Art. 52) is:

$$\tfrac{1}{12}\,ba^3=\tfrac{1}{12}[14\times(\tfrac{1}{2})^3]=0.15\text{ inches}^4;$$

and with respect to the axis AA (the area being 7 square inches) it is:

$$\mathbf{0.15}+7\times 3.95^2=109.37\text{ inches}^4.$$

Therefore the moment of inertia of the whole section with respect to AA is:

$$\mathbf{2\times 160.00+109.37=429.37\text{ inches}^4.}$$

2. It is required to compute the moment of inertia of the "hollow rectangle" of Fig. 29 with respect to a line through the center of gravity and parallel to the short side.

The amount of inertia of the large rectangle with respect to the named axis (see Art. 52) is:

$$\tfrac{1}{12}(5 \times 10^3) = 416\tfrac{2}{3};$$

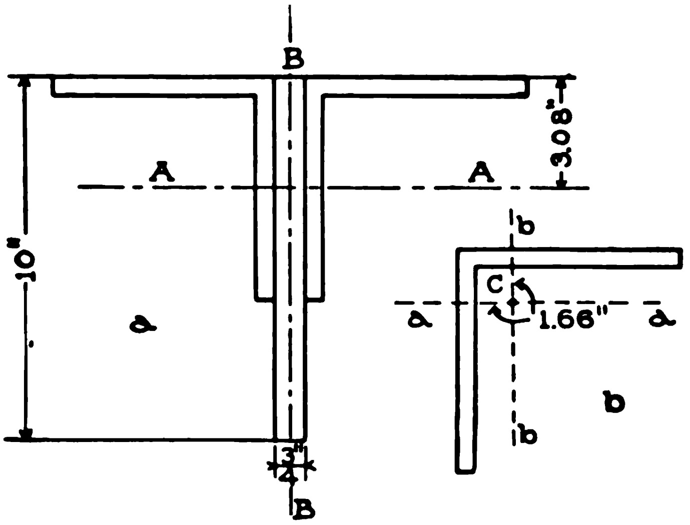


Fig. 31.

and the moment of inertia of the smaller one with respect to the same axis is:

$$\tfrac{1}{12}(4 \times 8^3) = 170\tfrac{2}{3};$$

hence the moment of inertia of the hollow section with respect to the axis is:

$$416\tfrac{2}{3} - 170\tfrac{2}{3} = 246 \text{ inches}^4.$$

EXAMPLES FOR PRACTICE.

1. Compute the moment of inertia of the section represented in Fig. 31, *a*, about the axis AA, it being 3.08 inches from the top. Given also that the area of one angle section is 5.06 square inches, its center of gravity C (Fig. 31, *b*) 1.66 inches from the top, and its moment of inertia with respect to the axis *aa* 17.68 inches⁴. Ans. 145.8 inches⁴.

2. Compute the moment of inertia of the section of Fig. 31, *a*,

with respect to the axis BB. Given that distance of the center of gravity of one angle from one side is 1.66 inches (see Fig. 31, *b*), and its moment of inertia with respect to *bb* 17.68 inches.

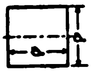
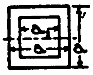
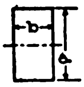
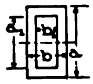
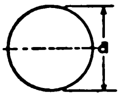
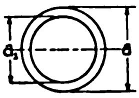
Ans. 77.618 inches^4.

55. Table of Centers of Gravity and Moments of Inertia. Column 2 in Table A below gives the formula for moment of inertia with respect to the horizontal line through the center of gravity. The numbers in the third column are explained in Art. 62; and those in the fourth, in Art. 80.

TABLE A.

Moments of Inertia, Section Moduli, and Radii of Gyration.

In each case the axis is horizontal and passes through the center of gravity.

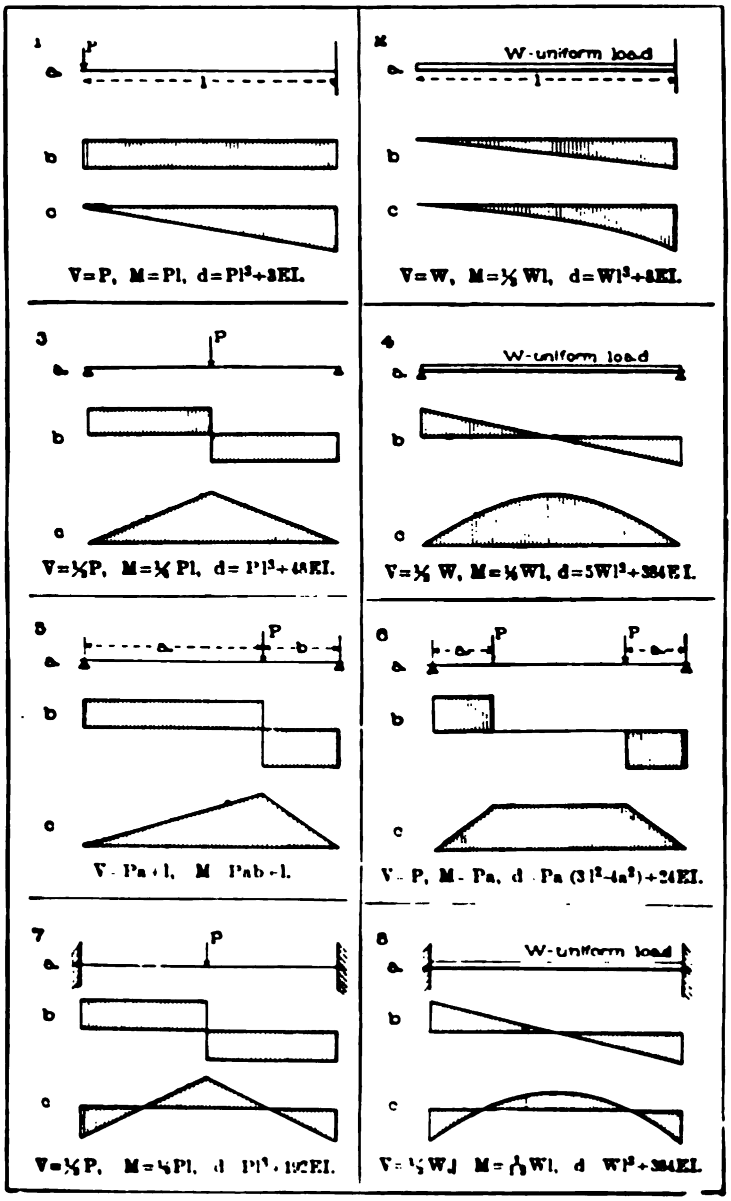
Section.	Moment of Inertia.	Section Modulus.	Radius of Gyration.
	$\frac{a^4}{12}$	$\frac{a^3}{6}$	$\frac{a}{\sqrt{12}}$
	$\frac{a^4 - a_1^4}{12}$	$\frac{a^4 - a_1^4}{6a}$	$\sqrt{\frac{a^2 + a_1^2}{12}}$
	$\frac{ba^3}{12}$	$\frac{ba^2}{6}$	$\frac{a}{\sqrt{12}}$
	$\frac{ba^3 - b_1a_1^3}{12}$	$\frac{ba^3 - b_1a_1^3}{6a}$	$\sqrt{\frac{ba^3 - b_1a_1^3}{12\,(ba - b_1a_1)}}$
	$0.049d^4$	$0.098d^3$	$\frac{d}{4}$
	$0.049\,(d^4 - d_1^4)$	$0.098\frac{d^4 - d_1^4}{d}$	$\frac{\sqrt{d^2 + d_1^2}}{4}$

STRENGTH OF BEAMS.

56. Kinds of Loads Considered. The loads that are applied to a horizontal beam are usually vertical, but sometimes forces are applied otherwise than at right angles to beams. Forces acting on beams at right angles are called **transverse forces**; those applied

TABLE B.

Shear Diagrams (b) and Moment Diagrams (c) for Eight Different Cases (a). Also Values of Maximum Shear (V), Bending Moment (M), and Deflection (d).



1. $V = P,\ M = Pl,\ d = Pl^3 + 3EI.$

2. $V = W,\ M = \frac{1}{2} Wl,\ d = Wl^3 + 8EI.$

3. $V = \frac{1}{2} P,\ M = \frac{1}{4} Pl,\ d = Pl^3 + 48EI.$

4. $V = \frac{1}{2} W,\ M = \frac{1}{8} Wl,\ d = 5Wl^3 + 384EI.$

5. $V = Pa + l,\ M = Pab + l.$

6. $V = P,\ M = Pa,\ d = Pa\,(3l^2 - 4a^2) + 24EI.$

7. $V = \frac{1}{2} P,\ M = \frac{1}{8} Pl,\ d = Pl^3 + 192EI.$

8. $V = \frac{1}{2} W,\ M = \frac{1}{12} Wl,\ d = Wl^3 + 384EI.$

parallel to a beam are called **longitudinal forces**; and others are called **inclined forces.** For the present we deal only with beams subjected to transverse forces (loads and reactions).

57. Neutral Surface, Neutral Line, and Neutral Axis. When a beam is loaded it may be wholly convex up (concave down), as a cantilever; wholly convex down (concave up), as a simple beam on end supports; or partly convex up and partly convex down, as a simple beam with overhanging ends, a restrained beam, or a continuous beam. Two vertical parallel lines drawn close together on the side of a beam before it is loaded will not be parallel after it is loaded and bent. If they are on a convex-down portion of a beam, they will be closer at the top and farther apart below than when drawn (Fig. 32*a*), and if they are on a convex-up portion, they will be closer below and farther apart above than when drawn (Fig. 32*b*).

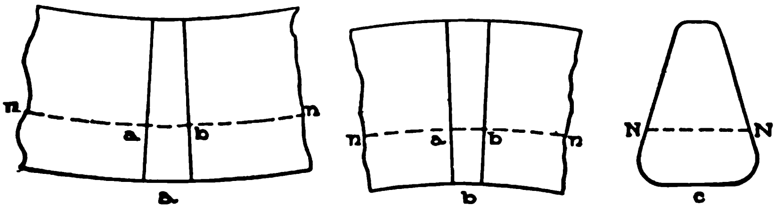


Fig. 32.

The "fibres" on the convex side of a beam are stretched and therefore under tension, while those on the concave side are shortened and therefore under compression. Obviously there must be some intermediate fibres which are neither stretched nor shortened, *i. e.*, **under neither tension nor compression.** These make up a sheet of fibres and define a surface in the beam, which surface is called the **neutral surface** of the beam. The intersection of the neutral surface with either side of the beam is called the **neutral line,** and its intersection with any cross-section of the beam is called the **neutral axis** of that section. Thus, if *ab* is a fibre that has been neither lengthened nor shortened with the bending of the beam, then *nn* is a portion of the neutral line of the beam; and, if Fig. 32*c* be taken to represent a cross-section of the beam, NN is the neutral axis of the section.

It can be proved that *the neutral axis of any cross-section of* *a loaded beam passes through the center of gravity of that section,* provided that all the forces applied to the beam are transverse, and that the tensile and compressive stresses at the cross-section are all within the elastic limit of the material of the beam.

58. Kinds of Stress at a Cross-section of a Beam. It has already been explained in the preceding article that there are tensile and compressive stresses in a beam, and that the tensions are on the convex side of the beam and the compressions on the concave (see Fig. 33). The forces T and C are exerted upon the portion of the beam represented by the adjoining portion to the right (not shown). These, the student is reminded, are often called **fibre stresses.**

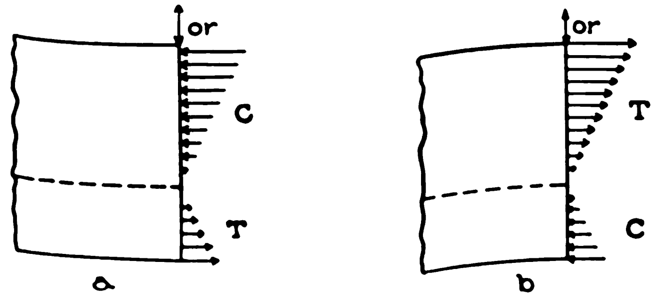


Fig. 33.

Besides the fibre stresses there is, in general, a shearing stress at every cross-section of a beam. This may be proved as follows:

Fig. 34 represents a simple beam on end supports which has actually been cut into two parts as shown. The two parts can maintain loads when in a horizontal position, if forces are applied at the cut ends equivalent to the forces that would act there if the beam were not cut. Evidently in the solid beam there are at the section a compression above and a tension below, and such forces can be applied in the cut beam by means of a short block C and a chain or cord T, as shown. The block furnishes the compressive forces and the chain the tensile forces. At first sight it appears as if the beam would stand up under its load after the block and chain have been put into place. Except in certain cases[*], however, it would not remain in a horizontal position, as would the solid beam. This shows that the forces exerted by the block and chain (horizontal compression and tension) are not equivalent to the actual stresses in the solid beam. What is needed is a vertical force at each cut end.

---

[*] When the external shear for the section is zero.

Suppose that R_1 is less than $L_1 + L_2 +$ weight of A, *i. e.*, that the external shear for the section is negative; then, if vertical pulls be applied at the cut ends, upward on A and downward on B, the beam will stand under its load and in a horizontal position, just as a solid beam. These pulls can be supplied, in the model of the beam, by means of a cord S tied to two brackets fastened on A and B, as shown. In the solid beam the two parts act upon each other directly, and the vertical forces are shearing stresses, since they act in the plane of the surfaces to which they are applied.

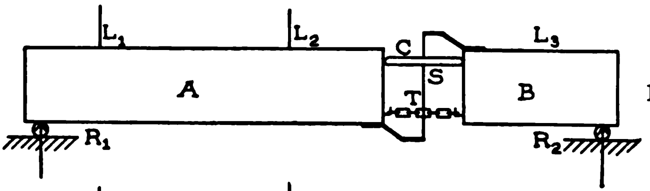


Fig. 34.

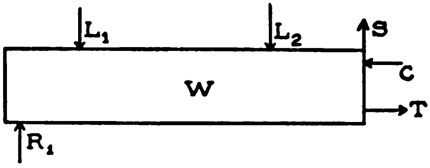


Fig. 35.

59. Relation Between the Stress at a Section and the Loads and Reactions on Either Side of It. Let Fig. 35 represent the portion of a beam on the left of a section; and let R_1 denote the left reaction; L_1 and L_2 the loads; W the weight of the left part; C, T, and S the compression, tension, and shear respectively which the right part exerts upon the left.

Since the part of the beam here represented is at rest, all the forces exerted upon it are balanced; and when a number of horizontal and vertical forces are balanced, then

1. The algebraic sum of the horizontal forces equals zero.
2. " " " " " vertical " " "
3. " " " " " moments of all the forces with respect to any point equals zero.

To satisfy condition 1, since the tension and compression are the only horizontal forces, *the tension must equal the compression.*

To satisfy condition 2, S (the internal shear) must equal the algebraic sum of all the other vertical forces on the portion, that is, must equal the external shear for the section; also, S must act up or down according as the external shear is negative or positive. In other words, briefly expressed, *the internal and external shears at a section are equal and opposite.*

To satisfy condition 3, the algebraic sum of the moments of the fibre stresses about the neutral axis must be equal to the sum of the moments of all the other forces acting on the portion about the same line, and the signs of those sums must be opposite. (The moment of the shear about the neutral axis is zero.) Now, the sum of the moments of the loads and reactions is called the bending moment at the section, and if we use the term **resisting moment** to signify the sum of the moments of the fibre stresses (tensions and compressions) about the neutral axis, then we may say briefly that *the resisting and the bending moments at a section are equal, and the two moments are opposite in sign.*

60. The Fibre Stress. As before stated, the fibre stress is not a uniform one, that is, it is not uniformly distributed over the section on which it acts. At any section, the compression is most "intense" (or the unit-compressive stress is greatest) on the concave side; the tension is most intense (or the unit-tensile stress is greatest) on the convex side; and the unit-compressive and unit-tensile stresses decrease toward the neutral axis, at which place the unit-fibre stress is zero.

If the fibre stresses are within the elastic limit, then the two straight lines on the side of a beam referred to in Art. 57 will still be straight after the beam is bent; hence the elongations and shortenings of the fibres vary directly as their distance from the neutral axis. Since the stresses (if within the elastic limit) and deformations in a given material are proportional, *the unit-fibre stress varies as the distance from the neutral axis.*

Let Fig. 36*a* represent a portion of a bent beam, 36*b* its cross-section, *nn* the neutral line, and NN the neutral axis. The way in which the unit-fibre stress on the section varies can be represented graphically as follows: Lay off *ac*, by some scale, to represent the unit-fibre stress in the top fibre, and join *c* and *n*, extending the line to the lower side of the beam; also make bc' equal to bc'' and draw nc'. Then the arrows represent the unit-fibre stresses, for their lengths vary as their distances from the neutral axis.

61. Value of the Resisting Moment. If S denotes the unit-fibre stress in the fibre farthest from the neutral axis (the greatest unit-fibre stress on the cross-section), and c the distance from the neutral axis to the remotest fibre, while S_1, S_2, S_3, etc., denote the unit-fibre stresses at points whose distances from the neutral axis are, respectively, y_1, y_2, y_3, etc. (see Fig. 36*b*), then

$$S : S_1 :: c : y_1; \text{ or } S_1 = \frac{S}{c} y_1.$$

Also, $$S_2 = \frac{S}{c} y_2; \; S_3 = \frac{S}{c} y_3, \text{ etc.}$$

Let a_1, a_2, a_3, etc., be the areas of the cross-sections of the fibres whose distances from the neutral axis are, respectively, y_1, y_2, y_3, etc. Then the stresses on those fibres are, respectively,

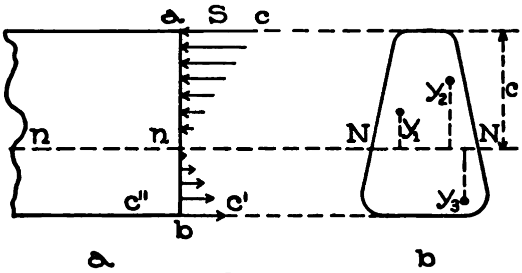


Fig. 36.

$$S_1 a_1, \; S_2 a_2, \; S_3 a_3, \text{ etc.};$$

or, $$\frac{S}{c} y_1 a_1, \; \frac{S}{c} y_2 a_2, \; \frac{S}{c} y_3 a_3, \text{ etc.}$$

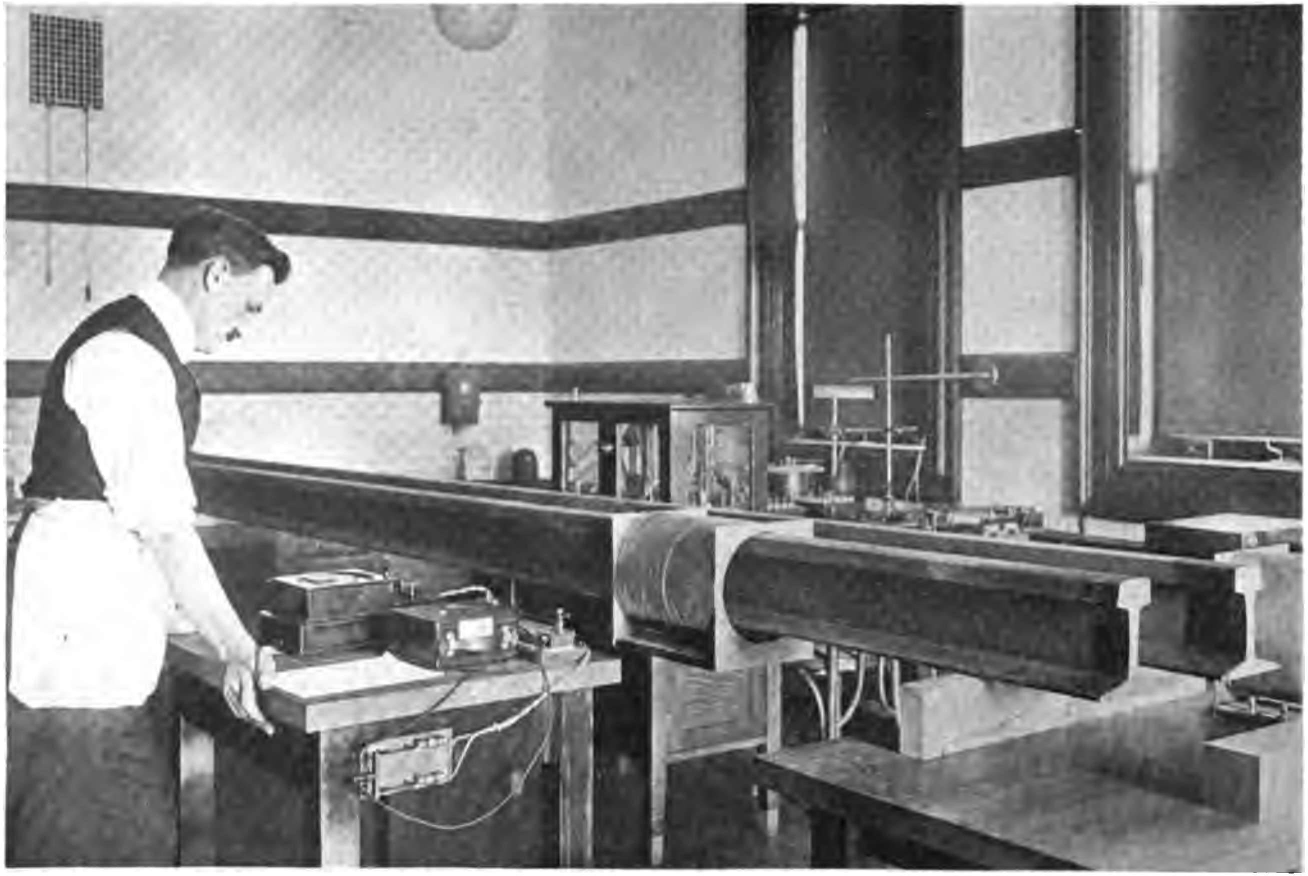
The arms of these forces or stresses with respect to the neutral axis are, respectively, y_1, y_2, y_3, etc.; hence their moments are

$$\frac{S}{c} a_1 y_1^2, \; \frac{S}{c} a_2 y_2^2, \; \frac{S}{c} a_3 y_3^2, \text{ etc.},$$

and the sum of the moments (that is, the resisting moment) is

$$\frac{S}{c} a_1 y_1^2 + \frac{S}{c} a_2 y_2^2 + \text{etc.} = \frac{S}{c}(a_1 y_1^2 + a_2 y_2^2 + \text{etc.})$$

Now $a_1 y_1^2 + a_2 y_2^2 + \text{etc.}$ is the sum of the products obtained by multiplying each infinitesimal part of the area of the cross-section by the square of its distance from the neutral axis; hence, it is the moment of inertia of the cross-section with respect to the neutral axis. If this moment is denoted by I, then the value of the resisting moment is $\frac{SI}{c}$.



TESTING ELECTRICALLY FOR FLAWS IN STEEL RAILS, NATIONAL BUREAU OF STANDARDS

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STRENGTH OF MATERIALS.

PART II.

STRENGTH OF BEAMS—(Concluded).

62. First Beam Formula. As shown in the preceding article, the resisting and bending moments for any section of a beam are equal; hence

$$\frac{SI}{c} = M, \qquad (6)$$

all the symbols referring to the same section. This is the most important formula relating to beams, and will be called the "first beam formula."

The ratio I ÷ c is now quite generally called the **section modulus.** Observe that for a given beam it depends only on the dimensions of the cross-section, and not on the material or anything else. Since I is the product of four lengths (see article 51), I ÷ c is the product of three; and hence a section modulus can be expressed in units of volume. The cubic inch is practically always used; and in this connection it is written thus, inches³. See Table A, page 52, for values of the section moduli of a few simple sections.

63. Applications of the First Beam Formula. There are three principal applications of equation 6, which will now be explained and illustrated.

64. *First Application.* The dimensions of a beam and its manner of loading and support are given, and it is required to compute the greatest unit tensile and compressive stresses in the beam.

This problem can be solved by means of equation 6, written in this form,

$$S = \frac{Mc}{I} \text{ or } \frac{M}{I \div c}. \qquad (6')$$

Unless otherwise stated, we assume that the beams are uniform in cross-section, as they usually are; then the section modulus (I ÷ c) is the same for all sections, and S (the unit-fibre stress on the remotest fibre) varies just as M varies, and is therefore greatest where M is a maximum.* Hence, to compute the value of the greatest unit-fibre stress in a given case, *substitute the values of the section modulus and the maximum bending moment in the preceding equation, and reduce.*

If the neutral axis is equally distant from the highest and lowest fibres, then the greatest tensile and compressive unit-stresses are equal, and their value is S. If the neutral axis is unequally distant from the highest and lowest fibres, let c' denote its distance from the nearer of the two, and S′ the unit-fibre stress there. Then, since the unit-stresses in a cross-section are proportional to the distances from the neutral axis,

$$\frac{S'}{S} = \frac{c'}{c}, \text{ or } S' = \frac{c'}{c}S.$$

If the remotest fibre is on the convex side of the beam, S is tensile and S′ compressive; if the remotest fibre is on the concave side, S is compressive and S′ tensile.

Examples. 1. A beam 10 feet long is supported at its ends, and sustains a load of 4,000 pounds two feet from the left end (Fig. 37, *a*). If the beam is 4 × 12 inches in cross-section (the long side vertical as usual), compute the maximum tensile and compressive unit-stresses.

The section modulus of a rectangle whose base and altitude are b and a respectively (see Table A, page 52), is $\frac{1}{6}ba^2$; hence, for the beam under consideration, the modulus is

$$\frac{1}{6} \times 4 \times 12^2 = 96 \text{ inches}^3.$$

To compute the maximum bending moment, we have, first, to find the dangerous section. This section is where the shear changes sign (see article 45); hence, we have to construct the shear diagram, or as much thereof as is needed to find where the change of sign occurs. Therefore we need the values of the reaction. Neglecting the weight of the beam, the moment equation with origin at C (Fig. 37, *a*) is

$$R_1 \times 10 - 4{,}000 \times 8 = 0, \text{ or } R_1 = 3{,}200 \text{ pounds}$$

---

* NOTE. Because S is greatest in the section where M is maximum, this section is usually called the "dangerous section" of the beam.

Then, constructing the shear diagram, we see (Fig. 37, *b*) that the change of sign of the shear (also the dangerous section) is at the load. The value of the bending moment there is

$$3{,}200 \times 2 = 6{,}400 \text{ foot-pounds,}$$

or

$$6{,}400 \times 12 = 76{,}800 \text{ inch-pounds.}$$

Substituting in equation 6', we find that

$$S = \frac{76{,}800}{96} = 800 \text{ pounds per square inch.}$$

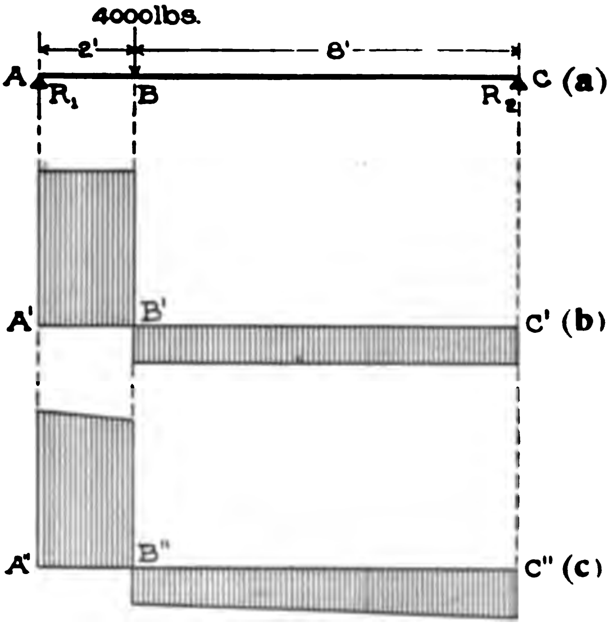


Fig. 37.

2. It is desired to take into account the weight of the beam in the preceding example, supposing the beam to be wooden.

The volume of the beam is

$$\frac{4 \times 12}{144} \times 10 = 3\tfrac{1}{3} \text{ cubic feet;}$$

and supposing the timber to weigh 45 pounds per cubic foot, the beam weighs 150 pounds (insignificant compared to the load). The left reaction, therefore, is

$$3{,}200 + \frac{1}{2} \times 150 = 3{,}275;$$

and the shear diagram looks like Fig. 37, *c*, the shear changing sign at the load as before. The weight of the beam to the left of the dangerous section is 30 pounds; hence the maximum bending moment equals

$$3{,}275 \times 2 - 30 \times 1 = 6{,}520 \text{ foot-pounds,}$$

or

$$6{,}520 \times 12 = 78{,}240 \text{ inch-pounds.}$$

Substituting in equation 6′, we find that

$$S = \frac{78{,}240}{96} = 815 \text{ pounds per square inch.}$$

The weight of the beam therefore increases the unit-stress produced by the load at the dangerous section by 15 pounds per square inch.

3. A T-bar (see Fig. 38) 8 feet long and supported at each end, bears a uniform load of 1,200 pounds. The moment of inertia of its cross-section with respect to the neutral axis being 2.42 inches⁴, compute the maximum tensile and compressive unit-stresses in the beam

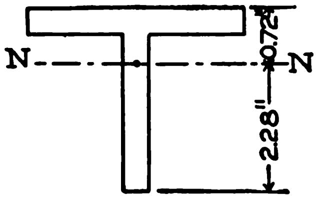


Fig. 38.

Evidently the dangerous section is in the middle, and the value of the maximum bending moment (see Table B, page 53, Part I) is $\frac{1}{8}$ Wl, W and l denoting the load and length respectively. Here

$$\frac{1}{8} Wl = \frac{1}{8} \times 1{,}200 \times 8 = 1{,}200 \text{ foot-pounds,}$$

or

$$1{,}200 \times 12 = 14{,}400 \text{ inch-pounds.}$$

The section modulus equals 2.42 ÷ 2.28 = 1.06; hence

$$S = \frac{14{,}400}{1.06} = 13{,}585 \text{ pounds per square inch.}$$

This is the unit-fibre stress on the lowest fibre at the middle section, and hence is tensile. On the highest fibre at the middle section the unit-stress is compressive, and equals (see page 60):

$$S' = \frac{c'}{c} S = \frac{0.72}{2.28} \times 13{,}585 = 4{,}290 \text{ pounds per square inch.}$$

EXAMPLES FOR PRACTICE.

1. A beam 12 feet long and 6 × 12 inches in cross-section rests on end supports, and sustains a load of 3,000 pounds in the middle. Compute the greatest tensile and compressive unit-stresses in the beam, neglecting the weight of the beam.

Ans. 750 pounds per square inch.

2. Solve the preceding example taking into account the weight of the beam, 300 pounds.

Ans. 787.5 pounds per square inch.

3. Suppose that a built-in cantilever projects 5 feet from the wall and sustains an end load of 250 pounds. The cross-section of the cantilever being represented in Fig. 38, compute the greatest tensile and compressive unit stresses, and tell at what places they occur. (Neglect the weight.)

Ans. { Tensile, 4,471 pounds per square inch.
{ Compressive, 14,150 " " " "

4. Compute the greatest tensile and compressive unit-stresses in the beam of Fig. 18, *a*, due to the loads and the weight of beam (400 pounds). (A moment diagram is represented in Fig. 18, *b*; for description see example 2, Art. 44, p. 39.) The section of the beam is a rectangle 8 × 12 inches.

Ans. 580 pounds per square inch.

5. Compute the greatest tensile and compressive unit-stresses in the cantilever beam of Fig. 19, *a*, it being a steel I-beam whose section modulus is 20.4 inches³. (A bending moment diagram for it is represented in Fig. 19, *b*; for description, see Ex. 3, Art. 44.)

Ans. 11,470 pounds per square inch.

6. Compute the greatest tensile and compressive unit-stresses in the beam of Fig. 10, neglecting its weight, the cross-sections being rectangular 6 × 12 inches. (See example for practice 1, Art. 43.)

Ans. 600 pounds per square inch.

65. *Second Application.* The dimensions and the working strengths of a beam are given, and it is required to determine its safe load (the manner of application being given).

This problem can be solved by means of equation 6 written in this form,

$$M = \frac{SI}{c} \qquad (6'')$$

We substitute for S the given working strength for the material of the beam, and for I and c their values as computed from the given dimensions of the cross-section; then reduce, thus obtaining the value of the safe resisting moment of the beam, which equals the greatest safe bending moment that the beam can stand. We next compute the value of the maximum bending moment in terms of the unknown load; equate this to the value of the resisting moment previously found; and solve for the unknown load.

In cast iron, the tensile and compressive strengths are very different; and the smaller (the tensile) should always be used if the neutral surface of the beam is midway between the top and bottom of the beam; but if it is unequally distant from the top and bottom, proceed as in example 4, following.

Examples. 1. A wooden beam 12 feet long and 6 × 12 inches in cross-section rests on end supports. If its working strength is 800 pounds per square inch, how large a load uniformly distributed can it sustain?

The section modulus is $\frac{1}{6}ba^2$, b and a denoting the base and altitude of the section (see Table A, page 52); and here

$$\frac{1}{6}ba^2 = \frac{1}{6} \times 6 \times 12^2 = 144 \text{ inches}^3.$$

Hence $$S\frac{I}{c} = 800 \times 144 = 115{,}200 \text{ inch-pounds}.$$

For a beam on end supports and sustaining a uniform load, the maximum bending moment equals $\frac{1}{8}Wl$ (see Table B, page 55), W denoting the sum of the load and weight of beam, and l the length. If W is expressed in pounds, then

$$\frac{1}{8}Wl = \frac{1}{8}W \times 12 \text{ foot-pounds} = \frac{1}{8}W \times 144 \text{ inch-pounds}.$$

Hence, equating the two values of maximum bending moment and the safe resisting moment, we get

$$\frac{1}{8}W \times 144 = 115{,}200;$$

or, $$W = \frac{115{,}200 \times 8}{144} = 6{,}400 \text{ pounds}.$$

The safe load for the beam is 6,400 pounds minus the weight of the beam.

2. A steel I-beam whose section modulus is 20.4 inches³ rests on end supports 15 feet apart. Neglecting the weight of the beam, how large a load may be placed upon it 5 feet from one end, if the working strength is 16,000 pounds per square inch?

The safe resisting moment is

$$\frac{SI}{c} = 16,000 \times 20.4 = 326,400 \text{ inch-pounds;}$$

hence the bending moment must not exceed that value. The dangerous section is under the load; and if P denotes the unknown value of the load in pounds, the maximum moment (see Table B, page 53, Part I) equals $\frac{2}{3}$ P × 5 foot-pounds, or $\frac{2}{3}$ P × 60 inch-pounds. Equating values of bending and resisting moments, we get

$$\frac{2}{3} P \times 60 = 326,400;$$

or,

$$P = \frac{326,400 \times 3}{2 \times 60} = 8,160 \text{ pounds.}$$

3. In the preceding example, it is required to take into account the weight of the beam, 375 pounds.

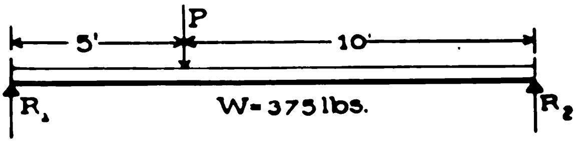


Fig. 39.

As we do not know the value of the safe load, we cannot construct the shear diagram and thus determine where the dangerous section is. But in cases like this, where the distributed load (the weight) is small compared with the concentrated load, the dangerous section is practically always where it is under the concentrated load alone; in this case, at the load. The reactions due to the weight equal $\frac{1}{2} \times 375 = 187.5$; and the reactions due to the load equal $\frac{1}{3}$ P and $\frac{2}{3}$ P, P denoting the value of the load. The larger reaction R_1 (Fig. 39) hence equals $\frac{2}{3} P + 187.5$. Since the weight of the beam per foot is 375 ÷ 15 = 25 pounds, the maximum bending moment (at the load) equals

$$\left(\frac{2}{3}P + 187.5\right)5 - (25 \times 5)\,2\tfrac{1}{2} =$$

$$\frac{10}{3}P + 937.5 - 312.5 = \frac{10}{3}P + 625.$$

This is in foot-pounds if P is in pounds.

The safe resisting moment is the same as in the preceding illustration, 326,400 inch-pounds; hence

$$\left(\frac{10}{3}P + 625\right)12 = 326{,}400.$$

Solving for P, we have

$$\frac{10}{3}P + 625 = \frac{326{,}400}{12};$$

$$10P + 625 \times 3 = \frac{326{,}400 \times 3}{12} = 81{,}600;$$

$$10P = 79{,}725;$$

or, $P = 7{,}972.5$ pounds.

It remains to test our assumption that the dangerous section is at the load. This can be done by computing R_1 (with $P = 7{,}972.5$), constructing the shear diagram, and noting where the shear changes sign. It will be found that the shear changes sign at the load, thus verifying the assumption.

4. A cast-iron built-in cantilever beam projects 8 feet from the wall. Its cross-section is represented in Fig. 40, and the moment of inertia with respect to the neutral axis is 50 inches^4; the working strengths in tension and compression are 2,000 and 9,000 pounds per square inch respectively. Compute the safe uniform load which the beam can sustain, neglecting the weight of the beam.

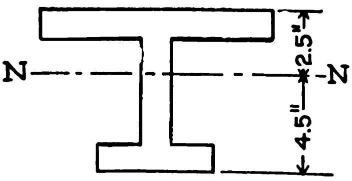


Fig. 40.

The beam being convex up, the upper fibres are in tension and the lower in compression. The resisting moment (SI ÷ c), as determined by the compressive strength, is

$$\frac{9,000 \times 50}{4.5} = 100,000 \text{ inch-pounds;}$$

and the resisting moment, as determined by the tensile strength, is

$$\frac{2,000 \times 50}{2.5} = 40,000 \text{ inch-pounds.}$$

Hence the safe resisting moment is the lesser of these two, or 40,000 inch-pounds. The dangerous section is at the wall (see Table B, page 53), and the value of the maximum bending moment is $\frac{1}{2}$ Wl, W denoting the load and l the length. If W is in pounds, then

$$M = \frac{1}{2} W \times 8 \text{ foot-pounds} = \frac{1}{2} W \times 96 \text{ inch-pounds.}$$

Equating bending and resisting moments, we have

$$\frac{1}{2} W \times 96 = 40,000;$$

or,
$$W = \frac{40,000 \times 2}{96} = 833 \text{ pounds.}$$

EXAMPLES FOR PRACTICE.

1. An 8 × 8-inch timber projects 8 feet from a wall. If its working strength is 1,000 pounds per square inch, how large an end load can it safely sustain?

Ans. 890 pounds.

2. A beam 12 feet long and 8 × 16 inches in cross-section, on end supports, sustains two loads P, each 3 feet from its ends respectively. The working strength being 1,000 pounds per square inch, compute P (see Table B, page 53).

Ans. 9,480 pounds.

3. An I-beam weighing 25 pounds per foot rests on end supports 20 feet apart. Its section modulus is 20.4 inches³, and its working strength 16,000 pounds per square inch. Compute the safe uniform load which it can sustain.

Ans. 10,880 pounds.

66. *Third Application.* The loads, manner of support, and working strength of beam are given, and it is required to determine the size of cross-section necessary to sustain the load safely, that is, to "design the beam."

To solve this problem, we use the first beam formula (equation 6), written in this form,

$$\frac{I}{c} = \frac{M}{S}. \qquad (6''')$$

We first determine the maximum bending moment, and then substitute its value for M, and the working strength for S. Then we have the value of the section modulus ($I \div c$) of the required beam. Many cross-sections can be designed, all having a given section modulus. Which one is to be selected as most suitable will depend on the circumstances attending the use of the beam and on considerations of economy.

Examples. 1. A timber beam is to be used for sustaining a uniform load of 1,500 pounds, the distance between the supports being 20 feet. If the working strength of the timber is 1,000 pounds per square inch, what is the necessary size of cross-section ?

The dangerous section is at the middle of the beam; and the maximum bending moment (see Table B, page 53) is

$$\frac{1}{8}Wl = \frac{1}{8} \times 1{,}500 \times 20 = 3{,}750 \text{ foot-pounds,}$$

or $$3{,}750 \times 12 = 45{,}000 \text{ inch-pounds.}$$

Hence $$\frac{I}{c} = \frac{45{,}000}{1{,}000} = 45 \text{ inches}^3.$$

Now the section modulus of a rectangle is $\frac{1}{6}ba^2$ (see Table A, page 54, Part I); therefore, $\frac{1}{6}ba^2 = 45$, or $ba^2 = 270$.

Any wooden beam (safe strength 1,000 pounds per square inch) whose breadth times its depth square equals or exceeds 270, is strong enough to sustain the load specified, 1,500 pounds.

To determine a size, we may choose any value for b or a, and solve the last equation for the unknown dimension. It is best, however, to select a value of the breadth, as 1, 2, 3, or 4 inches, and solve for a. Thus, if we try $b = 1$ inch, we have

$$a^2 = 270, \text{ or } a = 16.43 \text{ inches.}$$

This would mean a board 1 × 18 inches, which, if used, would have to be supported sidewise so as to prevent it from tipping or "buckling." Ordinarily, this would not be a good size.

Next try $b = 2$ inches; we have

$$2 \times a^2 = 270; \text{ or } a = \sqrt{270 \div 2} = 11.62 \text{ inches.}$$

This would require a plank 2 × 12, a better proportion than the first. Trying $b = 3$ inches, we have

$$3 \times d^2 = 270; \text{ or } d = \sqrt{270 \div 3} = 9.49 \text{ inches.}$$

This would require a plank 3 × 10 inches; and a choice between a 2 × 12 and a 3 × 10 plank would be governed by circumstances in the case of an actual construction.

It will be noticed that we have neglected the weight of the beam. Since the dimensions of wooden beams are not fractional, and we have to select a commercial size next larger than the one computed (12 inches instead of 11.62 inches, for example), the additional depth is usually sufficient to provide strength for the weight of the beam. If there is any doubt in the matter, we can settle it by computing the maximum bending moment including the weight of the beam, and then computing the greatest unit-fibre stress due to load and weight. If this is less than the safe strength, the section is large enough; if greater, the section is too small.

Thus, let us determine whether the 2 × 12-inch plank is strong enough to sustain the load and its own weight. The plank will weigh about 120 pounds, making a total load of

$$1{,}500 + 120 = 1{,}620 \text{ pounds.}$$

Hence the maximum bending moment is

$$\frac{1}{8}\text{W}l = \frac{1}{8}1{,}620 \times 20 \times 12 = 48{,}600 \text{ inch-pounds.}$$

Since $\frac{\text{I}}{c} = \frac{1}{6}bd^2 = \frac{1}{6} \times 2 \times 12^2 = 48$, and $\text{S} = \frac{\text{M}}{\text{I} \div c}$,

$$\text{S} = \frac{48{,}600}{48} = 1{,}013 \text{ pounds per square inch.}$$

Strictly, therefore, the 2 × 12-inch plank is not large enough; but as the greatest unit-stress in it would be only 13 pounds per square inch too large, its use would be permissible.

2. What size of steel I-beam is needed to sustain safely the loading of Fig. 9 if the safe strength of the steel is 16,000 pounds per square inch?

The maximum bending moment due to the loads was found in example 1, Art. 43, to be 8,800 foot-pounds, or 8,800 × 12 = 105,600 inch-pounds.

Hence $\frac{\text{I}}{c} = \frac{105{,}600}{16{,}000} = 6.6 \text{ inches}^3.$

That is, an I-beam is needed whose section modulus is a little larger than 6.6, to provide strength for its own weight.

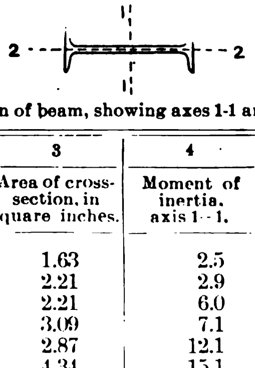
To select a size, we need a descriptive table of I-beams, such as is published in handbooks on structural steel.

Below is an abridged copy of such a table. (The last two columns contain information for use later.) The figure illustrates a cross-section of an I-beam, and shows the axes referred to in the table.

It will be noticed that two sizes are given for each depth; these are the lightest and heaviest of each size that are made, but intermediate sizes can be secured. In column 5 we find 7.3 as the next larger section modulus than the one required (6.6); and this corresponds to a 12$\frac{1}{4}$-pound 6-inch I-beam, which is probably the proper size. To ascertain whether the excess ($7.3 - 6.6 = 0.70$) in the section modulus is sufficient to provide for the weight of the beam, we might proceed as in example 1. In this case, however, the excess is quite large, and the beam selected is doubtless safe.

TABLE C.

Properties of Standard I-Beams



Section of beam, showing axes 1-1 and 2-2.

1	2	3	4	5	6
Depth of Beam, in inches.	Weight per foot, in pounds.	Area of cross-section, in square inches.	Moment of inertia, axis 1—1.	Section modulus, axis 1—1.	Moment of inertia, axis 2—2.
3	5.50	1.63	2.5	1.7	0.46
3	7.50	2.21	2.9	1.9	.60
4	7.50	2.21	6.0	3.0	.77
4	10.50	3.09	7.1	3.6	1.01
5	9.75	2.87	12.1	4.8	1.23
5	14.75	4.34	15.1	6.1	1.70
6	12.25	3.61	21.8	7.3	1.85
6	17.25	5.07	26.2	8.7	2.36
7	15.00	4.42	36.2	10.4	2.67
7	20.00	5.88	42.2	12.1	3.24
8	18.00	5.33	56.9	14.2	3.78
8	25.25	7.43	68.0	17.0	4.71
9	21.00	6.31	84.9	18.9	5.16
9	35.00	10.29	111.8	24.8	7.31
10	25.00	7.37	122.1	24.4	6.89
10	40.00	11.76	158.7	31.7	9.50
12	31.50	9.26	215.8	36.0	9.50
12	40.00	11.76	245.9	41.0	10.95
15	42.00	12.48	441.8	58.9	14.62
15	60.00	17.65	538.6	71.8	18.17
18	55.00	15.93	795.6	88.4	21.19
18	70.00	20.59	921.2	102.4	24.62
20	65.00	19.08	1,169.5	117.0	27.86
20	75.00	22.06	1,268.8	126.9	30.25
24	80.00	23.32	2,087.2	173.9	42.86
24	100.00	29.41	2,379.6	198.3	48.55

EXAMPLES FOR PRACTICE.

1. Determine the size of a wooden beam which can safely sustain a middle load of 2,000 pounds, if the beam rests on end supports 16 feet apart, and its working strength is 1,000 pounds per square inch. Assume width 6 inches.

Ans. 6 × 10 inches.

2. What sized steel I-beam is needed to sustain safely a uniform load of 200,000 pounds, if it rests on end supports 10 feet apart, and its working strength is 16,000 pounds per square inch?

Ans. 95-pound 24-inch.

3. What sized steel I-beam is needed to sustain safely the loading of Fig. 10, if its working strength is 16,000 pounds per square inch?

Ans. 14.75-pound 5-inch.

67. Laws of Strength of Beams. The strength of a beam is measured by the bending moment that it can safely withstand; or, since bending and resisting moments are equal, by its safe resisting moment ($SI \div c$). Hence the **safe strength** of a beam varies (1) directly as the working fibre strength of its material, and (2) directly as the section modulus of its cross-section. For beams rectangular in cross-section (as wooden beams), the section modulus is $\frac{1}{6}ba^2$, b and a denoting the breadth and altitude of the rectangle. Hence the strength of such beams varies also directly as the breadth, and as the square of the depth. Thus, doubling the breadth of the section for a rectangular beam doubles the strength, but doubling the depth quadruples the strength.

The **safe load** that a beam can sustain varies directly as its resisting moment, and depends on the way in which the load is distributed and how the beam is supported. Thus, in the first four and last two cases of the table on page 55,

$M = Pl$, hence $P = SI \div lc$,
$M = \frac{1}{2} Wl$, " $W = 2SI \div lc$,
$M = \frac{1}{4} Pl$, " $P = 4SI \div lc$,
$M = \frac{1}{8} Wl$, " $W = 8SI \div lc$,
$M = \frac{1}{8} Pl$, " $P = 8SI \div lc$,
$M = \frac{1}{12} Wl$, " $W = 12SI \div lc$,

Therefore the safe load in all cases varies **inversely with the** length; and for the different cases the safe loads **are as 1, 2, 4, 8,** 8, and 12 respectively.

Example. What is the ratio of the strengths **of a plank** 2 × 10 inches when placed edgewise and when placed **flatwise on its** supports?

When placed edgewise, the section modulus **of the plank is** $\frac{1}{6} \times 2 \times 10^2 = 33\frac{1}{3}$, and when placed flatwise it is $\frac{1}{6} \times 10 \times 2^2 = 6\frac{2}{3}$; hence its strengths in the two positions are **as $33\frac{1}{3}$ to $6\frac{2}{3}$** respectively, or as 5 to 1.

EXAMPLE FOR PRACTICE.

What is the ratio of the safe loads for two **beams of wood,** one being 10 feet long, 3 × 12 inches in section, and **having its load** in the middle; and the other 8 feet long and 2 × 8 **inches in section,** with its load uniformly distributed.

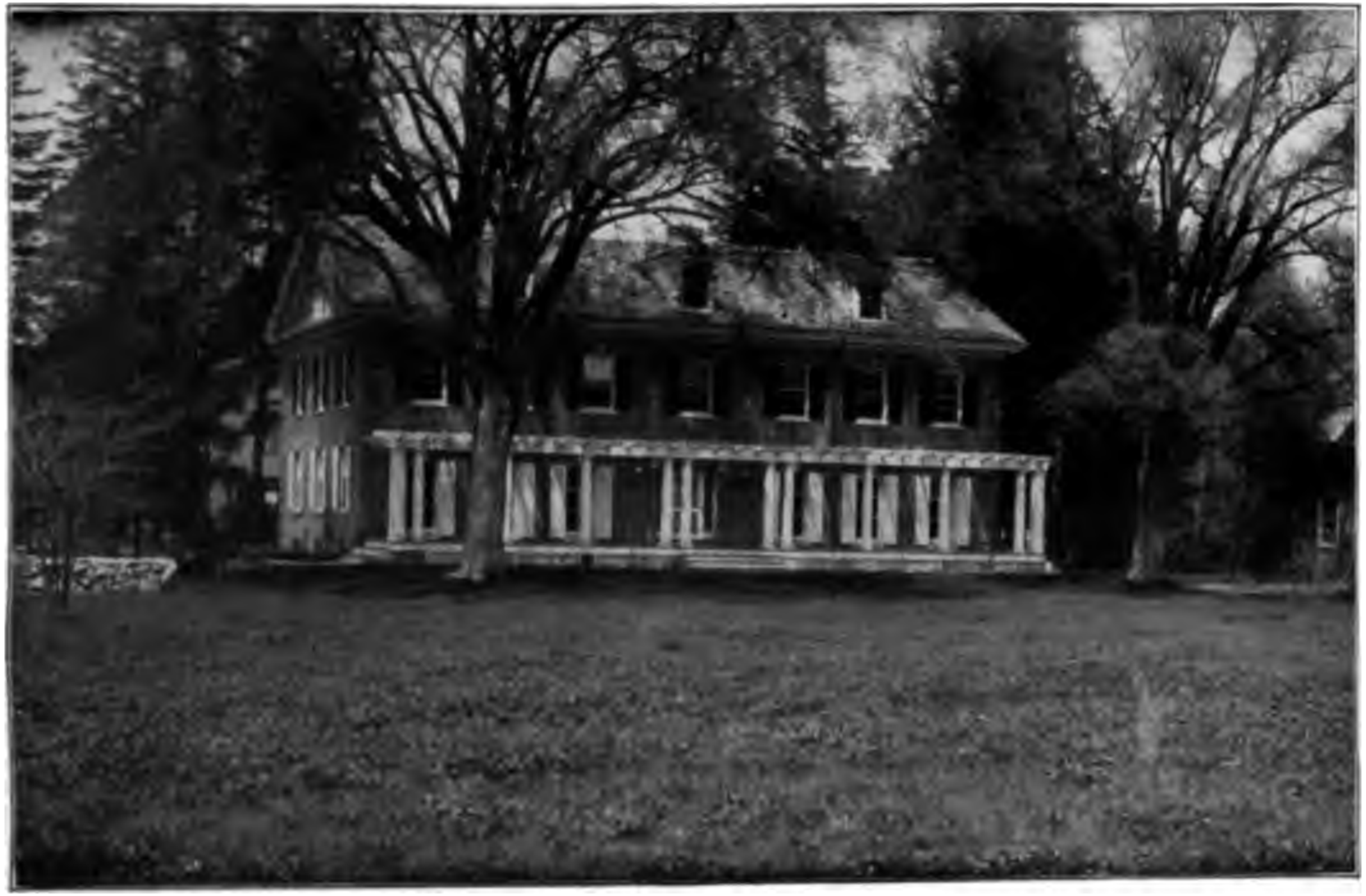
Ans. As 28.8 to 21.3

68. Modulus of Rupture. If a beam is loaded to **destruction,** and the value of the bending moment for the **rupture stage is** computed and substituted for M in the formula $SI \div c = M$, **then** the value of S computed from the equation is the **modulus of rupture** for the material of the beam. Many experiments **have** been performed to ascertain the moduli of rupture for **different** materials and for different grades of the same material. **The following** are fair values, all in pounds per square inch:

TABLE D.

Moduli of Rupture.

Timber:				
Spruce	4,000 -	7,000,	average	5,000
Hemlock	3,500	7,000,	"	4,500
White pine	5,500	10,500,	"	8,000
Long leaf pine	10,000	16,000,	"	12,500
Short-leaf pine	8,000	14,000,	"	10,000
Douglas spruce	4,000	12,000,	"	8,000
White oak	7,500	18,500,	"	13,000
Red oak	9,000	15,000,	"	11,500
Stone:				
Sandstone	400 -	1,200,		
Limestone	400	1,000.		
Granite	800	1,400.		
Cast iron:	One and one half to two and one-quarter times its ultimate tensile strength.			
Hard steel:	Varies from 100,000 to 150,000			



GARDEN FRONT OF RESIDENCE AT DEADHAM, MASSACHUSETTS

Frank Chouteau Brown, Architect, Boston, Massachusetts

For street front and plans see pages 394 and 410

Wrought iron and structural steels have no modulus of rupture, as specimens of those materials will "bend double," but not break. The modulus of rupture of a material is used principally as a basis for determining its working strength. *The factor of safety of a loaded beam is computed by dividing the modulus of rupture of its material by the greatest unit-fibre stress in the beam.*

69. The Resisting Shear. The shearing stress on a cross-section of a loaded beam is not a uniform stress; that is, it is not uniformly distributed over the section. In fact the intensity or unit-stress is actually zero on the highest and lowest fibres of a cross-section, and is greatest, in such beams as are used in practice, on fibres at the neutral axis. In the following article we explain how to find the maximum value in two cases—cases which are practically important.

70. Second Beam Formula. Let S_s denote the average value of the unit-shearing stress on a cross-section of a loaded beam, and A the area of the cross-section. Then the value of the whole shearing stress on the section is:

$$\text{Resisting shear} = S_s A.$$

Since the resisting shear and the external shear at any section of a beam are equal (see Art. 59),

$$S_s A = V. \qquad (7)$$

This is called the "second beam formula." It is used to investigate and to design for shear in beams.

In beams uniform in cross-section, A is constant, and S_s is greatest in the section for which V is greatest. Hence the greatest unit-shearing stress in a loaded beam is at the neutral axis of the section at which the external shear is a maximum. There is a formula for computing this maximum value in any case, but it is not simple, and we give a simpler method for computing the value in the two practically important cases:

1. In wooden beams (rectangular or square in cross-section), the greatest unit shearing stress in a section is 50 per cent larger than the average value S_s.

2. In I beams, and in others with a thin vertical web, the greatest unit shearing stress in a section practically equals S_s, as given by equation 7 if the area of the web is substituted for A.

Examples. 1. What is the greatest value of the unit-shearing stress in a wooden beam 12 feet long and 6 × 12 inches in cross-section when resting on end supports and sustaining a uniform load of 6,400 pounds? (This is the safe load as determined by working fibre stress; see example 1, Art. 65.)

The maximum external shear equals one-half the load (see Table B, page 53), and comes on the sections near the supports.

Since $A = 6 \times 12 = 72$ square inches;

$$S_s = \frac{3{,}200}{72} = 44 \text{ pounds per square inch,}$$

and the greatest unit-shearing stress equals

$$\frac{3}{2} S_s = \frac{3}{2} 44 = 66 \text{ pounds per square inch.}$$

Apparently this is very insignificant; but it is not negligible, as is explained in the next article.

2. A steel I-beam resting on end supports 15 feet apart sustains a load of 8,000 pounds 5 feet from one end. The weight of the beam is 375 pounds, and the area of its web section is 3.2 square inches. (This is the beam and load described in examples 2 and 3, Art. 65.) What is the greatest unit-shearing stress?

The maximum external shear occurs near the support where the reaction is the greater, and its value equals that reaction. Calling that reaction R, and taking moments about the other end of the beam, we have

$$R \times 15 - 375 \times 7\frac{1}{2} - 8{,}000 \times 10 = 0;$$

therefore $15\,R = 80{,}000 + 2{,}812.5 = 82{,}812.5;$

or, $R = 5{,}520.8$ pounds.

Hence $S_s = \frac{5{,}520.8}{3.2} = 1{,}725$ pounds per square inch.

EXAMPLES FOR PRACTICE.

1. A wooden beam 10 feet long and 2 × 10 inches in cross-section sustains a middle load of 1,000 pounds. Neglecting the weight of the beam, compute the value of the greatest unit-shearing stress.

Ans. 37.5 pounds per square inch.

2. Solve the preceding example taking into account the weight of the beam, 60 pounds.

Ans. 40 pounds per square inch.

3. A wooden beam 12 feet long and 4 × 12 inches in cross-section sustains a load of 3,000 pounds 4 feet from one end. Neglecting the weight of the beam, compute the value of the greatest shearing unit-stress.

Ans. 62.5 pounds per square inch.

71. Horizontal Shear. It can be proved that there is a shearing stress on every horizontal section of a loaded beam. An experimental explanation will have to suffice here. Imagine a pile of six boards of equal length supported so that they do not bend. If the intermediate supports are removed, they will bend and their ends will not be flush but somewhat as represented in Fig. 41. This indicates that the boards slid over each other during the bending, and hence there was a rubbing and a frictional resistance exerted by the boards upon each other. Now, when a solid beam is being bent, there is an exactly similar tendency for the horizontal layers to slide over each other; and, instead of a frictional resistance, there exists shearing stress on all horizontal sections of the beam.

In the pile of boards the amount of slipping is different at different places between any two boards, being greatest near the supports and zero midway between them. Also, in any cross-section the slippage is least between the upper two and lower two boards, and is greatest between the middle two. These facts indicate that the shearing unit-stress on horizontal sections of a solid beam is greatest in the neutral surface at the supports.

It can be proved that at any place in a beam the shearing unit-stresses on a horizontal and on a vertical section are equal.

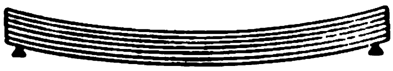


Fig. 41.

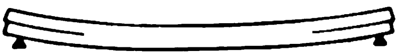


Fig. 42.

It follows that the horizontal shearing unit-stress is greatest at the neutral axis of the section for which the external shear (V) is a maximum. Wood being very weak in shear along the grain, timber beams sometimes fail under shear, the "rupture" being two horizontal cracks along the neutral surface somewhat as represented in Fig. 42. It is therefore necessary, when dealing with timber beams, to give due attention to their strength as determined by the working strength of the material in shear along the grain.

Example. A wooden beam 3 × 10 inches in cross-section rests on end supports and sustains a uniform load of 4,000 pounds Compute the greatest horizontal unit-stress in the beam.

The maximum shear equals one-half the load (see **Table B**, page 55), or 2,000 pounds. Hence, by equation 7, since $A = 3 \times 10 = 30$ square inches,

$$S_s = \frac{2,000}{30} = 66\frac{2}{3} \text{ pounds per square inch.}$$

This is the average shearing unit-stress on the cross-sections near the supports; and the greatest value equals

$$\frac{3}{2} \times 66\frac{2}{3} = 100 \text{ pounds per square inch.}$$

According to the foregoing, this is also the value of the greatest horizontal shearing unit-stress. (If of white pine, for example, the beam would not be regarded as safe, since the ultimate shearing strength along the grain of selected pine is only about 400 pounds per square inch.)

72. Design of Timber Beams. In any case we may proceed as follows:—(1) Determine the dimensions of the cross-section of the beam from a consideration of the fibre stresses as explained in Art. 66. (2) With dimensions thus determined, compute the value of the greatest shearing unit-stress from the formula,

$$\text{Greatest shearing unit-stress} = \frac{3}{2} V \div ab,$$

where V denotes the maximum external shear in the beam, and b and a the breadth and depth of the cross-section.

If the value of the greatest shearing unit-stress so computed does not exceed the working strength in shear along the grain, then the dimensions are large enough; but if it exceeds that value, then a or b, or both, should be increased until $\frac{3}{2}V \div ab$ is less than the working strength. Because timber beams are very often "season checked" (cracked) along the neutral surface, it is advisable to take the working strength of wooden beams, in shear along the grain, quite low. One-twentieth of the working fibre strength has been recommended* for all pine beams.

If the working strength in shear is taken equal to one-twentieth the working fibre strength, then it can be shown that,

1. For a beam on end supports loaded in the middle, the safe load depends on the shearing or fibre strength according as the ratio of length to depth ($l \div d$) is less or greater than 10.

2. For a beam on end supports uniformly loaded, the safe load depends on the shearing or fibre strength according as $l \div d$ is less or greater than 20.

Examples. 1. It is required to design a timber beam to sustain loads as represented in Fig. 11, the working fibre strength being 550 pounds and the working shearing strength 50 pounds per square inch.

The maximum bending moment (see example for practice 3, Art. 43; and example for practice 2, Art. 44) equals practically 7,000 foot-pounds or, 7,000 × 12 = 84,000 inch-pounds. Hence, according to equation 6‴,

$$\frac{I}{c} = \frac{84{,}000}{550} = 152.7 \text{ inches}^3.$$

Since for a rectangle

$$\frac{I}{c} = \frac{1}{6} bd^2,$$

$$\frac{1}{6} bd^2 = 152.7, \text{ or } bd^2 = 916.2.$$

Now, if we let $b = 4$, then $d^2 = 229$;

or, $d = 15.1$ (practically 16) inches.

If, again, we let $b = 6$, then $d^2 = 152.7$;

or $d = 12.4$ (practically 14) inches.

Either of these sizes will answer so far as fibre stress is concerned, but there is more "timber" in the second.

The maximum external shear in the beam equals 1,556 pounds, neglecting the weight of the beam (see example 3, Art. 37; and example 2, Art. 38). Therefore, for a 4 × 16-inch beam,

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* See "Materials of Construction." JOHNSON. Page 55.

$$\text{Greatest shearing unit-stress} = \frac{3}{2} \times \frac{1,556}{4 \times 16}$$
$$= 36.5 \text{ pounds per square inch;}$$

and for a 6 × 14-inch beam, it equals

$$\frac{3}{2} \times \frac{1,556}{6 \times 14} = 27.7 \text{ pounds per square inch.}$$

Since these values are less than the working strength in shear, either size of beam is safe as regards shear.

If it is desired to allow for weight of beam, one of the sizes should be selected. First, its weight should be computed, then the new reactions, and then the unit-fibre stress may be computed as in Art. 64, and the greatest shearing unit-stress as in the foregoing. If these values are within the working values, then the size is large enough to sustain safely the load and the weight of the beam.

2. What is the safe load for a white pine beam 9 feet long and 2 × 12 inches in cross-section, if the beam rests on end supports and the load is at the middle of the beam, the working fibre strength being 1,000 pounds and the shearing strength 50 pounds per square inch.

The ratio of the length to the depth is less than 10; hence the safe load depends on the shearing strength of the material Calling the load P, the maximum external shear (see Table B, page 53) equals $\frac{1}{2}$ P, and the formula for greatest shearing unit stress becomes

$$50 = \frac{3}{2} \times \frac{\frac{1}{2} P}{2 \times 12}; \text{ or } P = 1,600 \text{ pounds.}$$

EXAMPLES FOR PRACTICE.

1. What size of wooden beam can safely sustain loads as in Fig. 12, with shearing and fibre working strength equal to 50 and 1,000 pounds per square inch respectively ?

Ans. 6 × 12 inches

2. What is the safe load for a wooden beam 4 × 14 inches, and 18 feet long, if the beam rests on end supports and the load is uniformly distributed, with working strengths as in example 1 ?

Ans. 3,730 pounds

73. Kinds of Loads and Beams. We shall now discuss the strength of beams under **longitudinal** forces (acting parallel to the beam) and **transverse loads.** The longitudinal forces are supposed to be applied at the ends of the beams and along the axis* of the beam in each case. We consider only beams resting on end supports.

The transverse forces produce **bending** or **flexure,** and the longitudinal or end forces, if pulls, produce **tension** in the beam; if pushes, they produce **compression.** Hence the cases to be considered may be called "Combined Flexure and Tension" and "Combined Flexure and Compression."

74. Flexure and Tension. Let Fig. 43, *a*, represent a beam subjected to the transverse loads L_1, L_2 and L_3, and to two equal end pulls P and P. The reactions R_1 and R_2 are due to the transverse loads and can be computed by the methods of moments just as though there were no end pulls. To find the stresses at any cross-section, we determine those due to the transverse forces (L_1, L_2, L_3, R_1 and R_2) and those due to the longitudinal; then combine these stresses to get the total effect of all the applied forces.

The stress due to the transverse forces consists of a shearing stress and a fibre stress; it will be called the **flexural stress.** The fibre stress is compressive above and tensile below. Let M denote the value of the bending moment at the section considered; c_1 and c_2 the distances from the neutral axis to the highest and the lowest fibre in the section; and S_1 and S_2 the corresponding unit-fibre stresses due to the transverse loads. Then

$$S_1 = \frac{Mc_1}{I}; \text{ and } S_2 = \frac{Mc_2}{I}.$$

The stress due to the end pulls is a simple tension, and it equals P; this is sometimes called the **direct stress.** Let S_0 denote the unit-tension due to P, and A the area of the cross-section; then

$$S_0 = \frac{P}{A}.$$

Both systems of loads to the left of a section between L_1 and

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* NOTE. By "axis of a beam" is meant the line through the centers of gravity of all the cross sections.

L_2 are represented in Fig. 43, *b*; also the stresses caused by them at that section. Clearly the effect of the end pulls is to increase the tensile stress (on the lower fibres) and to decrease the compressive stress (on the upper fibres) due to the flexure. Let S_c denote the total (resultant) unit-stress on the upper fibre, and S_t that on the lower fibre, due to all the forces acting on the beam. In combining the stresses there are two cases to consider:

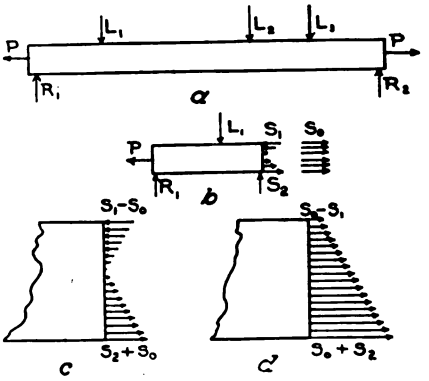

Fig. 43.

(1) The flexural compressive unit-stress on the upper fibre is greater than the direct unit-stress; that is, S_1 is greater than S_0. The resultant stress on the upper fibre is

$$S_c = S_1 - S_0 \text{ (compressive)};$$

and that on the lower fibre is

$$S_t = S_2 + S_0 \text{ (tensile)}.$$

The combined stress is as represented in Fig. 43, *c*, part tensile and part compressive.

(2) The flexural compressive unit-stress is less than the direct unit-stress; that is, S_1 is less than S_0. Then the combined unit-stress on the upper fibre is

$$S_c = S_0 - S_1 \quad \text{(tensile)};$$

and that on the lower fibre is

$$S_t = S_2 + S_0 \quad \text{(tensile)}.$$

The combined stress is represented by Fig. 43, *d*, and is all tensile.

Example. A steel bar 2 × 6 inches, and 12 feet long, is subjected to end pulls of 45,000 pounds. It is supported at each end, and sustains, as a beam, a uniform load of 6,000 pounds. It is required to compute the combined unit-fibre stresses.

Evidently the dangerous section is at the middle, and $M = \frac{1}{8} Wl$; that is,

$$M = \frac{1}{8} \times 6{,}000 \times 12 = 9{,}000 \text{ foot-pounds,}$$

or $$9{,}000 \times 12 = 108{,}000 \text{ inch-pounds.}$$

The bar being placed with the six-inch side vertical,

$$c_1 = c_2 = 3 \text{ inches, and}$$

$$I = \frac{1}{12} \times 2 \times 6^3 = 36 \text{ inches}^4. \quad \text{(See Art. 52.)}$$

Hence $$S_c = S_t = \frac{108{,}000 \times 3}{36} = 9{,}000 \text{ pounds per square inch.}$$

Since $$A = 2 \times 6 = 12 \text{ square inches,}$$

$$S_0 = \frac{45{,}000}{12} = 3{,}750 \text{ pounds per square inch.}$$

The greatest value of the combined compressive stress is

$S_c - S_0 = 9{,}000 - 3{,}750 = 5{,}250$ pounds per square inch,

and it occurs on the upper fibres of the middle section. The greatest value of the combined tensile stress is

$S_t + S_0 = 9{,}000 + 3{,}750 = 12{,}750$ pounds per square inch,

and it occurs on the lowest fibres of the middle section.

EXAMPLE FOR PRACTICE.

Change the load in the preceding illustration to one of **6,000** pounds placed in the middle, and then solve.

Ans. $\begin{cases} S_c = 14{,}250 \text{ pounds per square inch.} \\ S_t = 21{,}750 \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{"} \end{cases}$

75. Flexure and Compression. Imagine the arrowheads on P reversed; then Fig. 43, *a*, will represent a beam under combined flexural and compressive stresses. The flexural unit-stresses are computed as in the preceding article. The direct stress is a compression equal to P, and the unit-stress due to P is computed as in the preceding article. Evidently the effect of P is to increase the compressive stress and decrease the tensile stress due to the flexure. In combining, we have two cases as before:

(1) The flexural tensile unit-stress is greater than the direct unit-stress; that is, S_t is greater than S_0. Then the combined unit-stress on the lower fibre is

$$S_t = S_2 - S_0 \text{ (tensile)};$$

and that on the upper fibre is

$$S_c = S_1 + S_0 \text{ (compressive)}.$$

The combined fibre stress is represented by Fig. 44, *a*, and is part tensile and part compressive.

(2) The flexural unit-stress on the lower fibre is less than the direct unit-stress; that is, S_2 is less than S_0. Then the combined unit-stress on the lower fibre is

$$S_t = S_0 - S_2 \text{ (compressive)};$$

and that on the upper fibre is

$$S_c = S_0 + S_1 \text{ (compressive)}.$$

The combined fibre stress is represented by Fig. 44, *b*, and is all compressive.

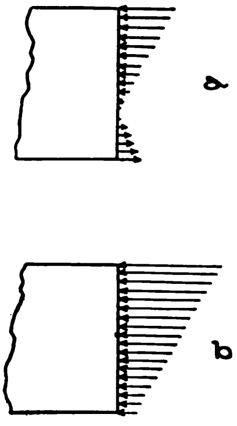

Fig. 44.

Example. A piece of timber 6 × 6 inches, and 10 feet long, is subjected to end pushes of 9,000 pounds. It is supported in a horizontal position at its ends, and sustains a middle load of 400 pounds. Compute the combined fibre stresses.

Evidently the dangerous section is at the middle, and $M = \frac{1}{4} Pl$; that is,

$$M = \frac{1}{4} \times 400 \times 10 = 1{,}000 \text{ foot-pounds},$$

or $1{,}000 \times 12 = 12{,}000$ inch-pounds.

Since $c_1 = c_2 = 3$ inches, and

$$I = \frac{1}{12} ba^3 = \frac{1}{12} \times 6 \times 6^3 = 108 \text{ inches}^4,$$

$$S_1 = S_2 = \frac{12{,}000 \times 3}{108} = 333\frac{1}{3} \text{ pounds per square inch.}$$

Since $A = 6 \times 6 = 36$ square inches,

$$S_0 = \frac{9{,}000}{36} = 250 \text{ pounds per square inch.}$$

Hence the greatest value of the combined compressive stress is

$$S_0 + S_1 = 333\frac{1}{3} + 250 = 583\frac{1}{3} \text{ pounds per square inch.}$$

It occurs on the upper fibres of the middle section. The greatest value of the combined tensile stress is

$$S_t - S_c = 333\frac{1}{3} - 250 = 83\frac{1}{3} \text{ pounds per square inch.}$$

It occurs on the lowest fibres of the middle section.

EXAMPLE FOR PRACTICE.

Change the load of the preceding illustration to a uniform load and solve.

Ans. $\begin{cases} S_c — 417 \text{ pounds per square inch.} \\ S_t \quad 83 \text{ " " " " (compression).} \end{cases}$

76. Combined Flexural and Direct Stress by More Exact Formulas. The results in the preceding articles are only approximately correct. Imagine the beam represented in Fig. 45, *a*, to be first loaded with the transverse loads alone. They cause the beam to bend more or less, and produce certain flexural stresses at each section of the beam. Now, if end pulls are applied they tend to straighten the beam and hence diminish the flexural stresses. This effect of the end pulls was omitted in the discussion of Art. 74, and the results there given are therefore only approximate, the value of the greatest combined fibre unit-stress (S_t) being too large. On the other hand, if the end forces are pushes, they increase the bending, and therefore increase the flexural fibre stresses already caused by the transverse forces (see Fig. 45, *b*). The results indicated in Art. 75 must therefore in this case also be regarded as only approximate, the value of the greatest unit-fibre stress (S_c) being too small.

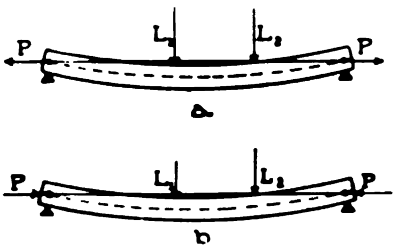

Fig. 45.

For beams loaded in the middle or with a uniform load, the following formulas, which take into account the flexural effect of the end forces, may be used:

M denotes bending moment at the middle section of the beam;

I denotes the moment of inertia of the middle section with respect to the neutral axis;

S_1, S_2, c_1 and c_2 have the same meanings as in Arts. 74 and 75, but refer always to the middle section;

l denotes length of the beam;

E is a number depending on the stiffness of the material, the average values of which are, for timber, 1,500,000; and for structural steel 30,000,000.*

$$S_1 = \frac{Mc_1}{I \pm \frac{Pl^2}{10E}}, \text{ and } S_2 = \frac{Mc_2}{I \pm \frac{Pl^2}{10E}}. \quad (8)$$

The plus sign is to be used when the end forces P are pulls, and the minus sign when they are pushes.

It must be remembered that S_1 and S_2 are flexural unit-stresses. The combination of these and the direct unit-stress is made exactly as in articles 74 and 75.

Examples. 1. It is required to apply the formulas of this article to the example of article 74.

As explained in the example referred to, M = 108,000 inch-pounds; $c_1 = c_2 = 3$ inches; and I = 36 inches^4.
Now, since l = 12 feet = 144 inches,

$$S_1 = S_2 = \frac{108{,}000 \times 3}{36 + \frac{45{,}000 \times 144^2}{10 \times 30{,}000{,}000}} = \frac{324{,}000}{36 + 3.11} = 8{,}284 \text{ pounds}$$

per square inch, as compared with 9,000 pounds per square inch, the result reached by the use of the approximate formula.
As before, S_o = 3,750 pounds per square inch; hence

S_c = 8,284 - 3,750 = 4,534 pounds per square inch;
and S_t = 8,284 + 3,750 = 12,034 " " " "

2. It is required to apply the formulas of this article to the example of article 75.

As explained in that example,

M = 12,000 inch-pounds;
$c_1 = c_2$ = 3 inches, and I = 108 inches^4.

Now, since l = 120 inches,

$$S_1 = S_2 = \frac{12{,}000 \times 3}{108 - \frac{9{,}000 \times 120^2}{10 \times 1{,}500{,}000}} = \frac{36{,}000}{108 - 8.64} = 362 \text{ pounds}$$

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* Note. This quantity "E" is more fully explained in Article 95.

per square inch, as compared with $333\frac{1}{3}$ pounds per square inch, the result reached by use of the approximate method.

As before, $S_o = 250$ pounds per square inch; hence

$S_c = 362 + 250 = 612$ pounds per square inch; and
$S_t = 362 - 250 = 112$ " " " ".

EXAMPLES FOR PRACTICE.

1. Solve the example for practice of Art. 74 by the formulas of this article.

Ans. $\begin{cases} S_c = 12{,}820 \text{ pounds per square inch.} \\ S_t = 20{,}320 \text{ " " " "} \end{cases}$

2. Solve the example for practice of Art. 75 by the formulas of this article.

Ans. $\begin{cases} S_c = 430 \text{ pounds per square inch.} \\ S_t = 70 \text{ " " " " (compression).} \end{cases}$

STRENGTH OF COLUMNS.

A stick of timber, a bar of iron, etc., when used to sustain end loads which act lengthwise of the pieces, are called **columns, posts,** or **struts** if they are so long that they would bend before breaking. When they are so short that they would not bend before breaking, they are called **short blocks,** and their compressive strengths are computed by means of equation 1. The strengths of columns cannot, however, be so simply determined, and we now proceed to explain the method of computing them.

77. End Conditions. The strength of a column depends in part on the way in which its ends bear, or are joined to other parts of a structure, that is, on its "end conditions." There are practically but three kinds of end conditions, namely:

1. "Hinge" or "pin" ends,
2. "Flat" or "square" ends, and
3. "Fixed" ends.

(1) When a column is fastened to its support at one end by means of a pin about which the column could rotate if the other end were free, it is said to be "hinged" or "pinned" at the former end. Bridge posts or columns are often hinged at the ends.

(2) A column either end of which is flat and perpendicular to its axis and bears on other parts of the structure at that surface, is said to be "flat" or "square" at that end.

(3) Columns are sometimes riveted near their ends directly to other parts of the structure and do not bear directly on their ends; such are called "fixed ended." A column which bears on its flat ends is often fastened near the ends to other parts of the structure, and such an end is also said to be "fixed." The fixing of an end of a column stiffens and therefore strengthens it more or less, but the strength of a column with fixed ends is computed as though its ends were flat. Accordingly we have, so far as strength is concerned, the following classes of columns:

78. Classes of Columns. (1) Both ends hinged or pinned; (2) one end hinged and one flat; (3) both ends flat.

Other things being the same, columns of these three classes are unequal in strength. Columns of the first class are the weakest, and those of the third class are the strongest.

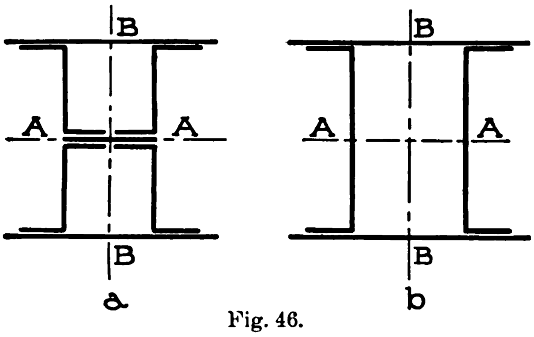


Fig. 46.

79. Cross-sections of Columns. Wooden columns are usually solid, square, rectangular, or round in section; but sometimes they are "built up" hollow. Cast-iron columns are practically always made hollow, and rectangular or round in section. Steel columns are made of single rolled shapes—angles, zees, channels, etc.; but the larger ones are usually "built up" of several shapes. Fig. 46, *a*, for example, represents a cross-section of a "Z-bar" column; and Fig. 46, *b*, that of a "channel" column.

80. Radius of Gyration. There is a quantity appearing in almost all formulas for the strength of columns, which is called "radius of gyration." It depends on the form and extent of the cross-section of the column, and may be defined as follows:

The radius of gyration of any plane figure (as the section of a column) with respect to any line, is such a length that the square of this length multiplied by the area of the figure equals the moment of inertia of the figure with respect to the given line.

Thus, if A denotes the area of a figure; I, its moment of inertia with respect to some line; and r, the radius of gyration with respect to that line; then

$$r^2A = I; \text{ or } r = \sqrt{I \div A}. \qquad (9)$$

In the column formulas, the radius of gyration always refers to an axis through the center of gravity of the cross-section, and usually to that axis with respect to which the radius of gyration (and moment of inertia) is least. (For an exception, see example 3, Art. 83.) Hence the radius of gyration in this connection is often called for brevity the "least radius of gyration," or simply the "least radius."

Examples. 1. Show that the value of the radius of gyration given for the square in Table A, page 52, is correct.

The moment of inertia of the square with respect to the axis is $\frac{1}{12}a^4$. Since A = a^2, then, by formula 9 above,

$$r = \sqrt{\frac{1}{12}a^4 \div a^2} = \sqrt{\frac{1}{12}a^2} = a\sqrt{\frac{1}{12}}.$$

2. Prove that the value of the radius of gyration given for the hollow square in Table A, page 54, is correct.

The value of the moment of inertia of the square with respect to the axis is $\frac{1}{12}(a^4 - a_1^4)$. Since A = $a^2 - a_1^2$,

$$r = \sqrt{\frac{\frac{1}{12}(a^4 - a_1^4)}{a^2 - a_1^2}} = \sqrt{\frac{1}{12}(a^2 + a_1^2)}.$$

EXAMPLE FOR PRACTICE.

Prove that the values of the radii of gyration of the other figures given in Table A, page 52, are correct. The axis in each case is indicated by the line through the center of gravity.

81. Radius of Gyration of Built-up Sections. The radius of gyration of a built-up section is computed similarly to that of any other figure. First, we have to compute the moment of inertia of the section, as explained in Art. 54; and then we use formula 9, as in the examples of the preceding article.

Example. It is required to compute the radius of gyration of the section represented in Fig. 30 (page 52) with respect to the axis AA.

In example 1, Art. 54, it is shown that the moment of inertia of the section with respect to the axis AA is 429 inches^4. The area of the whole section is

$$2 \times 6.03 + 7 = 19.06;$$

hence the radius of gyration r is

$$r = \sqrt{\frac{429}{19.06}} = 4.74 \text{ inches.}$$

EXAMPLE FOR PRACTICE.

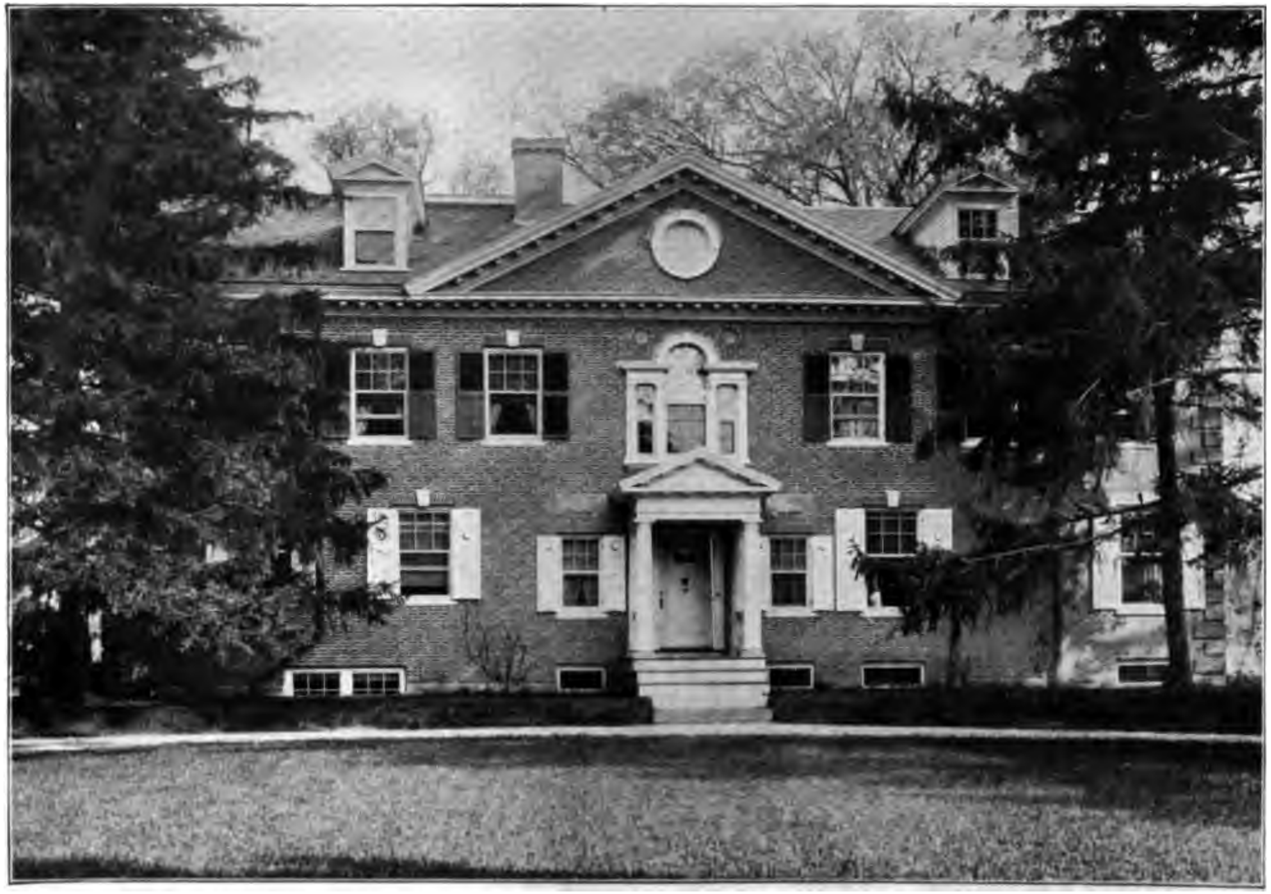
Compute the radii of gyration of the section represented in Fig. 31, *a*, with respect to the axes AA and BB. (See examples for practice 1 and 2, Art. 54.)

$$\text{Ans.} \begin{cases} 2.87 \text{ inches.} \\ 2.09 \quad " \end{cases}$$

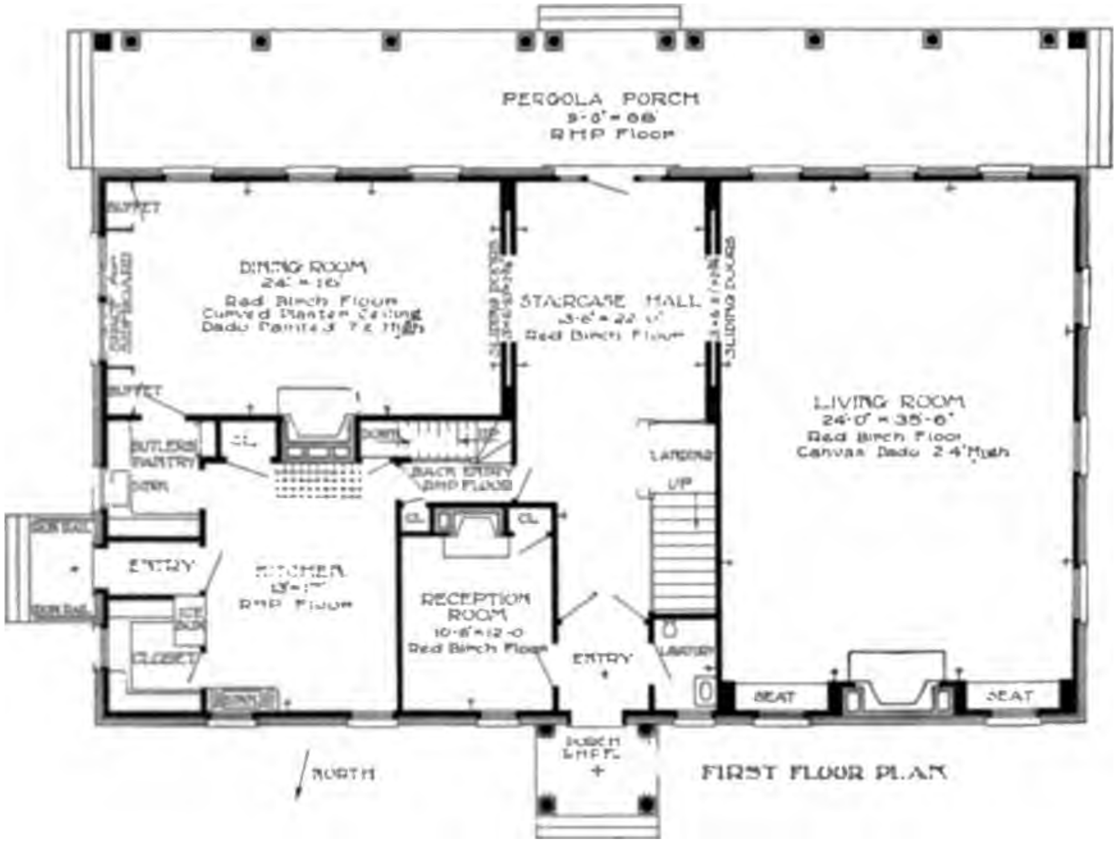
82. Kinds of Column Loads. When the loads applied to a column are such that their resultant acts through the center of gravity of the top section and along the axis of the column, the column is said to be **centrally loaded.** When the resultant of the loads does not act through the center of gravity of the top section, the column is said to be **eccentrically loaded.** All the following formulas refer to columns centrally loaded.

83. Rankine's Column Formula. When a perfectly straight column is centrally loaded, then, if the column does not bend and if it is homogeneous, the stress on every cross-section is a uniform compression. If P denotes the load and A the area of the cross-section, the value of the unit-compression is P ÷ A.

On account of lack of straightness or lack of uniformity in material, or failure to secure exact central application of the load, the load P has what is known as an "arm" or "leverage" and bends the column more or less. There is therefore in such a column a bending or flexural stress in addition to the direct compressive stress above mentioned; this bending stress is compressive



STREET FRONT OF RESIDENCE AT DEADHAM, MASSACHUSETTS
Frank Chouteau Brown, Architect, Boston, Massachusetts
For garden front see page 379; for plans see pages 395 and 410



FIRST FLOOR PLANS OF RESIDENCE AT DEADHAM, MASSACHUSETTS

Frank Chouteau Brown, Architect, Boston, Massachusetts

For other plans see opposite page, 379 and 410

on the concave side and tensile on the convex. The value of the stress per unit-area (unit-stress) on the fibre at the concave side, according to equation 6, is $Mc \div I$, where M denotes the bending moment at the section (due to the load on the column), c the distance from the neutral axis to the concave side, and I the moment of inertia of the cross-section with respect to the neutral axis. (Notice that this axis is perpendicular to the plane in which the column bends.)

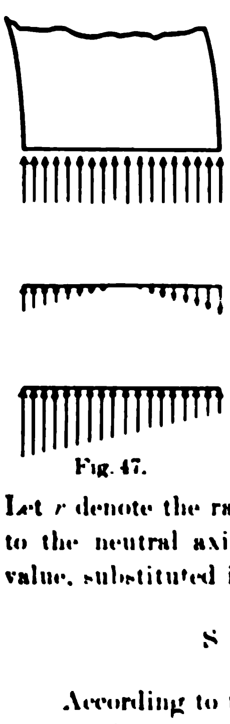


Fig. 47.

The upper set of arrows (Fig. 47) represents the direct compressive stress; and the second set the bending stress if the load is not excessive, so that the stresses are within the elastic limit of the material. The third set represents the combined stress that actually exists on the cross-section. The greatest combined unit-stress evidently occurs on the fibre at the concave side and where the deflection of the column is greatest. The stress is compressive, and its value S per unit-area is given by the formula,

$$S = \frac{P}{A} + \frac{Mc}{I}.$$

Now, the bending moment at the place of greatest deflection equals the product of the load P and its arm (that is, the deflection). Calling the deflection d, we have $M = Pd$; and this value of M, substituted in the last equation, gives

$$S = \frac{P}{A} + \frac{Pdc}{I}.$$

Let r denote the radius of gyration of the cross-section with respect to the neutral axis. Then $I = Ar^2$ (see equation 9); and this value, substituted in the last equation, gives

$$S = \frac{P}{A} + \frac{Pdc}{Ar^2} = \frac{P}{A}\left(1 + \frac{dc}{r^2}\right).$$

According to the theory of the stiffness of beams on end supports, deflections vary directly as the square of the length l, and inversely as the distance c from the neutral axis to the remotest fibre of the cross-section. Assuming that the deflections of columns follow the same laws, we may write $d = k\ (l^2 \div c)$, where k is some constant depending on the material of the column and on the end conditions. Substituting this value for d in the last equation, we find that

$$\left.\begin{aligned} S &= \frac{P}{A}\left(1 + k\frac{l^2}{r^2}\right); \\ \frac{P}{A} &= \frac{S}{1 + k\dfrac{l^2}{r^2}}; \\ \text{and}\qquad P &= \frac{SA}{1 + k\dfrac{l^2}{r^2}}. \end{aligned}\right\} \qquad (10)$$

Each of these (usually the last) is known as "Rankine's formula."

For *mild-steel* columns a certain large steel company uses $S = 50{,}000$ pounds per square inch, and the following values of k:

1.	Columns with two pin ends,	$k = 1 \div 18{,}000$.
2.	" " one flat and one pin end,	$k = 1 \div 24{,}000$.
3.	" " both ends flat,	$k = 1 \div 36{,}000$.

With these values of S and k, P of the formula means the **ultimate load**, that is, the load causing failure. The safe load equals P divided by the selected factor of safety—a factor of 4 for steady loads, and 5 for moving loads, being recommended by the company referred to. The same unit is to be used for l and r.

Cast-iron columns are practically always made hollow with comparatively thin walls, and are usually circular or rectangular in cross-section. The following modifications of Rankine's formula are sometimes used:

$$\left.\begin{aligned} &\text{For circular sections,} && \frac{P}{A} = \frac{80{,}000}{1 + \dfrac{l^2}{800\,d^2}} \\ &\text{For rectangular sections,} && \frac{P}{A} = \frac{80{,}000}{1 + \dfrac{l^2}{1{,}000\,d^2}} \end{aligned}\right\} \qquad (10')$$

In these formulas d denotes the outside diameter of the circular sections or the length of the lesser side of the rectangular sections. The same unit is to be used for l and d.

Examples. 1. A 40-pound 10-inch steel I-beam 8 feet long is used as a flat-ended column. Its load being 100,000 pounds, what is its factor of safety?

Obviously the column tends to bend in a plane perpendicular to its web. Hence the radius of gyration to be used is the one with respect to that central axis of the cross-section which is in the web, that is, axis 2 – 2 (see figure accompanying table, page 72). The moment of inertia of the section with respect to that axis, according to the table, is 9.50 inches⁴; and since the area of the section is 11.76 square inches,

$$r^2 = \frac{9.50}{11.76} = 0.808.$$

Now, l = 8 feet = 96 inches; and since $k = 1 \div 36{,}000$, and S = 50,000, the breaking load for this column, according to Rankine's formula, is

$$P = \frac{50{,}000 \times 11.76}{1 + \dfrac{96^2}{36{,}000 \times 0.808}} = 446{,}790 \text{ pounds.}$$

Since the factor of safety equals the ratio of the breaking load to the actual load on the column, the factor of safety in this case is

$$\frac{446{,}790}{100{,}000} = 4.5 \text{ (approx.)}.$$

2. What is the safe load for a cast-iron column 10 feet long with square ends and a hollow rectangular section, the outside dimensions being 5 × 8 inches; the inner, 4 × 7 inches; and the factor of safety, 6?

In this case l = 10 feet = 120 inches; A = 5 × 8 – 4 × 7 = 12 square inches; and d = 5 inches. Hence, according to formula 10′, for rectangular sections, the breaking load is

$$P = \frac{80{,}000 \times 12}{1 + \dfrac{120^2}{1{,}000 \times 5^2}} = 610{,}000 \text{ pounds.}$$

Since the safe load equals the breaking load divided by the factor of safety, in this case the safe load equals

$$\frac{610{,}000}{6} = 101{,}700 \text{ pounds.}$$

3. A channel column (see Fig. 46, b) is pin-ended, the pins being perpendicular to the webs of the channel (represented by AA in the figure), and its length is 16 feet (distance between axes

of the pins). If the sectional area is 23.5 square inches, and the moment of inertia with respect to AA is 386 inches^4 and with respect to BB 214 inches^4, what is the safe load with a factor of safety of 4?

The column is liable to bend in one of two ways, namely, in the plane perpendicular to the axes of the two pins, or in the plane containing those axes.

(1) For bending in the first plane, the strength of the column is to be computed from the formula for a pin-ended column. Hence, for this case, $r^2 = 386 \div 23.5 = 16$; and the breaking load is

$$P = \frac{50{,}000 \times 23.5}{1 + \frac{(16 \times 12)^2}{18{,}000 \times 16}} = 1{,}041{,}600 \text{ pounds.}$$

The safe load for this case equals $\frac{1{,}041{,}600}{4} = 260{,}400$ pounds.

(2) If the supports of the pins are rigid, then the pins stiffen the column as to bending in the plane of their axes, and the strength of the column for bending in that plane should be computed from the formula for the strength of columns with flat ends. Hence, $r^2 = 214 \div 23.5 = 9.11$, and thebreaking load is

$$P = \frac{50{,}000 \times 23.5}{1 + \frac{(16 \times 12)^2}{36{,}000 \times 9.11}} = 1{,}056{,}000 \text{ pounds.}$$

The safe load for this case equals $\frac{1{,}056{,}000}{4} = 264{,}000$ pounds.

EXAMPLES FOR PRACTICE.

1. A 40-pound 12-inch steel I-beam 10 feet long is used as a column with flat ends sustaining a load of 100,000 pounds. What is its factor of safety?

Ans. 4.1

2. A cast-iron column 15 feet long sustains a load of 150,000 pounds. Its section being a hollow circle, 9 inches outside and 7 inches inside diameter, what is the factor of safety?

Ans. 8.9

3. A steel Z-bar column (see Fig. 46, *a*) is 24 feet long and has square ends; the least radius of gyration of its cross-section is

with respect to that central axis of the cross-section which is in the web, that is, axis 2 – 2 (see figure accompanying table, page 72). The moment of inertia of the section with respect to that axis, according to the table, is 9.50 inches^4; and since the area of the section is 11.76 square inches,

$$r^2 = \frac{9.50}{11.76} = 0.808.$$

Now, l = 8 feet = 96 inches; and since $k = 1 \div 36{,}000$, and S = 50,000, the breaking load for this column, according to Rankine's formula, is

$$P = \frac{50{,}000 \times 11.76}{1 + \dfrac{96^2}{36{,}000 \times 0.808}} = 446{,}790 \text{ pounds.}$$

Since the factor of safety equals the ratio of the breaking load to the actual load on the column, the factor of safety in this case is

$$\frac{446{,}790}{100{,}000} = 4.5 \text{ (approx.).}$$

2. What is the safe load for a cast-iron column 10 feet long with square ends and a hollow rectangular section, the outside dimensions being 5 × 8 inches; the inner, 4 × 7 inches; and the factor of safety, 6?

In this case l = 10 feet = 120 inches; A = 5 × 8 – 4 × 7 = 12 square inches; and d = 5 inches. Hence, according to formula 10', for rectangular sections, the breaking load is

$$P = \frac{80{,}000 \times 12}{1 + \dfrac{120^2}{1{,}000 \times 5^2}} = 610{,}000 \text{ pounds.}$$

he safe load equals the breaking load divided by the factor y, in this case the safe load equals

$$\frac{610{,}000}{6} = 101{,}700 \text{ pounds.}$$

A channel column (see Fig. 46, b) is pin-ended, the pins rpendicular to the webs of the channel (represented by he figure), and its length is 16 feet (distance between axes etc., represents the formula. Such a curve, besides representing the formula to one's eye, can be used for finding the value of P ÷ A for any value of $l \div r$; or the value of $l \div r$ for any value of P ÷ A. The use herein made is in explaining other column formulas in succeeding articles.

85. Combination Column Formulas. Many columns have been tested to destruction in order to discover in a practical way the laws relating to the strength of columns of different kinds. The results of such tests can be most satisfactorily represented graphically by plotting a point in a diagram for each test. Thus, suppose that a column whose $l \div r$ was 80 failed under a load of 276,000 pounds, and that the area of its cross-section was 7.12 square inches. This test would be represented by laying off O*a*, Fig. 49, equal to 80, according to some scale; and then *ab* equal to 276,000 ÷ 7.12 (P ÷ A), according to some other convenient scale. The point *b* would then represent the result of this particular test. All the dots in the figure represent the way in which the results of a series of tests appear when plotted.

It will be observed at once that the dots do not fall upon any one curve, as the curve of Rankine's formula. Straight lines and curves simpler than the curve of Rankine's formula have been fitted to represent the average positions of the dots as determined by actual tests, and the formulas corresponding to such lines have been deduced as column formulas. These are explained in the following articles.

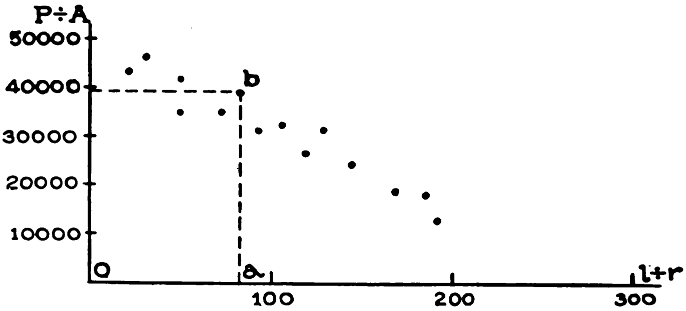


Fig. 49.

86. Straight-Line and Euler Formulas. It occurred to Mr. T. H. Johnson that most of the dots corresponding to ordinary lengths of columns agree with a straight line just as well as with a curve. He therefore, in 1886, made a number of such plats or diagrams as Fig. 49, fitted straight lines to them, and deduced the formula corresponding to each line. These have become known as "straight-line formulas," and their general form is as follows:

$$\frac{P}{A} = S - m\frac{l}{r}, \qquad (11)$$

P, A, l, and r having meanings as in Rankine's formula (Art. 83), and S and m being constants whose values according to Johnson are given in Table E below.

For the slender columns, another formula (Euler's, long since deduced) was used by Johnson. Its general form is—

$$\frac{P}{A} = \frac{n}{(l \div r)^2}, \qquad (12)$$

n being a constant whose values, according to Johnson, are given in the following table:

TABLE E.

Data for Mild-Steel Columns.

	S	m	Limit ($l \div r$)	n
Hinged ends.....	52,500	220	160	444,000,000
Flat ends	52,500	180	195	666,000,000

The numbers in the fourth column of the table mark the point of division between columns of ordinary length and slender columns. For the former kind, the straight line formula applies; and for the second, Euler's. That is, if the ratio $l \div r$ for a steel column with hinged end, for example, is less than 160, we must use the straight line formula to compute its safe load, factor of safety, etc; but if the ratio is greater than 160, we must use Euler's formula.

For *cast iron columns* with flat ends, S = 34,000, and m = 88; and since they should never be used "slender," there is no use of Euler's formula for cast iron columns.

The line AB, Fig. 50, represents Johnson's straight-line formula; and BC, Euler's formula. It will be noticed that the two lines are tangent; the point of tangency corresponds to the "limiting value" $l \div r$, as indicated in the table.

Examples. 1. A 40-pound 10-inch steel I-beam column 8 feet long sustains a load of 100,000 pounds, and the ends are flat. Compute its factor of safety according to the methods of this article.

The first thing to do is to compute the ratio $l \div r$ for the column, to ascertain whether the straight-line formula or Euler's formula should be used. From Table C, on page 70, we find that the moment of inertia of the column about the neutral axis of its cross-section is 9.50 inches⁴, and the area of the section is 11.76 square inches. Hence

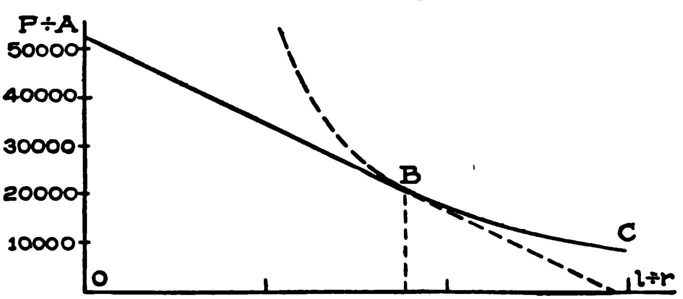


Fig. 50.

$$r^2 = \frac{9.50}{11.76} = 0.81; \text{ or } r = 0.9 \text{ inch.}$$

Since l = 8 feet = 96 inches,

$$\frac{l}{r} = \frac{96}{0.9} = 106\frac{2}{3}$$

This value of $l \div r$ is less than the limiting value (195) indicated by the table for steel columns with flat ends (Table E, p. 97), and we should therefore use the straight-line formula; hence

$$\frac{P}{11.76} = 52{,}500 - 180 \times 106\frac{2}{3};$$

or, $$P = 11.76\,(52{,}500 - 180 \times 106\frac{2}{3}) = 391{,}600 \text{ pounds.}$$

This is the breaking load for the column according to the straight-line formula; hence the factor of safety is

$$\frac{391{,}600}{100{,}000} = 3.9$$

2. Suppose that the length of the column described in the preceding example were 16 feet. What would its factor of safety be?

Since l = 16 feet = 192 inches; and, as before, r = 0.9 inch, $l \div r = 213\frac{1}{3}$. This value is greater than the limiting value (195) indicated by Table E (p. 97) for flat-ended steel columns; hence Euler's formula is to be used. Thus

$$\frac{P}{11.76} = \frac{666{,}000{,}000}{(213\frac{1}{3})^2};$$

or, $$P = \frac{11.76 \times 666{,}000{,}000}{(213\frac{1}{3})^2} = 172{,}100 \text{ pounds.}$$

This is the breaking load; hence the factor of safety is

$$\frac{172{,}100}{100{,}000} = 1.7$$

3. What is the safe load for a cast-iron column 10 feet long with square ends and hollow rectangular section, the outside dimensions being 5×8 inches and the inside 4×7 inches, with a factor of safety of 6?

Substituting in the formula for the radius of gyration given in Table A, page 52, we get

$$r = \sqrt{\frac{8 \times 5^3 - 7 \times 4^3}{12\ (8 \times 5 - 7 \times 4)}} = 1.96 \text{ inches.}$$

Since l = 10 feet = 120 inches,

$$\frac{l}{r} = \frac{120}{1.96} = 61.22$$

According to the straight-line formula for cast iron, A being equal to 12 square inches,

$$\frac{P}{12} = 34{,}000 - 88 \times 61.22;$$

or, $$P = 12\ (34{,}000 - 88 \times 61.22) = 343{,}360 \text{ pounds.}$$

This being the breaking load, the safe load is

$$\frac{343{,}360}{6} = 57{,}227 \text{ pounds.}$$

EXAMPLES FOR PRACTICE.

1. A 40-pound 12-inch steel I-beam 10 feet long is used as a flat-ended column. Its load being 100,000 pounds, compute the factor of safety by the formulas of this article.

Ans. 3.5

2. A cast-iron column 15 feet long sustains a load of 150,000 pounds. Its section being a hollow circle of 9 inches outside and 7 inches inside diameter, compute the factor of safety by the straight-line formula.

Ans. 4.8

3. A steel Z-bar column (see Fig. 46, *a*) is 24 feet long and has square ends; the least radius of gyration of its cross-section is 3.1 inches; and the area of the cross-section is 24.5 square inches. Compute the safe load for the column by the formulas of this article, using a factor of safety of 4.

Ans. 219,000 pounds.

4. A hollow cast-iron column 13 feet long has a circular cross-section, and is 7 inches outside and $5\frac{1}{2}$ inches inside in diameter. Compute its safe load by the formulas of this article, using a factor of safety of 6.

Ans. 68,500 pounds

5. Compute by the methods of this article the safe load for a 40-pound 12-inch steel I-beam used as a column with flat ends, if the length is 17 feet and the factor of safety 5.

Ans. 35,100 pounds.

87. Parabola-Euler Formulas. As better fitting the results of tests of the strength of columns of "ordinary lengths," Prof. J. B. Johnson proposed (1892) to use parabolas instead of straight lines. The general form of the "parabola formula" is

$$\frac{P}{A} = S - m\left(\frac{l}{r}\right)^2, \qquad (13)$$

P, A, l and r having the same meanings as in Rankine's formula, Art. 83; and S and m denoting constants whose values, according to Professor Johnson, are given in Table F below.

Like the straight-line formula, the parabola formula should not be used for slender columns, but the following (Euler's) is applicable:

$$\frac{P}{A} = \frac{n}{(l \div r)^2}, \qquad (14)$$

the values of n (Johnson) being given in the following table:

TABLE F.

Data for Mild Steel Columns.

	S	m	Limit ($l \div r$)	n
Hinged ends......	42,000	0.97	150	456,000,000
Flat ends........	42,000	0.62	190	712,000,000

The point of division between columns of ordinary length and slender columns is given in the fourth column of the table. That is, if the ratio $l \div r$ for a column with hinged ends, for example, is less than 150, the parabola formula should be used to compute the safe load, factor of safety, etc.; but if the ratio is greater than 150, then Euler's formula should be used.

The line AB, Fig. 51, represents the parabola formula; and the line BC, Euler's formula. The two lines are tangent, and the point of tangency corresponds to the "limiting value" $l \div r$ of the table.

For *wooden columns* square in cross-section, it is convenient to replace r by d, the latter denoting the length of the sides of the square. The formula becomes

$$\frac{P}{A} = S - m\left(\frac{l}{d}\right)^2,$$

S and m for flat-ended columns of various kinds of wood having the following values according to Professor Johnson:

For White pine, S=2,500, $m = 0.6$;
" Short leaf yellow pine, S=3,300, $m = 0.7$;
" Long-leaf yellow pine, S=4,000, $m = 0.8$;
" White oak, S=3,500, $m = 0.8$.

The preceding formula applies to any wooden column whose ratio, $l \div d$, is less than 60, within which limit columns of practice are included.

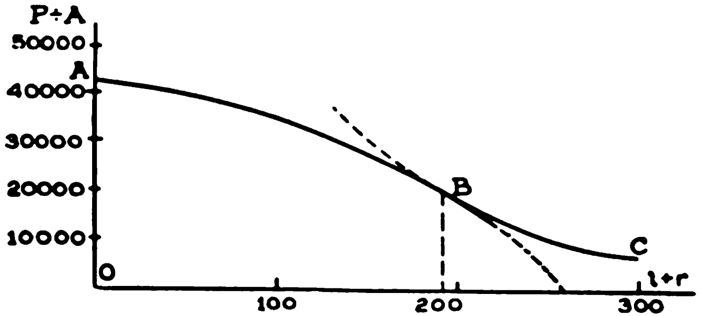


Fig. 51.

Examples. 1. A 40-pound 10-inch steel I-beam column

8 feet long sustains a load of 100,000 pounds, and its ends are flat. Compute its factor of safety according to the methods of this article.

The first thing to do is to compute the ratio $l \div r$ for the column, to ascertain whether the parabola formula or Euler's formula should be used. As shown in example 1 of the preceding article, $l \div r = 106\frac{2}{3}$. This ratio being less than the limiting value, 190, of the table, we should use the parabola formula. Hence, since the area of the cross-section is 11.76 square inches (see Table C, page 70),

$$\frac{P}{11.76} = 42{,}000 - 0.62\ (106\tfrac{2}{3})^2;$$

or, $$P = 11.76\ [42{,}000 - 0.62\ (106\tfrac{2}{3})^2] = 410{,}970 \text{ pounds.}$$

This is the breaking load according to the parabola formula; hence the factor of safety is

$$\frac{410{,}970}{100{,}000} = 4.1$$

2. A white pine column 10 × 10 inches in cross-section and 18 feet long sustains a load of 40,000 pounds. What is its factor of safety?

The length is 18 feet or 216 inches; hence the ratio $l \div d =$ 21.6, and the parabola formula is to be applied.
Now, since $A = 10 \times 10 = 100$ square inches,

$$\frac{P}{100} = 2{,}500 - 0.6 \times 21.6^2;$$

or, $$P = 100\ (2{,}500 - 0.6 \times 21.6^2) = 222{,}000 \text{ pounds.}$$

This being the breaking load according to the parabola formula, the factor of safety is

$$\frac{222{,}000}{40{,}000} = 5.5$$

3. What is the safe load for a long-leaf yellow pine column 12 × 12 inches square and 30 feet long, the factor of safety being 5?

The length being 30 feet or 360 inches,

$$\frac{l}{d} = \frac{360}{12} = 30;$$

hence the parabola formula should be used. Since A = 12 × 12 = 144 square inches,

$$\frac{P}{144} = 4{,}000 - 0.8 \times 30^2;$$

or, $$P = 144\,(4{,}000 - 0.8 \times 30^2) = 472{,}320 \text{ pounds.}$$

This being the breaking load according to the parabola formula, the safe load is

$$\frac{472{,}320}{5} = 94{,}465 \text{ pounds.}$$

EXAMPLES FOR PRACTICE.

1. A 40-pound 12-inch steel I-beam 10 feet long is used as a flat-ended column. Its load being 100,000 pounds, compute its factor of safety by the formulas of this article.

Ans. 3.8

2. A white oak column 15 feet long sustains a load of 30,000 pounds. Its section being 8 × 8 inches, compute the factor of safety by the parabola formula.

Ans. 6.6

3. A steel Z-bar column (see Fig. 46, *a*) is 24 feet long and has square ends; the least radius of gyration of its cross-section is 3.1 inches; and the area of its cross-section is 24.5 square inches. Compute the safe load for the column by the formulas of this article, using a factor of safety of 4.

Ans. 224,500 pounds.

4. A short-leaf yellow pine column 14 × 14 inches in section is 20 feet long. What load can it sustain, with a factor of safety of 6?

Ans. 101,100 pounds.

88. "Broken Straight-Line" Formula. A large steel company computes the strength of its flat-ended steel columns by two formulas represented by two straight lines AB and BC, Fig. 52. The formulas are

$$\frac{P}{A} = 48{,}000,$$

and $$\frac{P}{A} = 68{,}400 - 228\,\frac{l}{r},$$

P, A, *l*, and *r* having the same meanings as in Art. 83.

The point B corresponds very nearly to the ratio $l \div r = 90$. Hence, for columns for which the ratio $l \div r$ is less than 90, the first formula applies; and for columns for which the ratio is greater than 90, the second one applies. The point C corresponds to the ratio $l \div r = 200$, and the second formula does not apply to a column for which $l \div r$ is greater than that limit.

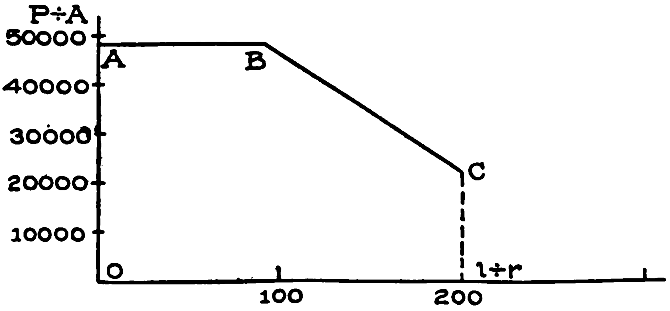


Fig. 52.

The ratio $l \div r$ for steel columns of practice rarely exceeds 150, and is usually less than 100.

Fig. 53 is a combination of Figs. 49, 50, 51 and 52, and represents graphically a comparison of the Rankine, straight-line, Euler, parabola-Euler, and broken straight-line formulas for flat-ended mild-steel columns, It well illustrates the fact that our knowledge of the strength of columns is not so exact as that, for example, of the strength of beams.

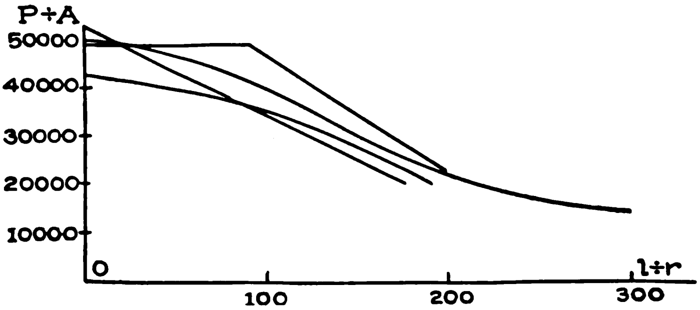


Fig. 53.

89. Design of Columns. All the preceding examples relating to columns were on either (1) computing the factor of safety of a given loaded column, or (2) computing the safe load for a given column. A more important problem is to design a column to sustain a given load under given conditions. A complete discussion of this problem is given in a later paper on design. We show here merely how to compute the *dimensions* of the cross-section of the column after the *form* of the cross-section has been decided upon.

In only a few cases can the dimensions be computed directly (see example 1 following), but usually, when a column formula is applied to a certain case, there will be two unknown quantities in it, A and r or d. Such cases can best be solved by trial (see examples 2 and 3 below).

Example. 1. What is the proper size of white pine column to sustain a load of 80,000 pounds with a factor of safety of 5, when the length of the column is 22 feet?

We use the parabola formula (equation 13). Since the safe load is 80,000 pounds and the factor of safety is 5, the breaking load P is

$$80{,}000 \times 5 = 400{,}000 \text{ pounds.}$$

The unknown side of the (square) cross-section being denoted by d, the area A is d^2. Hence, substituting in the formula, since l = 22 feet = 264 inches, we have

$$\frac{400{,}000}{d^2} = 2{,}500 - 0.6\frac{264^2}{d^2}.$$

Multiplying both sides by d^2 gives

$$400{,}000 = 2{,}500\,d^2 - 0.6 \times 264^2,$$

or

$$2{,}500\,d^2 = 400{,}000 + 0.6 \times 264^2 = 441{,}817.6.$$

Hence

$$d^2 = 176.73, \text{ or } d = 13.3 \text{ inches.}$$

2. What size of cast-iron column is needed to sustain a load of 100,000 pounds with a factor of safety of 10, the length of the column being 14 feet?

We shall suppose that it has been decided to make the cross-section circular, and shall compute by Rankine's formula modified for cast-iron columns (equation 10′). The breaking load for the column would be

$$100{,}000 \times 10 = 1{,}000{,}000 \text{ pounds.}$$

The length is 14 feet or 168 inches; hence the formula becomes

$$\frac{1{,}000{,}000}{\text{A}} = \frac{80{,}000}{1 + \frac{168^2}{800d^2}};$$

or, reducing by dividing both sides of the equation by 10,000, and then clearing of fractions, we have

$$100\left[1 + \frac{168^2}{800d^2}\right] = 8\text{A}.$$

There are two unknown quantities in this equation, d and A, and we cannot solve directly for them. Probably the best way to proceed is to assume or guess at a practical value of d, then solve for A, and finally compute the thickness or inner diameter. Thus, let us try d equal to 7 inches, first solving the equation for A as far as possible. Dividing both sides by 8 we have

$$\text{A} = \frac{100}{8}\left[1 + \frac{168^2}{800d^2}\right],$$

and, combining,

$$\text{A} = 12.5 + \frac{441}{d^2}.$$

Now, substituting 7 for d, we have

$$\text{A} = 12.5 + \frac{441}{49} = 21.5 \text{ square inches.}$$

The area of a hollow circle whose outer and inner diameters are d and d_1 respectively, is $0.7854\ (d^2 - d_1^2)$. Hence, to find the inner diameter of the column, we substitute 7 for d in the last expression, equate it to the value of A just found, and solve for d_1. Thus,

$$0.7854\ (49 - d_1^2) = 21.5\cdot$$

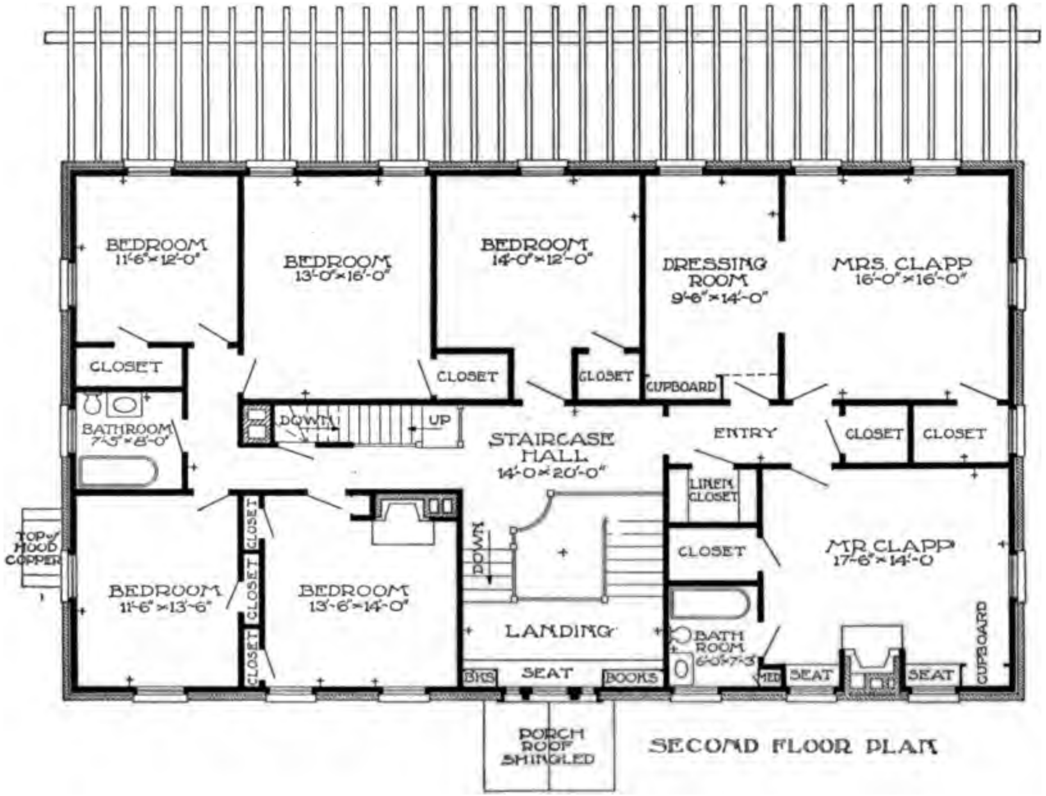
hence

$$49 - d_1^2 = \frac{21.5}{0.7854} = 27.37;$$

and $\quad d_1^2 = 49 - 27.37 = 21.63 \text{ or } d_1 = 4.65.$

This value of d makes the thickness equal to

$$\tfrac{1}{2}\ (7 - 4.65) = 1.175 \text{ inches,}$$



SECOND FLOOR PLANS OF RESIDENCE AT DEADHAM, MASSACHUSETTS
Frank Chouteau Brown, Architect, Boston, Massachusetts
For other views see pages 379 and 394

which is safe. It might be advisable in an actual case to try d equal to 8 repeating the computation.*

EXAMPLE FOR PRACTICE.

1. What size of white oak column is needed to sustain a load of 45,000 pounds with a factor of safety of 6, the length of the column being 12 feet.

Ans. d = 8½, practically a 10 × 10-inch section

STRENGTH OF SHAFTS.

A **shaft** is a part of a machine or system of machines, and is used to transmit power by virtue of its torsional strength, or resistance to twisting. Shafts are almost always made of metal and are usually circular in cross-section, being sometimes made hollow.

90. Twisting Moment. Let AF, Fig. 54, represent a shaft with four pulleys on it. Suppose that D is the driving pulley and that B, C and E are pulleys from which power is taken off to drive machines. The portions of the shafts between the pulleys are twisted when it is transmitting power; and by the twisting moment at any cross-section of the shaft is meant the algebraic sum of the moments of all the forces acting on the shaft on either

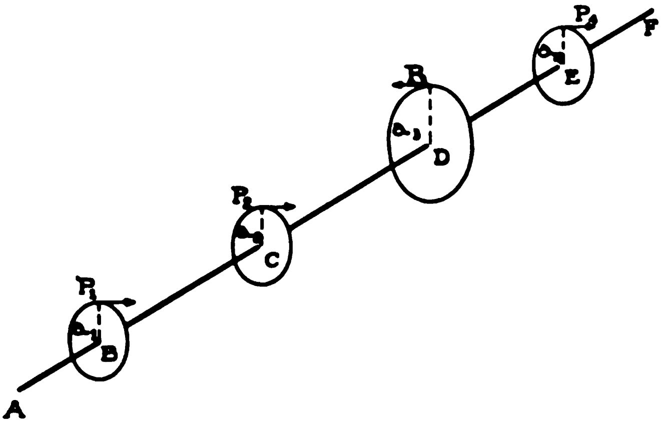


Fig. 54.

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* Note. The structural steel handbooks contain extensive tables by means of which the design of columns of steel or cast iron is much facilitated. The difficulties encountered in the use of formulæ are well illustrated in this example.

side of the section, the moments being taken with respect to the axis of the shaft. Thus, if the forces acting on the shaft (at the pulleys) are P_1, P_2, P_3, and P_4 as shown, and if the arms of the forces or radii of the pulleys are a_1, a_2, a_3, and a_4 respectively, then the twisting moment at any section in

$$\begin{aligned}&\text{BC is } P_1\,a_1,\\ &\text{CD is } P_1\,a_1 + P_2\,a_2,\\ &\text{DE is } P_1\,a_1 + P_2\,a_2 - P_3\,a_3.\end{aligned}$$

Like bending moments, twisting moments are usually expressed in inch-pounds.

Example. Let $a_1 = a_2 = a_4 = 15$ inches, $a_3 = 30$ inches, $P_1 = 400$ pounds, $P_2 = 500$ pounds, $P_3 = 750$ pounds, and $P_4 = 600$ pounds.* What is the value of the greatest twisting moment in the shaft?

At any section between the first and second pulleys, the twisting moment is

$$400 \times 15 = 6{,}000 \text{ inch-pounds;}$$

at any section between the second and third it is

$$400 \times 15 + 500 \times 15 = 13{,}500 \text{ inch-pounds; and}$$

at any section between the third and fourth it is

$$400 \times 15 + 500 \times 15 - 750 \times 30 = -9{,}000 \text{ inch-pounds.}$$

Hence the greatest value is 13,500 inch-pounds.

91. Torsional Stress. The stresses in a twisted shaft are called "torsional" stresses. The torsional stress on a cross-section of a shaft is a shearing stress, as in the case illustrated by Fig. 55, which represents a flange coupling in a shaft. Were it not for the bolts, one flange would slip over the other when either part of the shaft is turned; but the bolts prevent the slipping. Obviously there is a tendency to shear the bolts off unless they are screwed up very tight; that is, the material of the bolts is subjected to shearing stress.

Just so, at any section of the solid shaft there is a tendency for one part to slip past the other, and to prevent the slipping or shearing of the shaft, there arise shearing stresses at all parts of the cross-section. The shearing stress on the cross-section of a shaft is not a uniform stress, its value per unit-area being zero at the center of the section, and increasing toward the circumference. In circular sections, solid or hollow, the shearing stress per unit-area (unit-stress) varies directly as the distance from the center of the section, provided the elastic limit is not exceeded. Thus, if the shearing unit-stress at the circumference of a section is 1,000 pounds per square inch, and the diameter of the shaft is 2 inches, then, at ½ inch from the center, the unit-stress is 500 pounds per square inch; and at ¼ inch from the center it is 250 pounds per square inch. In Fig. 55 the arrows indicate the values and the directions of the shearing stresses on very small portions of the cross-section of a shaft there represented.

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* Note. These numbers were so chosen that the moment of P (driving moment) equals the sum of the moments of the other forces. This is always the case in a shaft rotating at constant speed; that is, the power given the shaft equals the power taken off.

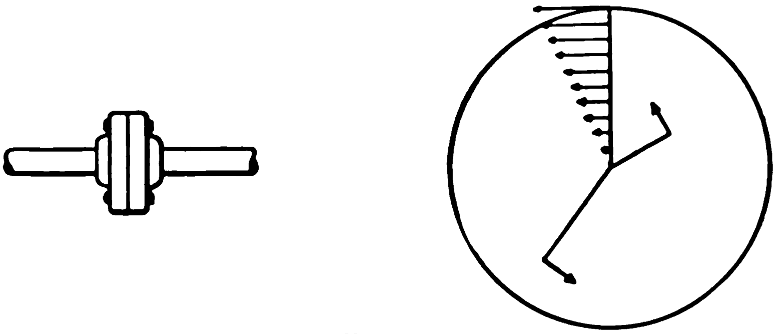

Fig. 55.

92. Resisting Moment. By "resisting moment" at a section of a shaft is meant the sum of the moments of the shearing stresses on the cross-section about the axis of the shaft.

Let S_s denote the value of the shearing stress per unit-area (unit-stress) at the outer points of a section of a shaft; d the diameter of the section (outside diameter if the shaft is hollow); and d_1 the inside diameter. Then it can be shown that the resisting moment is:

For a solid section, $0.1963\ S_s\ d^3$;

For a hollow section, $\dfrac{0.1963\ S_s\ (d^4 - d_1^4)}{d}$.

93. Formula for the Strength of a Shaft. As in the case of beams, the resisting moment equals the twisting moment at any section. If T be used to denote twisting moment, then we have the formulas:

$$\left.\begin{array}{l}\text{For solid circular shafts, } 0.1963\,S_s\,d^3 = T;\\ \text{For hollow circular shafts, } \dfrac{0.1963\,S_s\,(d^4 - d_1^4)}{d} = T.\end{array}\right\} \quad (15)$$

In any portion of a shaft of constant diameter, the unit-shearing stress S_s is greatest where the twisting moment is greatest. Hence, to compute the greatest unit-shearing stress in a shaft, we first determine the value of the greatest twisting moment, substitute its value in the first or second equation above, as the case may be, and solve for S_s. It is customary to express T in inch-pounds and the diameter in inches, S_s then being in pounds per square inch.

Examples. **1.** Compute the value of the greatest shearing unit-stress in the portion of the shaft between the first and second pulleys represented in Fig. 54, assuming values of the forces and pulley radii as given in the example of article 90. Suppose also that the shaft is solid, its diameter being 2 inches.

The twisting moment T at any section of the portion between the first and second pulleys is 6,000 inch-pounds, as shown in the example referred to. Hence, substituting in the first of the two formulas 15 above, we have

$$0.1963\,S_s \times 2^3 = 6{,}000;$$

or, $$S_s = \frac{6{,}000}{0.1963 \times 8} = 3{,}820 \text{ pounds per square inch.}$$

This is the value of the unit-stress at the outside portions of all sections between the first and second pulleys.

2. A hollow shaft is circular in cross-section, and its outer and inner diameters are 16 and 8 inches respectively. If the working strength of the material in shear is 10,000 pounds per square inch, what twisting moment can the shaft safely sustain?

The problem requires that we merely substitute the values of S_s, d, and d_1 in the second of the above formulas 15, and solve for T. Thus,

$$T = \frac{0.1963 \times 10{,}000\,(16^4 - 8^4)}{16} = 7{,}537{,}920 \text{ inch-pounds.}$$

EXAMPLES FOR PRACTICE.

1. Compute the greatest value of the shearing unit-stress in the shaft represented in Fig. 54, using the values of the forces and pulley radii given in the example of article 90, the diameter of the shaft being 2 inches.

Ans. 8,595 pounds per square inch

2. A solid shaft is circular in cross-section and is 9.6 inches in diameter. If the working strength of the material in shear is 10,000 pounds per square inch, how large a twisting moment can the shaft safely sustain? (The area of the cross-section is practically the same as that of the hollow shaft of example 2 preceding.)

Ans. 1,736,736 inch-pounds.

94. Formula for the Power Which a Shaft Can Transmit. The power that a shaft can safely transmit depends on the shearing working strength of the material of the shaft, on the size of the cross-section, and on the speed at which the shaft rotates.

Let H denote the amount of horse-power; S_s the shearing working strength in pounds per square inch; d the diameter (outside diameter if the shaft is hollow) in inches; d_1 the inside diameter in inches if the shaft is hollow; and n the number of revolutions of the shaft per minute. Then the relation between power transmitted, unit-stress, etc., is:

$$\left.\begin{aligned}\text{For solid shafts, } H &= \frac{S_s d^3 n}{321{,}000};\\ \text{For hollow shafts, } H &= \frac{S_s (d^4 - d_1^4) n}{321{,}000\, d}.\end{aligned}\right\} \quad (16)$$

Examples. 1. What horse-power can a hollow shaft 16 inches and 8 inches in diameter safely transmit at 50 revolutions per minute, if the shearing working strength of the material is 10,000 pounds per square inch?

We have merely to substitute in the second of the two formulas 16 above, and reduce. Thus,

$$H = \frac{10{,}000\,(16^4 - 8^4)\,50}{321{,}000 \times 16} = 6{,}000 \text{ horse-power (nearly).}$$

2. What size of solid shaft is needed to transmit 6,000 horse-power at 50 revolutions per minute if the shearing working strength of the material is 10,000 pounds per square inch?

We have merely to substitute in the first of the two formulas 16, and solve for *d*. Thus,

$$6{,}000 = \frac{10{,}000 \times d^3 \times 50}{321{,}000};$$

therefore $$d^3 = \frac{6{,}000 \times 321{,}000}{10{,}000 \times 50} = 3{,}852;$$

or, $$d = \sqrt[3]{3{,}852} = 15.68 \text{ inches.}$$

(A solid shaft of this diameter contains over 25% more material than the hollow shaft of example 1 preceding. There is therefore considerable saving of material in the hollow shaft.)

3. A solid shaft 4 inches in diameter transmits 200 horse-power while rotating at 200 revolutions per minute. What is the greatest shearing unit-stress in the shaft?

We have merely to substitute in the first of the equations 16, and solve for S_s. Thus,

$$200 = \frac{S_s \times 4^3 \times 200}{321{,}000};$$

or, $$S = \frac{200 \times 321{,}000}{4^3 \times 200} = 5{,}016 \text{ pounds per square inch.}$$

EXAMPLES FOR PRACTICE.

1. What horse-power can a solid shaft 9.6 inches in diameter safely transmit at 50 revolutions per minute, if its shearing working strength is 10,000 pounds per square inch?

Ans. 1,378 horse-power.

2. What size of solid shaft is required to transmit 500 horse-power at 150 revolutions per minute, the shearing working strength of the material being 8,000 pounds per square inch.

Ans. 5.1 inches.

3. A hollow shaft whose outer diameter is 14 and inner 6.7 inches transmits 5,000 horse-power at 60 revolutions per minute. What is the value of the greatest shearing unit-stress in the shaft?

Ans. 10,273 pounds per square inch.

STIFFNESS OF RODS, BEAMS, AND SHAFTS.

The preceding discussions have related to the *strength* of materials. We shall now consider principally the *elongation of rods*, *deflection of beams*, and *twist of shafts*.

95. Coefficient of Elasticity. According to Hooke's Law (Art. 9, p. 7), the elongations of a rod subjected to an increasing pull are proportional to the pull, provided that the stresses due to the pull do not exceed the elastic limit of the material. Within the elastic limit, then, the ratio of the pull and the elongation is constant; hence the ratio of the unit-stress (due to the pull) to the unit-elongation is also constant. This last-named ratio is called "coefficient of elasticity." If E denotes this coefficient, S the unit-stress, and s the unit-deformation, then

$$E = \frac{S}{s}. \qquad (17)$$

Coefficients of elasticity are usually expressed in pounds per square inch.

The preceding remarks, definition, and formula apply also to a case of compression, provided that the material being compressed does not bend, but simply shortens in the direction of the compressing forces. The following table gives the *average* values of the coefficient of elasticity for various materials of construction:

TABLE G.

Coefficients of Elasticity.

Material.	Average Coefficient of Elasticity.
Steel	30,000,000 pounds per square inch.
Wrought iron.....	27,500,000 " " " "
Cast iron.........	15,000,000 " " " "
Timber..........	1,800,000 " " " "

The coefficients of elasticity for steel and wrought iron, for different grades of those materials, are remarkably constant; but for different grades of cast iron the coefficients range from about 10,000,000 to 30,000,000 pounds per square inch. Naturally the coefficient has not the same value for the different kinds of wood; for the principal woods it ranges from 1,600,000 (for spruce) to 2,100,000 (for white oak).

Formula 17 can be put in a form more convenient for use, as follows:

Let P denote the force producing the deformation; A the area of the cross-section of the piece on which P acts; l the length of the piece; and D the deformation (elongation or shortening).

Then

$$S = P \div A \text{ (see equation 1),}$$

and

$$s = D \div l \text{ (see equation 2).}$$

Hence, substituting these values in equation 17, we have

$$E = \frac{Pl}{AD}; \text{ or } D = \frac{Pl}{AE}. \qquad (17')$$

The first of these two equations is used for computing the value of the coefficient of elasticity from measurements of a "test," and the second for computing the elongation or shortening of a given rod or bar for which the coefficient is known.

Examples. 1. It is required to compute the coefficient of elasticity of the material the record of a test of which is given on page 9.

Since the unit-stress S and unit-elongation s are already computed in that table, we can use equation 17 instead of the first of equations 17′. The elastic limit being between 40,000 and 45,000 pounds per square inch, we may use any value of the unit-stress less than that, and the corresponding unit-elongation.

Thus, with the first values given,

$$E = \frac{5{,}000}{0.00017} = 29{,}400{,}000.$$

With the second,

$$E = \frac{10{,}000}{0.00035} = 28{,}600{,}000.$$

This lack of constancy in the value of E as computed from different loads in a test of a given material, is in part due to errors in measuring the deformation, a measurement difficult to make. The value of the coefficient adopted from such a test, is the average of all the values of E which can be computed from the record.

2. How much will a pull of 5,000 pounds stretch a round steel rod 10 feet long and 1 inch in diameter?

We use the second of the two formulas 17′. Since $A = 0.7854 \times 1^2 = 0.7854$ square inches, $l = 120$ inches, and $E = 30{,}000{,}000$ pounds per square inch, the stretch is:

$$D = \frac{5{,}000 \times 120}{0.7854 \times 30{,}000{,}000} = 0.0254 \text{ inch.}$$

EXAMPLES FOR PRACTICE.

1. What is the coefficient of elasticity of a material if a pull of 20,000 pounds will stretch a rod 1 inch in diameter and 4 feet long 0.045 inch?

Ans. 27,000,000 pounds per square inch.

2. How much will a pull of 15,000 pounds elongate a round cast-iron rod 10 feet long and 1 inch in diameter?

Ans. 0.152 inch.

96. Temperature Stresses. In the case of most materials, when a bar or rod is heated, it lengthens; and when cooled, it shortens if it is free to do so. The **coefficient of linear expansion** of a material is the ratio which the elongation caused in a rod or bar of the material by a change of one degree in temperature bears to the length of the rod or bar. Its values for Fahrenheit degrees are about as follows:

For Steel,	0.0000065.
For Wrought iron,	.0000067.
For Cast iron,	.0000062.

Let K be used to denote this coefficient; t a change of temperature, in degrees Fahrenheit; l the length of a rod or bar; and D the change in length due to the change of temperature. Then

$$D = K t l. \qquad (18)$$

D and l are expressed in the same unit.

If a rod or bar is confined or restrained so that it cannot change its length when it is heated or cooled, then any change in its temperature produces a stress in the rod; such are called **temperature stresses.**

Examples. 1. A steel rod connects two solid walls and is screwed up so that the unit-stress in it is 10,000 pounds per square inch. Its temperature falls 10 degrees, and it is observed that the walls have not been drawn together. What is the temperature stress produced by the change of temperature, and what is the actual unit-stress in the rod at the new temperature?

Let l denote the length of the rod. Then the change in length which would occur if the rod were free, is given by formula 18, above, thus:

$$D = 0.0000065 \cdot 10 \cdot l = 0.000065\,l.$$

Now, since the rod could not shorten, it has a greater than normal length at the new temperature; that is, the fall in temperature has produced an effect equivalent to an elongation in the rod amounting to D, and hence a tensile stress. This tensile stress can be computed from the elongation D by means of formula 17. Thus,

$$S = E\,s;$$

and since s, the unit-elongation, equals

$$\frac{D}{l} = \frac{.0000065\cdot l}{l} = .0000065,$$

$S = 30{,}000{,}000 \times .0000065 = 195.0$ pounds per square inch. This is the value of the temperature stress; and the new unit-stress equals

$$10{,}000 + 195.0 = 10{,}195 \text{ pounds per square inch.}$$

Notice that the unit temperature stresses are independent of the length of the rod and the area of its cross-section.

2. Suppose that the change of temperature in the preceding example is a rise instead of a fall. What are the values of the temperature stress due to the change, and of the new unit-stress in the rod?

The temperature stress is the same as in example 1, that is, 1,950 pounds per square inch; but the rise in temperature releases, as it were, the stress in the rod due to its being screwed up, and the final unit stress is

$$10{,}000 - 1{,}950 = 8{,}050 \text{ pounds per square inch.}$$

EXAMPLE FOR PRACTICE.

1. The ends of a wrought-iron rod 1 inch in diameter are fastened to two heavy bodies which are to be drawn together, the temperature of the rod being 200 degrees when fastened to the objects. A fall of 120 degrees is observed not to move them. What is the temperature stress, and what is the pull exerted by the rod on each object?

Ans. { Temperature stress, 22,000 pounds per square inch.
Pull, 17,280 pounds.

97. Deflection of Beams. Sometimes it is desirable to know how much a given beam will deflect under a given load, or to design a beam which will not deflect more than a certain amount under a given load. In Table B, page 53, Part I, are given formulas for deflection in certain cases of beams and different kinds of loading.

In those formulas, d denotes deflection; I the moment of inertia of the cross-section of the beam with respect to the neutral axis, as in equation 6; and E the coefficient of elasticity of the material of the beam (for values, see Art. 95).

In each case, the load should be expressed in pounds, the length in inches, and the moment of inertia in biquadratic inches; then the deflection will be in inches.

According to the formulas for d, the deflection of a beam varies inversely as the coefficient of its material (E) and the moment of inertia of its cross-section (I); also, in the first four and last two cases of the table, the deflection varies directly as the cube of the length (l^3).

Example. What deflection is caused by a uniform load of 6,400 pounds (including weight of the beam) in a wooden beam on end supports, which is 12 feet long and 6 × 12 inches in cross-section? (This is the safe load for the beam; see example 1, Art. 65.)

The formula for this case (see Table B, page 53) is

$$d = \frac{5\ \mathrm{W}l^3}{384\ \mathrm{EI}}.$$

Here $\mathrm{W} = 6{,}400$ pounds; $l = 144$ inches; $\mathrm{E} = 1{,}800{,}000$ pounds per square inch; and

$$\mathrm{I} = \frac{1}{12}bd^3 = \frac{1}{12}6 \times 12^3 = 864 \text{ inches}^4.$$

Hence the deflection is

$$d = \frac{5 \times 6{,}400 \times 144^3}{384 \times 1{,}800{,}000 \times 864} = 0.16 \text{ inch}.$$

EXAMPLES FOR PRACTICE.

1. Compute the deflection of a timber built-in cantilever 8 × 8 inches which projects 8 feet from the wall and bears an end load of 900 pounds. (This is the safe load for the cantilever, see example 1, Art. 65.)

Ans. 0.43 inch.

2. Compute the deflection caused by a uniform load of 40,000

pounds on a 42-pound 15-inch steel I-beam which is 16 feet long and rests on end supports.

Ans. 0.28 inch.

98. Twist of Shafts. Let Fig. 57 represent a portion of a shaft, and suppose that the part represented lies wholly between two adjacent pulleys on a shaft to which twisting forces are applied (see Fig. 54). Imagine two radii *ma* and *nb* in the ends of the portion, they being parallel as shown when the shaft is not twisted. After the shaft is twisted they will not be parallel, *ma* having moved to *ma'*, and *nb* to *nb'*. The angle between the two lines in their twisted positions (*ma'* and *nb'*) is called the **angle of twist, or angle of torsion,** for the length *l*. If *a'a''* is parallel to *ab*, then the angle *a''nb'* equals the angle of torsion.

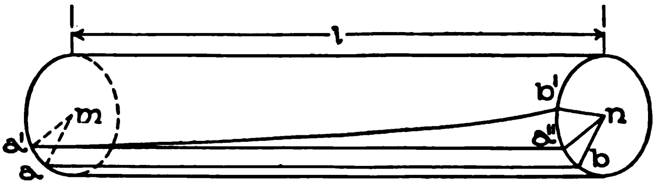

Fig. 57.

If the stresses in the portion of the shaft considered do not exceed the elastic limit, and if the twisting moment is the same for all sections of the portion, then the angle of torsion *a* (in degrees) can be computed from the following:

$$\left.\begin{aligned} &\text{For solid circular shafts,}\\ &a=\frac{584\,\mathrm{T}l}{\mathrm{E}^1\,d^4}=\frac{36{,}800{,}000\,\mathrm{H}l}{\mathrm{E}^1\,d^4 n}.\\ &\text{For hollow circular shafts,}\\ &a=\frac{584\,\mathrm{T}ld}{\mathrm{E}^1(d^4-d_1^4)}=\frac{36{,}800{,}000\,\mathrm{H}l}{\mathrm{E}^1\,(d^4-d_1^4)\,n}. \end{aligned}\right\}\quad(19)$$

Here T, *l*, *d*, d_1, H, and *n* have the same meanings as in Arts. 93 and 94, and should be expressed in the units there used. The letter E^1 stands for a quantity called **coefficient of elasticity for shear;** it is analogous to the coefficient of elasticity for tension and compression (E), Art. 95. The values of E^1 for a few materials average about as follows (roughly $E^1 = \frac{2}{5}E$):

For Steel, 11,000,000 pounds per square inch.
For Wrought iron, 10,000,000 " " " "
For Cast iron, 6,000,000 " " " "

Example. What is the value of the angle of torsion of a steel shaft 60 feet long when transmitting 6,000 horse-power at 50 revolutions per minute, if the shaft is hollow and its outer and inner diameters are 16 and 8 inches respectively?

Here $l = 720$ inches; hence, substituting in the appropriate formula (19), we find that

$$a = \frac{36,800,000 \times 6,000 \times 720}{11,000,000 \times (16^4 - 8^4)\,50} = 4.7 \text{ degrees.}$$

EXAMPLE FOR PRACTICE.

Suppose that the first two pulleys in Fig. 54 are 12 feet apart; that the diameter of the shaft is 2 inches; and that $P_1 = 400$ pounds, and $a_1 = 15$ inches. If the shaft is of wrought iron, what is the value of the angle of torsion for the portion between the first two pulleys?

Ans. 3.15 degrees.

99. Non-elastic Deformation. The preceding formulas for elongation, deflection, and twist hold only so long as the greatest unit-stress does not exceed the elastic limit. There is no theory, and no formula, for non-elastic deformations, those corresponding to stresses which exceed the elastic limit. It is well known, however, that non-elastic deformations are not proportional to the forces producing them, but increase much faster than the loads. The value of the ultimate elongation of a rod or bar (that is, the amount of elongation at rupture), is quite well known for many materials. This elongation, for eight-inch specimens of various materials (see Art. 16), is:

For Cast iron, about 1 per cent.
For Wrought iron (plates), 12 - 15 per cent.
For " " (bars), 20 - 25 " " .
For Structural steel, 22 - 28 " " .

Specimens of ductile materials (such as wrought iron and structural steel), when pulled to destruction, **neck down**, that is, diminish very considerably in cross-section at some place along the length of the specimen. The decrease in cross-sectional area is known as **reduction of area**, and its value for wrought iron and steel may be as much as 50 per cent.

RIVETED JOINTS.

100. Kinds of Joints. A **lap joint** is one in which the plates or bars joined overlap each other, as in Fig. 58, *a*. A **butt joint** is one in which the plates or bars that are joined butt against each other, as in Fig. 58, *b*. The thin side plates on butt joints are called **cover-plates**; the thickness of each is always made not less than one-half the thickness of the **main plates**, that is, the plates or bars that are joined. Sometimes butt joints are made with only one cover-plate; in such a case the thickness of the cover-plate is made not less than that of the main plate.

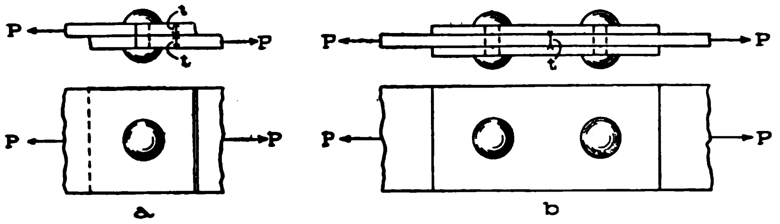

Fig. 58.

When wide bars or plates are riveted together, the rivets are placed in rows, always parallel to the "seam" and sometimes also perpendicular to the seam; but when we speak of a row of rivets, we mean a row parallel to the seam. A lap joint with a single row of rivets is said to be **single-riveted**; and one with two rows of rivets is said to be **double-riveted.** A butt joint with two rows of rivets (one on each side of the joint) is called "single-riveted," and one with four rows (two on each side) is said to be "double-riveted."

The distance between the centers of consecutive holes in a row of rivets is called **pitch.**

101. Shearing Strength, or Shearing Value, of a Rivet. When a lap joint is subjected to tension (that is, when P, Fig. 58, *a*, is a pull), and when the joint is subjected to compression (when P is a push), there is a tendency to cut or shear each rivet along the surface between the two plates. In butt joints with two cover-plates, there is a tendency to cut or shear each rivet on two surfaces (see Fig. 58, *b*). Therefore the rivets in the lap joint are said to be in **single shear**; and those in the butt joint (two covers) are said to be in **double shear.**

The "shearing value" of a rivet means the resistance which it can safely offer to forces tending to shear it on its cross-section. This value depends on the area of the cross-section and on the working strength of the material. Let d denote the diameter of the cross-section, and S_s the shearing working strength. Then, since the area of the cross-section equals $0.7854\ d^2$, the shearing strength of one rivet is:

For single shear, $0.7854\ d^2\ S_s$.
For double shear, $1.5708\ d^2\ S_s$.

102. Bearing Strength, or Bearing Value, of a Plate. When a joint is subjected to tension or compression, each rivet presses against a part of the sides of the holes through which it passes. By "bearing value" of a plate (in this connection) is meant the pressure, exerted by a rivet against the side of a hole in the plate, which the plate can safely stand. This value depends on the thickness of the plate, on the diameter of the rivet, and on the compressive working strength of the plate. Exactly how it depends on these three qualities is not known; but the bearing value is always computed from the expression $t\ d\ S_c$, wherein t denotes the thickness of the plate; d, the diameter of the rivet or hole; and S_c, the working strength of the plate.

103. Frictional Strength of a Joint. When a joint is subjected to tension or compression, there is a tendency to slippage between the faces of the plates of the joint. This tendency is overcome wholly or in part by frictional resistance between the plates. The frictional resistance in a well-made joint may be very large, for rivets are put into a joint hot, and are **headed** or **capped** before being cooled. In cooling they contract, drawing the plates of the joint tightly against each other, and producing a great pressure between them, which gives the joint a correspondingly large frictional strength. It is the opinion of some that all well-made joints perform their service by means of their frictional strength; that is to say, the rivets act only by pressing the plates together and are not under shearing stress, nor are the plates under compression at the sides of their holes. The "frictional strength" of a joint, however, is usually regarded as uncertain, and generally no allowance is made for friction in computations on the strength of riveted joints.

104. Tensile and Compressive Strength of Riveted Plates. The holes punched or drilled in a plate or bar weaken its tensile strength, and to compute that strength it is necessary to allow for the holes. By **net section**, in this connection, is meant the smallest cross-section of the plate or bar; this is always a section along a line of rivet holes.

If, as in the foregoing article, t denotes the thickness of the plates joined; d, the diameter of the holes; n_1, the number of rivets in a row; and w, the width of the plate or bar; then the net section $= (w - n_1 d)\ t$.

Let S_t denote the tensile working strength of the plate; then the strength of the unriveted plate is wtS_t, and the reduced tensile strength is $(w - n_1 d)\ t\ S_t$.

The compressive strength of a plate is also lessened by the presence of holes; but when they are again filled up, as in a joint, the metal is replaced, as it were, and the compressive strength of the plate is restored. No allowance is therefore made for holes in figuring the compressive strength of a plate.

105. Computation of the Strength of a Joint. The strength of a joint is determined by either (1) the shearing value of the rivets; (2) the bearing value of the plate; or (3) the tensile strength of the riveted plate if the joint is in tension. Let P_s denote the strength of the joint as computed from the shearing values of the rivets; P_c, that computed from the bearing value of the plates; and P_t, the tensile strength of the riveted plates. Then, as before explained,

$$\left.\begin{array}{l} P_t = (w - n_1 d)\ tS_t; \\ P_s = n_2\ 0.7854\ d^2 S_s;\ \text{and} \\ P_c = n_3 t d S_c; \end{array}\right\} \qquad (20)$$

n_2 denoting the total number of rivets in the joint; and n_3 denoting the total number of rivets in a lap joint, and one-half the number of rivets in a butt joint.

Examples. 1. Two half-inch plates 7½ inches wide are connected by a single lap joint double-riveted, six rivets in two rows. If the diameter of the rivets is $\frac{3}{4}$ inch, and the working strengths are as follows: $S_t = 12,000$, $S_s = 7,500$, and $S_c = 15,000$ pounds per square inch, what is the safe tension which the joint can transmit?

Here $n_1 = 3$, $n_2 = 6$, and $n_3 = 6$; hence

$$P_t = (7\frac{1}{2} - 3 \times \frac{3}{4}) \times \frac{1}{2} \times 12,000 = 31,500 \text{ pounds};$$

$$P_s = 6 \times 0.7854 \times (\frac{3}{4})^2 \times 7,500 = 19,880 \text{ pounds};$$

$$P_c = 6 \times \frac{1}{2} \times \frac{3}{4} \times 15,000 = 33,750 \text{ pounds}.$$

Since P_s is the least of these three values, the strength of the joint depends on the shearing value of its rivets, and it equals 19,880 pounds.

2. Suppose that the plates described in the preceding example are joined by means of a butt joint (two cover-plates), and 12 rivets are used, being spaced as before. What is the safe tension which the joint can bear?

Here $n_1 = 3$, $n_2 = 12$, and $n_3 = 6$; hence, as in the preceding example,

$$P_t = 31,500; \text{ and } P_c = 33,750 \text{ pounds; but}$$

$$P_s = 12 \times 0.7854 \times (\frac{3}{4})^2 \times 7,500 = 39,760 \text{ pounds}$$

The strength equals 31,500 pounds, and the joint is stronger than the first.

3. Suppose that in the preceding example the rivets are arranged in rows of two. What is the tensile strength of the joint?

Here $n_1 = 2$, $n_2 = 12$, and $n_3 = 6$; hence, as in the preceding example,

$$P_s = 39,760; \text{ and } P_c = 33,750 \text{ pounds; but}$$

$$P_t = (7\frac{1}{2} - 2 \times \frac{3}{4}) \times \frac{1}{2} \times 12,000 = 36,000 \text{ pounds}.$$

The strength equals 33,750 pounds, and this joint is stronger than either of the first two.

EXAMPLES FOR PRACTICE.

Note. Use working strengths as in example 1, above.

$S_t = 12{,}000$, $S_s = 7{,}500$, and $S_c = 15{,}000$ pounds per square inch.

1. Two half-inch plates 5 inches wide are connected by a lap joint, with two ¾-inch rivets in a row. What is the safe strength of the joint?

Ans. 6,625 pounds.

2. Solve the preceding example supposing that four ¾-inch rivets are used, in two rows.

Ans. 13,250 pounds.

3. Solve example 1 supposing that three 1-inch rivets are used, placed in a row lengthwise of the joint.

Ans. 17,670 pounds.

4. Two half-inch plates 5 inches wide are connected by a butt joint (two cover-plates), and four ¾-inch rivets are used, in two rows. What is the strength of the joint?

Ans. 11,250 pounds.

106. Efficiency of a Joint. The ratio of the strength of a joint to that of the solid plate is called the "efficiency of the joint." If ultimate strengths are used in computing the ratio, then the efficiency is called **ultimate efficiency;** and if working strengths are used, then it is called **working efficiency.** In the following, we refer to the latter. An efficiency is sometimes expressed as a per cent. To express it thus, multiply the ratio *strength of joint ÷ strength of solid plate*, by 100.

Example. It is required to compute the efficiencies of the joints described in the examples worked out in the preceding article.

In each case the plate is ½ inch thick and 7½ inches wide; hence the tensile working strength of the solid plate is

$$7\frac{1}{2} \times \frac{1}{2} \times 12{,}000 = 45{,}000 \text{ pounds.}$$

Therefore the efficiencies of the joints are:

$$(1) \quad \frac{19{,}880}{45{,}000} = 0.44, \text{ or } 44 \text{ per cent;}$$

$$(2) \quad \frac{31{,}500}{45{,}000} = 0.70, \text{ or } 70 \text{ per cent;}$$

$$(3) \quad \frac{33{,}750}{45{,}000} = 0.75, \text{ or } 75 \text{ per cent.}$$

REVIEW QUESTIONS

REVIEW QUESTIONS

ON THE SUBJECT OF

STRUCTURAL DRAFTING

PART I

---

1. Define *engineer of plant, chief draftsman, squad boss,* and *tracers,* and give their duties.

2. How is the work handled in a drafting room?

3. Describe the "laying out" of a plate.

4. Give briefly the methods of ordering of material.

5. Show method of making allowances for angle bends.

6. What are shop bills?

7. Make a straight line alphabet and show by arrows how the several strokes should be made.

8. What abbreviations are used in making drawings? Give them.

9. Show correct and incorrect methods of making arrow heads.

10. Show correct and incorrect methods of placing dimensions.

11. How are field rivets detailed?

12. Give an example of material notation for angles, plates, and I-beams.

13. Give the conventional rivet signs.

14. What is the minimum spacing for ¾- and ⅞-inch rivets?

15. What are gauge lines?

16. What are "crimped" angles, and what is the rule regarding their rivet spacing?

17. Define "rivet grip".

18. Give several cases of "rivet clearance".

19. Tell how to determine the value of a rivet in tension.

REVIEW QUESTIONS

ON THE SUBJECT OF

STRUCTURAL DRAFTING

PART II

1. Give the maximum and minimum spacings on one gauge line, rivets $\frac{7}{8}$ inch.
2. Show how to detail a clip angle.
3. What is a "working point"?
4. Show how to detail a lateral connection plate.
5. How are bevels indicated?
6. Describe a beam sheet. What are its advantages?
7. Define the term "cope".
8. Make sketches showing a 12-inch beam coped to a 9-inch beam flush at bottoms.
9. What are standard connections?
10. Make a sketch of the detail of the joint angle, of a roof truss.
11. What information should be on the stress sheet of a plate girder?
12. Show with sketches the correct manner of cutting the ends of a diagonal member.
13. How many sheets are required to detail a plate girder properly, and what are they?
14. Make a sketch showing the cross-section of a plate girder.
15. Give a brief description of the detailing of stiffeners.
16. Make a detail of a web splice.
17. How close may rivets be placed to stiffener angles?
18. Make sketch of a cast-steel bearing.
19. What are lattice bars? How are they detailed?

REVIEW QUESTIONS

ON THE SUBJECT OF

STATICS

1. Define concurrent and non-concurrent forces, equilibrant and resultant.

2. What do you understand by the "Triangle law"?

3. What is the total snow load for the truss if the snow weighs 20 pounds per square foot (horizontal)?

4. Compute the total wind load for the truss of Question 9, when the wind blows 75 miles per hour.

5. Make a complete record of the stresses as determined in answer to the preceding question for cases *a*, *b*, and *c*, and for snow load as computed in answer to Question 10. Compute from the record the value of the greatest stress which can come upon each member due to combinations of loads, assuming that the wind and snow loads will not act at the same time.

6. Find the magnitude of the resultant of two forces making an angle of 70° with each other, one being 30 pounds and the other 45 pounds.

7. The lines of action of two forces of 65 and 35 pounds, respectively, make an angle of 120°. Find the magnitude of the force that holds them in equilibrium, and the angles that it makes with each given force.

8. Draw a force polygon for five forces in equilibrium and prove that any diagonal of the polygon is the resultant of the forces on one side and holds in equilibrium those on the other.

REVIEW QUESTIONS

ON THE SUBJECT OF

ROOF TRUSSES

1. Name and describe the three classes of roof trusses, and give a sketch of one truss of each class.

2. Give a sketch of the Fink truss and the modified Fink truss.

3. Given $W = \frac{3}{4}\ al\ (1 + \frac{l}{10})$, compute the dead panel load of an eight-panel Fink truss of 80-foot span, if the distances between trusses is 20 feet and the roof covering is composed of corrugated steel.

4. Tell how a felt and asphalt roof is made and laid.

5. What is a non-condensation roofing?

6. Design the purlins if they are to be spaced 6 feet apart and the trusses are to be spaced 16 feet apart. They are to carry 40 pounds per square foot of roof surface.

7. Write one page on the economical pitch and spacing of roof trusses.

8. If the trusses are of 80-foot span and are spaced 20 feet center to center, and are eight-panel Fink, compute the stresses in the knee-braces if they join the columns 8 feet from the top. The columns are 25 feet long; the normal wind pressure on the roof is 12 pounds; the pitch of the roof is $\frac{1}{4}$; the normal wind pressure on the side of the building is 25 pounds; and the columns are considered free.

9. In the trusses of Question 8, above, compute the bending moment in the posts.

10. If in Question 8, above, the girts are placed 4 feet apart, design them.

11. If a post is 25 feet long, has a total direct stress of 20 000 pounds which acts at the center of the column, and a crane load of 10 000 pounds which acts 12 inches from the face of the column, design the section of the column if it consists of four angles and a plate.

REVIEW QUESTIONS

ON THE SUBJECT OF

STRENGTH OF MATERIALS

PART I

1. When a ¾-inch round rod sustains a pull of 10,000 pounds, what is the value of the unit-tensile stress in the rod?

2. What do you understand by Hooke's Law?

3. What are the dimensions of a square white pine post, needed to support a steady load of 6,500 pounds with a factor of safety of 8?

4. How large a force is required to punch a 1-inch hole through a ¾-inch plate of wrought iron, if the ultimate shearing strength of the material is 40,000 pounds per square inch?

5. Compare the ultimate strengths of wood along and across the grain; also the ultimate tensile and compressive strengths of cast iron.

6. Make a sketch of a beam 20 feet long resting on end supports, and represent loads of 6,000, 3,000, 1,000, and 4,000 pounds at points 2, 5, 11, and 16 feet from the left end, respectively. What is the value and sign of the moment of each of these loads about the middle of the beam? Also about the left end?

7. A beam 15 feet long is supported at two points, 2 feet from the right end, and 3 feet from the left end. If the beam sustains a uniform load of 400 pounds per foot, what are the values of the reactions?

8. Compute the values of the external shear and bending moment for the loaded beam described in Question 6, at sections 1, 4, 10, and 15 feet from the left end.

9. Draw shear and moment diagrams to scale for the beam described in Question 7.

10. Suppose a T-bar 2 inches deep, has a flange 3 inches wide and is ½ inch thick throughout. Locate the center of gravity by computation.

REVIEW QUESTIONS

ON THE SUBJECT OF

STRENGTH OF MATERIALS

PART II

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1. A cantilever beam 6 feet in length projects from a wall and sustains an end load of 300 pounds. The cross-section being as in Fig. 38, find the greatest tensile and compressive unit-stresses and state where they occur.

2. An I-beam weighing 30 pounds per foot, rests on end supports 25 feet apart. Its section modulus is 20.4 inches, and its working strength 16,000 pounds per square inch. Calculate weight of the beam.

3. A wooden beam 15 feet long, 4×14 inches in cross-section sustains a load of 4,000 pounds 5 feet from one end, and 2,000 pounds at the middle. Compute the greatest unit shearing stress.

4. What do you know about radius of gyration? Give an example.

5. Find the factor of safety of a 24-inch 80-pound steel I-beam 15 feet long, used as a flat-ended column to sustain a load of 150,000 pounds. Note—Use "Rankine's Formula".

6. A steel Z-bar is 20 feet long and has square ends; the least radius of gyration of its cross-section is 3.1 inches, and its area of cross-section is 24.5 square inches. Calculate the safe load with a factor of safety of 6. Note—Use "Rankine's Formula".

7. Make sketches of the following:

   Lap joint single-riveted;
   Lap joint double-riveted;
   Butt joint single-riveted;
   Butt joint double-riveted.

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